Detecting Structural Breaks in Tail Behavior —From the Perspective of Fitting the Generalized Pareto Distribution

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Theme

- Extreme value theory is heavily applied in modeling tail behavior
- Previous literature uses the tail index to test for structural breaks in the tails.
- This study relies on the outperformance of the generalized Pareto distribution (GPD) in modeling tails.
- The transformed GPD is treated as a classical ordinary least square regression and four major tests are applied to detect the existence and the number of structural break(s):
  - supLM test (Andrews and Ploberger 1994)
  - OLS-based CUSUM test (Ploberger and Kramer 1992)
  - Nyblom-Hansen test (1989; 1992)
  - generalized M-fluctuation test Zeileis (2006).
- multiple structural breaks at extremal quantile levels of all the three exchange return series considered (UK pound, Japanese Yen, and New Taiwan Dollar, all vs. US Dollar).
- EVT should be used with caution for risk management purposes.

Power law

- assumption that the tails follow a power-like distribution or a regularly varying distribution
- The rate of decay is thus represented by the tail index, which is the inverse of the shape parameter
- the tail behavior is governed by the tail index.

Power law?

- Candelon and Straetmans (2006) and Galbraith and Zernov (2004), Werner and Upper (2004), and Quintos, Fan, and Phillips (2001) test the multiple regimes in the tails of return series of currency, equity, and futures, respectively.
- However, this way of treatment is suspected of being incomplete and deficient in several aspects.

Why not Power law?

- The curves for the tails are usually far from being smooth as the power-like distributions predict
- assumed that the tail index serves as a summary measure of extreme behavior but the approaches to its estimation exhibit significant deficiencies- the Hill estimator
- three major types of limiting distribution of maximum-domain of attraction－generalized extreme value distribution (GEV): Frecht, Gumbel, and Weibull.
  - If the structural change test is based on the tail index, it implicitly assumes that the tails follows a Frechet distribution or exhibits an exponential decay rate.
  - this assumption gives partial observation and the latter is hardly supported by the actual data

Hill estimator

Consider the order statistics $x_1 \ge x_2 \ge ... \ge x_T$, for a positive integer k. The Hill

estimator of shape parameter is defined as

$$\hat{\xi}(k) = \frac{1}{k}\sum_{j=1}^{k}\left(\log x_j - \log x_k\right)$$

, then the Hill estimator of tail index is

$$\hat{\alpha}(k) = \frac{1}{\hat{\xi}(k)}$$

Hill estimator?

- inadequate if the tail of the underlying density function is not regularly varying
- For a slowly varying function, the actual distribution is usually unknown
- dataset of the tails is usually not sufficiency large and the tail is not heavy enough, i.e. the shape parameter is not big
- very sensitive to choice of smoothing parameter
- tail index estimate is hardly satisfactory irrespective of estimation method
- considerable room for suspicion of the tests on structural change in tail behavior which are based on tail index

Argument

- the traditional way to study structural breaks in the tails is severely bounded by its partial observation and deficient estimation.
- the conclusion is susceptible to question and criticism.

generalized Pareto distribution

- more appropriate for modeling the tails than the three generalized extreme value distributions
- If normalized maxima converge to a GEV distribution, the threshold exceedances can converge to the GPD as the threshold is raised (Pickands-Balkema-de Haan theorem).
- GPD is evaluated as the canonical distribution for modeling threshold exceedances
- more advisable to study structural breaks from the perspective of GPD fitting

GPD—threshold, shape, and scale

- For large threshold, the distribution of the values x over threshold y can be approximated by cumulative distribution function

$$G_{\alpha,\lambda}(x) = \begin{cases} 1-\left(1+\frac{\alpha}{\lambda}x\right)^{-1/\alpha} & if\,\alpha \neq 0 \\ 1-\exp\left(-\frac{x}{\lambda}\right) & if\,\alpha = 0 \end{cases}$$

α and λ denote shape parameter and scale factor, respectively

Tests on Structural Break

- In a classical OLS regression setup
- assumed m breakpoints in the parameter estimates in

$$y_i = x_i' \beta_j + \varepsilon_i \qquad i = i_{j-1}+1,\dots,i_j;\ j = 1,\dots,m+1$$

where i and j denote data and segment index, respectively

the null hypothesis $H_0 : \beta_i = \beta_0$

Tests on Structural Break

- Four types
  - F statistics: Andrew (1993) and Andrews and Ploberger (1994
  - maximum likelihood scores: Nyblom (1989), Hansen (1992), and Hjort and Koning (2002)
  - OLS residuals based fluctuation tests: recursive CUSUM (Brown, Durbin, and Evans 1975) and OLS-based CUSUM (Ploberger and Kramer 1992)
  - generalized M-fluctuation tests: Zeileis (2006)
- following Zeileis (2005) and Zeileis (2006), focuses on four major tests: supLM test, OLS-based CUSUM test, Nyblom-Hansen test, and generalized M-fluctuation test
- The former two tests are built for detecting single structural break (of unknown timing) and the latter two are for multiple structural breaks with dating capacity.

Dataset

- Foreign exchange rate series are noticed for their volatility and structural change.
- Three foreign exchange rate series are selected for empirical examination: the UK pound, Japanese Yen, and New Taiwan Dollar, all vs. US Dollar.
- Coverage: January 5, 1993 and February 9, 2007.

Observations

- Both of the supLM test and the OLS-based CUSUM test confirm the existence of structural breaks for all the lower tails of the three series at 95% and 99% quantile levels with 5% significance level.
- The supLM tests show that the empirical fluctuation processes are significantly over their respective boundaries of red lines (Figures 4 and 5)
- OLS-based CUSUM processes fit the fluctuation process at 95% and 99% quantile levels, along with its boundaries (the two red lines) with 5% significance level (Figures 6 and 7).
- All the processes in the six panels exceed the boundary and multiple crossings indicate at least one structural break in the tail data

Observations

- Nyblom-Hansen tests: significant evidence confirms the existence of multiple structural breaks in the lower tail behavior of the three exchange series (Figures 8 and 9)
- generalized M-fluctuation test is known for its straightforward graphical presentation of test for structural break (Figures 10 and 11)
- these evidences confirm there are multiple structural changes in the lower tails of the three exchange series which are captured by GPD models

Observations

- BIC criteria for deciding the number of structural break
  - there are 3, 4, and 5 structural breaks in GBP, JPY, and NTD series, respectively (Figure 12).
  - Multiple structural breaks also exist at tails at 99% quantile level.
  - 2, 4, and 1 structural break(s) in GBP, JPY, and NTD series, respectively (Figures 13).

Conclusion

- analyses on the GPD modeling of the tails confirm that there are multiple structural breaks in each of the lower tails of the foreign exchange rate series