Event Study: A Change-Point Model Approach

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Introduction

- Event study:
  - A statistical method to assess the impact of events on the value of a firm
- Change-point model approach:
  - Bayesian method to determine latent variables (regime levels) and the times at which they change on the basis of related observations (e.g., asset prices)

Motivating Example: Bankruptcy of Lehman Brothers

Fundamental Definitions

- Direction: expected value of rate of return of equity price over time
- Volatility: standard deviation of rate of return of equity price over time
- Structural break: a significant departure in direction and volatility from the immediate past
- Regime: a distinguishable set consisting of a time interval, a direction value and a volatility value

Fundamental Problems

- What is the market price discovery process in relation to an event?
- If some traders know the information earlier than when it is announced, can we detect it?
- How can we estimate directions and volatilities in the regime?
- How do we detect structural breaks around key developments (events)?
- Are there any properties common to disparate key developments?

Objectives of the Proposed Study

- Propose a change-point model to detect structural breaks in an event window
- Show empirical evidence of structural breaks
- Discuss implications between the breaks and key developments, especially the ex-ante/ex-post contrast

Key Findings

- Price adjustments to key developments may begin before and end after the announcement
- Statistical correlation between successive key developments is not significant

Methodology: Overview

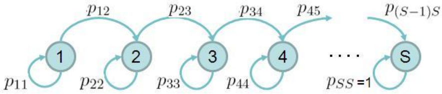
- Parametrization
- Gibbs sampling
- Markov chain expectation-maximization (MCEM)
- Bayes factor

Parametrization: Abnormal Return

- Abnormal return a_t at time t is defined by,

$$a_t := r_t - \left[\hat{\alpha} + \hat{\beta} r_t^m + \hat{s} SMB_t + \hat{h} HML_t\right] \tag{1}$$

- Regime s_t in a regime set $S_T = \{s_t\}, \{t = 1, \ldots, T\}$ follows discrete-state Markov chain where $p_{ij} = P(s_t = j | s_{(t-1)} = i)$ with transition diagram,



- a_t follows IID *Normal* with mean μ_s and variance σ_s^2 given the regime of time t is s,

$$a_t | s_t = s \sim \mathcal{N}(\mu_s, \sigma_s^2), \qquad t = 1, 2, \ldots, T, \qquad s = 1, 2, \ldots, S \tag{2}$$

- Prior densities of parameter $\pi(\mu_s, \sigma_s^2) = \pi(\mu_s | \sigma_s^2) \pi(\sigma_s^2)$ and regime set distribution $\pi(p_{ss})$ follow *Normal*, *Inverse Gamma*, and *Beta* respectively,

$$\sigma_s^2 \sim \mathcal{IG}\left(\frac{\nu_0}{2}, \frac{\sigma_0^2}{2}\right), \qquad \mu_s | \sigma_s^2 \sim \mathcal{N}\left(\mu_0, \frac{\sigma_s^2}{\kappa_0}\right), \qquad p_{ss} \sim \mathcal{B}(a_0, b_0) \tag{3}$$

Gibbs sampling: Estimating Structural Break Points

- Step 1: Generate $P \sim \pi(P|S_T)$

$$p_{ss}|S_T \sim \mathcal{B}(a_0 + \sum_{t=1}^{T} I(s_t = s), b_0 + 1) \qquad s = 1, \ldots, S-1 \tag{4}$$

- Step 2: Generate $\Theta \sim \pi(\Theta|S_T, A_T)$

$$\sigma_s^2|S_T, A_T \sim \mathcal{IG}\left(\frac{\nu_m}{2}, \frac{\sigma_m^2}{2}\right) \quad \mu_s|\sigma_s^2, S_T, A_T \sim \mathcal{N}\left(\mu_m, \frac{\sigma_s^2}{\kappa_m}\right) \quad s = 1, \ldots, S \tag{5}$$

- Step 3: Generate $S_T \sim \pi(S_T|\Theta, A_T, P)$

$$\pi(S_T|A_T, \Theta, P) = \prod_{t=2}^{T-1} \frac{\pi(s_t = k|A_t, \Theta, P)\pi(s_{t+1}|s_t, P)}{\sum_{l=1}^{S-1} \pi(s_t = l|A_t, \Theta, P)\pi(s_{t+1}|s_t = l, P)} \tag{6}$$

Gibbs sampling: Estimating Structural Break Points(Continued)

- After M iterations of Gibbs sampler, we find marginal posterior of regime set,

$$\begin{aligned} \pi(s_t = k|A_T) &:= \int \pi(s_t = k|A_{t-1}, \Theta, P)\pi(\Theta, P|A_T)d(\Theta, P) && (7) \\ &\approx \frac{1}{M}\sum_{m=1}^{M} \pi(s_t^{(m)} = k|A_{t-1}^{(m)}, \Theta^{(m)}, P^{(m)}) && (8) \end{aligned}$$

MCEM: Estimating Directions and Volatilities of Regimes

- Step 1: Expectation of log-scale complete data likelihood

$$E_{\Theta_{(i)}}(\ln f(A_T, S_T|\Theta)) \approx \frac{1}{M}\sum_{m=1}^{M}\left\{\ln f(A_T|S_T^{(m)}, \Theta) + \ln f(S_T^{(m)}|P)\right\} \quad (9)$$

- Step 2: maximization of Θ^*

$$\mu_s^* = \frac{1}{M}\sum_{m=1}^{M}\bar{a}_s^{(m)} = \frac{1}{M}\sum_{m=1}^{M}\left[\frac{1}{m_{ss}^{(m)}}\sum_{t=1}^{T}a_t I(s_t^{(m)} = s)\right] \quad (10)$$

$$\sigma_s^{2*} = \frac{1}{M}\sum_{m=1}^{M}\left[\frac{1}{m_{ss}^{(m)}}\sum_{t=1}^{T}I(s_t^{(m)} = s)(a_t - \bar{a}_s^{(m)})^2\right] \quad (11)$$

Bayes factor: Choosing a model with the Best Change Points

- Bayes factor comparing $\mathcal{M}_r$ and $\mathcal{M}_s$ is,

$$B_{rs} := \frac{m(A_T|\mathcal{M}_r)}{m(A_T|\mathcal{M}_s)} \tag{12}$$

- Where $m(A_T|\mathcal{M}_.)$ is the marginal likelihood,

$$m(A_T|\mathcal{M}_.) = \frac{f(A_T|\mathcal{M}_., \Theta^*, P^*)\pi(\Theta^*, P^*|\mathcal{M}_.)}{\pi(\Theta^*, P^*|A_T, \mathcal{M}_.)} \tag{13}$$

- Decomposing log-scale marginal likelihood as,

$$\ln m(A_T) = \ln f(A_T|\Theta^*) + \ln \pi(\Theta^*) + \ln \pi(P^*) - \ln \pi(\Theta^*|A_T) - \ln \pi(P^*|A_T, \Theta^*) \tag{14}$$

- Where we can find each component as,

$$\ln f(A_T|\Theta^*) = \sum_{t=1}^{T} \ln \left(\sum_{s=1}^{S} f(a_t|\theta^*, s_t = s)\pi(s_t = s|A_{t-1}, \theta^*) \right)$$

$$\pi(\Theta^*|A_T) \approx \frac{1}{G}\sum_{g=1}^{G} \pi(\Theta^*|A_T, S_T^{(g)}), \qquad \pi(P^*|A_T, \Theta^*) \approx \frac{1}{G}\sum_{g=1}^{G} \pi(P^*|S_T^{(g)})$$

Partial R code

```
#Gibbs sampling: marginal posterior of
      Sn
k <- 1 #repetition index
repeat{
MCMCpart1(1,k) #Generating P
MCMCpart2(1,k) #Generating Theta(muK or
      sigmaKsq)
MCMCpart3(1,k) #Generating Sn, Nii
if(k>=nsimMCMC+transient){ #terminating
      condition
    termMCMC <- 0
    break}
reportRecord(if(reportFlag[2]==TRUE){2}
      else{0})
k <- k+1
if(k> transient){#Storing Sn and Nii to
      be used in calculating marginal
      likelihood
    SnM[k-transient,] <- Sn
    NiiM[k-transient,] <- Nii}
}
MCMCTrials <- k
#marginal posterior of states Sn
marPostSnProb <- sumPsYtm1TP/nsimMCMC
reportExport(if(reportFlag[2]==TRUE){2}
      else{0})
```

```
#MCEM: Marginal likelihood
repeat{
MCMCpart1(2,k) #Generating MLE of P
MCMCpart2(2,k) #Generating MLE of Theta(
      muK, sigmaKsq)
if(k<=nsimMCEM/3){N <- Ncand[1] #
      reinitialize N, increasing as k
      increases
}else if(k<=2*nsimMCEM/3){N <- Ncand[2]
}else{N <- Ncand[3]}
Sn <- matrix(NA, nrow = N, ncol=length(
      Yn))
Nii <- matrix(NA, nrow = N, ncol=M[j])
MCMCpart3(2,k) #Part 3: Generating Sn,
      Nii
if(k>=nsimMCEM){ #terminating condition
    termMCEM <- 1
    break}
reportRecord(if(reportFlag[3]==TRUE){3}
      else{0})
k <- k+1
}
MCEMTrials <- k
MLEsigmaKsq <- sigmaKsq; MLEmuK <- muK;
      MLETrnPrb <- TrnPrb #MLEs
MLESn <- Sn; MLENii <- Nii; MLEN <- N
```

Partial R code(Continued)

```
#Marginal likelihood MLEPsYtm1TP <- MCMCpart3(3,k)
ytLikeAll <- matrix(NA, nrow = length(Yn), ncol = M[j])
sumPiTYS <- 0; sumPiPYT <- 0
sigmaKsqLikePr=muKLikePr<-rep(NA, times=M[j]) niiLikePr <-
rep(NA, times=M[j]-1) for(l in 1:M[j]){
  ytLikeAll[,l] <- dnorm(Yn,MLEmuK[l],sqrt(MLEsigmaKsq[l]))}
for(m in 1:nsimMCMC){
  Sn <- SnM[m,]; Nii <<- NiiM[m,] sumPiTYS <- sumPiTYS + MCMCpart2(3,k)}
if(M[j]==1){
  sumPiPYT <- nsimMCMC
}else{for(m in 1:MLEN){
  Nii <<- MLENii[m,]
  for(l in 1:(M[j]-1)){
    niiLikePr[l] <- dbeta(MLETrnPrb[l], a0+Nii[l], b0+1)}
    sumPiPYT <- sumPiPYT + prod(niiLikePr)}}
for(l in 1:M[j]){
  sigmaKsqLikePr[l] <- dinvgamma(MLEsigmaKsq[l], nu0/2,
  sigma0sq/2) #posterior MLE sigmaKsq likelihood on prior
  muKLikePr[l] <- dnorm(MLEmuK[l], mu0,
  sqrt(MLEsigmaKsq[l]/kappa0))} #posterior MLE mu likelihood on prior
margLike <- sum(log(rowSums(ytLikeAll*MLEPsYtm1TP))) +
log(prod(sigmaKsqLikePr)) + log(prod(muKLikePr)) -
log(sumPiTYS/nsimMCMC) - log(sumPiPYT/nsimMCMC)
```

Empirical Analysis: Data Specification

- Firm selection: Russell 3000, COMPUSTAT
- Key Development: Reuters Knowledge
- Abnormal Return: EVENTUS@WRDS
- Software: R and MySQL

Partial Summary of Abnormal Returns: by Industry

	Firms	Events	Market Cap. ($Bil.)	BTM Ratio	Event Window	Abnormal Return Mean (%)	Abnormal Return Std.Dev. (%)	$\hat{\beta}$ Estimate / t value / R^2	$\hat{\beta}$ Std. Err. / p value
Consumer Discretionary	26	17.7	7.08	0.42	$(-30, -2)$	0.025	1.711	-0.0190	0.0060
					$(-1, 0)$	0.219	3.345	-3.1538	0.0016
					$(1, 30)$	0.021	1.806	0.0004	
Health Care	25	21.2	18.33	0.43	$(-30, -2)$	-0.007	2.283	-0.0172	0.0056
					$(-1, 0)$	-0.165	4.195	-3.0689	0.0022
					$(1, 30)$	-0.02	2.446	0.0003	
Financials	26	8.5	4.95	0.41	$(-30, -2)$	-0.013	1.507	0.0277	0.0087
					$(-1, 0)$	0.171	2.313	3.1993	0.0014
					$(1, 30)$	0.003	1.653	0.0008	
Telecommunication Services	20	18.7	52.97	0.49	$(-30, -2)$	0.011	1.872	0.0341	0.0067
					$(-1, 0)$	0.008	2.532	5.1237	0.0000
					$(1, 30)$	0.031	1.929	0.0012	

- To check serial correlation, we perform AR(1) test,

$$a_t = \alpha + \beta a_{(t-1)} + \epsilon_t. \tag{15}$$

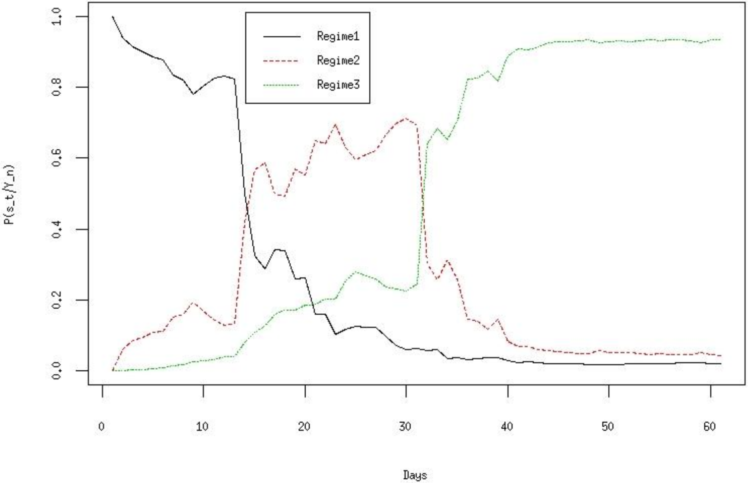
Summary of Abnormal Returns: by Market Cap. and BTM

	Firms	Events	Market Cap. ($Bil.)	BTM Ratio	Event Window	Abnormal Return Mean (%)	Abnormal Return Std.Dev. (%)	$\hat{\beta}$ Estimate / t value / R^2	$\hat{\beta}$ Std. Err. / p value
Big	136	19.4	23.03	0.44	$(-30, -2)$	0.004	1.784	-0.0134	0.0025
					$(-1, 0)$	0.016	3.059	-5.3353	0.0000
					$(1, 30)$	0.002	1.884	0.0002	
Small	101	9.8	1.08	0.45	$(-30, -2)$	0.003	2.23	-0.0184	0.0041
					$(-1, 0)$	0.161	3.889	-4.4889	0.0000
					$(1, 30)$	0.02	2.334	0.0003	
Value	82	14.9	7.94	0.71	$(-30, -2)$	0.006	1.8	-0.0322	0.0037
					$(-1, 0)$	-0.042	3.304	-8.7514	0.0000
					$(1, 30)$	-0.012	1.929	0.0010	
Mid	83	16.4	26.71	0.4	$(-30, -2)$	0.001	1.736	-0.0249	0.0035
					$(-1, 0)$	0.13	3.025	-7.1085	0.0000
					$(1, 30)$	0.026	1.782	0.0006	
Growth	72	14.6	15.17	0.19	$(-30, -2)$	0.005	2.24	0.0054	0.0040
					$(-1, 0)$	0.072	3.637	1.3707	0.1705
					$(1, 30)$	0.003	2.371	0.0000	

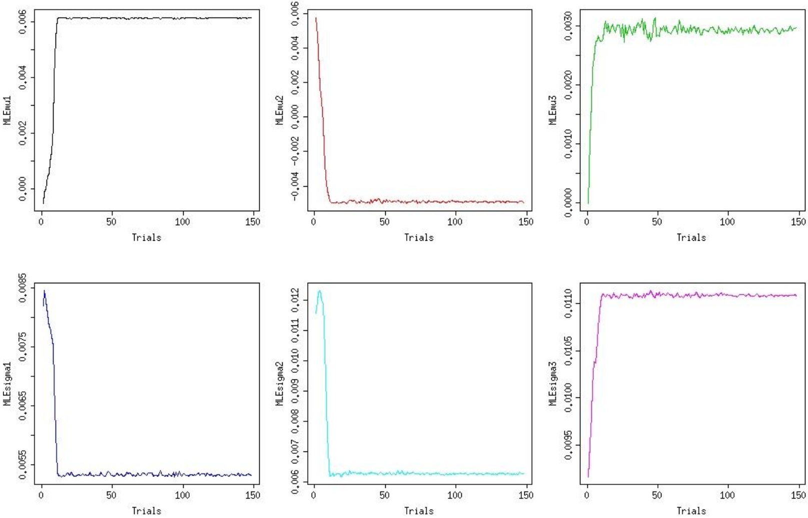
Announcement of Key Developments: Characteristics

	Key Development Arrivals	$\hat{\beta}$ Estimate / p value	Std. Err. / R^2	t value	KPSS: KPSS level / p value	KS-$\mathcal{E}$: D stat. / p value	KS-$\mathcal{N}$: D stat. / p value
Big	2713	0.25 0	0.02 0.07	14.38	0.17 0.1	0.05 0	0.17 0.00
Small	981	0.17 0	0.03 0.03	5.86	0.12 0.1	0.09 0	0.18 0.00
Value	1290	0.26 0	0.03 0.08	10.49	0.82 0.01	0.05 0	0.18 0.00
Mid	1347	0.23 0	0.03 0.05	8.67	0.78 0.01	0.06 0	0.19 0.00
Growth	1057	0.24 0	0.03 0.07	9.03	0.52 0.04	0.06 0	0.17 0.00

Estimating Break Points: Gibbs Sampling Example



Estimating Direction and Volatility: MCEM Example



Selecting the Best Change-Point Model: Bayes Factor Example

		Compared Model				Upper Confidence Limit				p value			
Ticker	Model	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$
AA	1	0.00	-0.28	16.78	-3.94	NA	2.15	58.06	1.92	0.00	0.06	0.73	0.05
	2	0.28	0.00	17.06	-3.66	2.71	NA	58.32	2.16	0.12	0.00	0.73	0.05
	3	-16.78	-17.06	0.00	-20.72	24.50	24.19	NA	19.54	0.22	0.22	0.00	0.17
	4	3.94	3.66	20.72	0.00	9.81	9.48	60.98	NA	0.71	0.68	0.78	0.00
JCP	1	0.00	30.63	28.71	12.44	NA	67.59	57.22	30.92	0.00	0.90	0.94	0.83
	2	-30.63	0.00	-1.92	-18.19	6.32	NA	33.23	23.93	0.07	0.00	0.43	0.21
	3	-28.71	1.92	0.00	-16.27	-0.21	37.07	NA	11.13	0.04	0.50	0.00	0.13
	4	-12.44	18.19	16.27	0.00	6.03	60.31	43.68	NA	0.10	0.74	0.81	0.00
All Firms	1	0.0000	12.7231	13.5696	29.5629	NA	17.5675	18.8539	36.8193	0.0000	0.9999	0.9998	1.0000
	2	-12.7231	0.0000	0.8466	16.8398	-7.8786	NA	6.4587	25.0420	0.0000	0.0000	0.3676	0.9985
	3	-13.5696	-0.8466	0.0000	15.9933	-8.2853	4.7655	NA	24.3442	0.0000	0.2019	0.0000	0.9970
	4	-29.5629	-16.8398	-15.9933	0.0000	-22.3065	-8.6377	-7.6423	NA	0.0000	0.0001	0.0002	0.0000

Selecting the Best Change-Point Model(Continued)

Ticker	$\mathcal{M}_r$	Ticker	$\mathcal{M}_r$	Ticker	$\mathcal{M}_r$	Ticker	$\mathcal{M}_r$
AA	4	FDX	3	PDX	1	TMO	4
AEP	1	FL	1	PG	1	TSN	1
BIIB	3	GCO	1	PLCE	1	UNH	2
BSX	3	JCP	1	PTRY	4	VZ	3
CAT	3	KFT	2	QCOM	1	WYE	2
CELG	4	MGM	1	S	1		
EGN	1	MON	1	T	1		

Estimating the Lengths of Regimes

Ticker	$\mathcal{M}_r$	$\hat{m}_{11}$	$\hat{m}_{22}$	$\hat{m}_{33}$	$\hat{m}_{44}$
AA	4	13.39	11.7	10.47	25.44
		(11.38, 15.4)	(10.35, 13.05)	(9.07, 11.87)	(22.75, 28.12)
CAT	3	19.76	11.55	29.7	
		(17, 22.52)	(9.09, 14)	(26.68, 32.71)	
FDX	3	17.61	11.39	32	
		(14.72, 20.5)	(9.71, 13.08)	(28.92, 35.08)	
KFT	2	20.62	40.38		
		(16.75, 24.48)	(36.52, 44.25)		
TMO	4	15.07	11.89	10.05	24
		(12.76, 17.38)	(10.6, 13.17)	(8.94, 11.15)	(21.79, 26.2)
VZ	3	17.53	12.61	30.85	
		(14.42, 20.65)	(11.68, 13.55)	(27.92, 33.79)	
WYE	2	22.51	38.49		
		(18.88, 26.14)	(34.86, 42.12)		

Direction and Volatility Differences in Consecutive Regimes

						Welch Two Sample t test(Confidence Limit)(p value)		
Ticker	$\mathcal{M}_r$	$\overline{\hat{\mu}}_1$	$\overline{\hat{\mu}}_2$	$\overline{\hat{\mu}}_3$	$\overline{\hat{\mu}}_4$	$\overline{\hat{\mu}}_1 - \overline{\hat{\mu}}_2$	$\overline{\hat{\mu}}_2 - \overline{\hat{\mu}}_3$	$\overline{\hat{\mu}}_3 - \overline{\hat{\mu}}_4$
AA	4	0.0017	-0.002	-0.0031	-0.0002	(-0.0037, 0.011) (0.3205)	(-0.0093, 0.0116) (0.8257)	(-0.0115, 0.0056) (0.4839)
CAT	3	-0.0016	-0.0053	0.0008		(-0.0049, 0.0123) (0.393)	(-0.0143, 0.0021) (0.1411)	
FDX	3	-0.0012	0.0011	-0.0009		(-0.006, 0.0014) (0.2173)	(-0.0015, 0.0055) (0.265)	
KFT	2	0.0018	0.0003			(-0.0033, 0.0063) (0.5324)		
TMO	4	0.001	-0.0072	0.0049	0.0007	(-0.0021, 0.0185) (0.1153)	(-0.0231, -0.001) (0.0328)	(-0.0011, 0.0094) (0.1191)
VZ	3	-0.0012	0.0027	0.0006		(-0.0068, -0.0009) (0.0115)	(-0.001, 0.005) (0.1827)	
WYE	2	0.0003	-0.0003			(-0.0014, 0.0025) (0.5856)		
		$\overline{\hat{\sigma}}_1$	$\overline{\hat{\sigma}}_2$	$\overline{\hat{\sigma}}_3$	$\overline{\hat{\sigma}}_4$	$\overline{\hat{\sigma}}_1 - \overline{\hat{\sigma}}_2$	$\overline{\hat{\sigma}}_2 - \overline{\hat{\sigma}}_3$	$\overline{\hat{\sigma}}_3 - \overline{\hat{\sigma}}_4$
AA	4	0.0079	0.0085	0.0088	0.0103	(-0.0042, 0.0031) (0.7646)	(-0.0039, 0.0033) (0.8652)	(-0.004, 0.0009) (0.2038)
CAT	3	0.0098	0.0081	0.0097		(-0.002, 0.0053) (0.3621)	(-0.0044, 0.0012) (0.26)	
FDX	3	0.0085	0.0083	0.0093		(-0.0029, 0.0033) (0.919)	(-0.004, 0.002) (0.5155)	
KFT	2	0.006	0.009			(-0.0049, -0.0011) (0.0022)		
TMO	4	0.0062	0.0081	0.0073	0.0082	(-0.0049, 0.0011) (0.2179)	(-0.0023, 0.0039) (0.6138)	(-0.0034, 0.0016) (0.47)
VZ	3	0.0059	0.006	0.0065		(-0.0021, 0.0018) (0.8924)	(-0.0023, 0.0014) (0.6252)	
WYE	2	0.0074	0.011			(-0.0059, -0.0013) (0.0023)		

Correlation and Serial Correlation of Directions and Volatilities

				$\hat{\mu}_j = \hat{\beta}_0 + \hat{\beta}_1\hat{\mu}_i + \hat{\beta}_2\hat{\sigma}_i + \hat{\beta}_3\hat{\sigma}_j$ $\hat{\sigma}_j = \hat{\beta}_0 + \hat{\beta}_1\hat{\mu}_i + \hat{\beta}_2\hat{\sigma}_i + \hat{\beta}_3\hat{\mu}_j$				p values				
Ticker	$\mathcal{M}_r$	(i,j)	$\hat{\mu}/\hat{\sigma}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	R^2
AA	4	(1, 2)	$\hat{\mu}$	0	-0.2445	-0.3351	0.1231	0.9975	0.5407	0.6096	0.7836	0.0227
			$\hat{\sigma}$	0.0054	-0.0311	0.3938	0.024	0.0549	0.8607	0.1672	0.7836	0.0799
		(2, 3)	$\hat{\mu}$	-0.0169	-0.0626	0.1476	1.4073	0.0613	0.7873	0.7733	0.0802	0.1237
			$\hat{\sigma}$	0.0086	-0.0073	0.0505	0.0804	0	0.8949	0.6801	0.0802	0.1249
		(3, 4)	$\hat{\mu}$	-0.0002	-0.0139	0.0395	-0.0306	0.9455	0.8044	0.864	0.9155	0.0028
			$\hat{\sigma}$	0.0097	-0.0376	0.0629	-0.0144	0	0.3239	0.6907	0.9155	0.0377
KFT	2	(1, 2)	$\hat{\mu}$	-0.0045	0.0195	0.014	0.5252	0.0001	0.3716	0.8333	0	0.5771
			$\hat{\sigma}$	0.0091	-0.0364	-0.061	1.0931	0	0.2451	0.5223	0	0.5903
VZ	3	(1, 2)	$\hat{\mu}$	0.0069	-0.0979	-0.2916	-0.4444	0.0313	0.7648	0.4326	0.196	0.1065
			$\hat{\sigma}$	0.0057	0.0377	0.1338	-0.1427	0.001	0.839	0.526	0.196	0.0964
		(2, 3)	$\hat{\mu}$	-0.0008	-0.0344	0.0681	0.1675	0.7743	0.7901	0.7721	0.5482	0.0171
			$\hat{\sigma}$	0.0076	0.0611	-0.2121	0.0838	0	0.5026	0.195	0.5482	0.1109
WYE	2	(1, 2)	$\hat{\mu}$	-0.0041	-0.0468	0.0712	0.3016	0.0143	0.6724	0.5335	0.0114	0.2363
			$\hat{\sigma}$	0.0117	-0.0248	-0.0662	0.7357	0	0.8861	0.7114	0.0114	0.2273

- Linear autoregressive panel data models are used,

$$\hat{\mu}_j = \beta_0^1 + \beta_1^1\hat{\mu}_i + \beta_2^1\hat{\sigma}_i + \beta_3^1\hat{\sigma}_j + \epsilon_j^1, \quad (16)$$
$$\hat{\sigma}_j = \beta_0^2 + \beta_1^2\hat{\mu}_i + \beta_2^2\hat{\sigma}_i + \beta_3^2\hat{\mu}_j + \epsilon_j^2, \quad (17)$$

- where $i = 1, \ldots, S-1$, $j = 2, \ldots, S$

Summary

- Estimated variable lengths of multiple regimes using Gibbs sampler
- Estimated directions and volatilities of abnormal returns by MCEM
- Evaluated models with various number of regimes
- Best model defined as having highest Bayes factor
- Not all key developments produce structural breaks
- Breakpoints may not necessarily coincide with announcement date
- Serial correlation between successive directions/volatilities are not statistically significant

Further Study

- Extend the analysis to liquidity changes in intraday data by accounting for
  - adverse selection component in bid-ask spreads as in [Itzhak and Lee, 1996]
  - regime changes in realized volatility as in [Liu and Maheu, 2008]

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