



**UW**

# **Using R for Hedge Fund of Funds Risk Management**

**R/Finance 2009: Applied Finance with R  
University of Illinois Chicago, April 25, 2009**

**Eric Zivot**

**Professor and Gary Waterman Distinguished Scholar,  
Department of Economics**

**Adjunct Professor, Departments of Finance and Statistics  
University of Washington**

# Outline

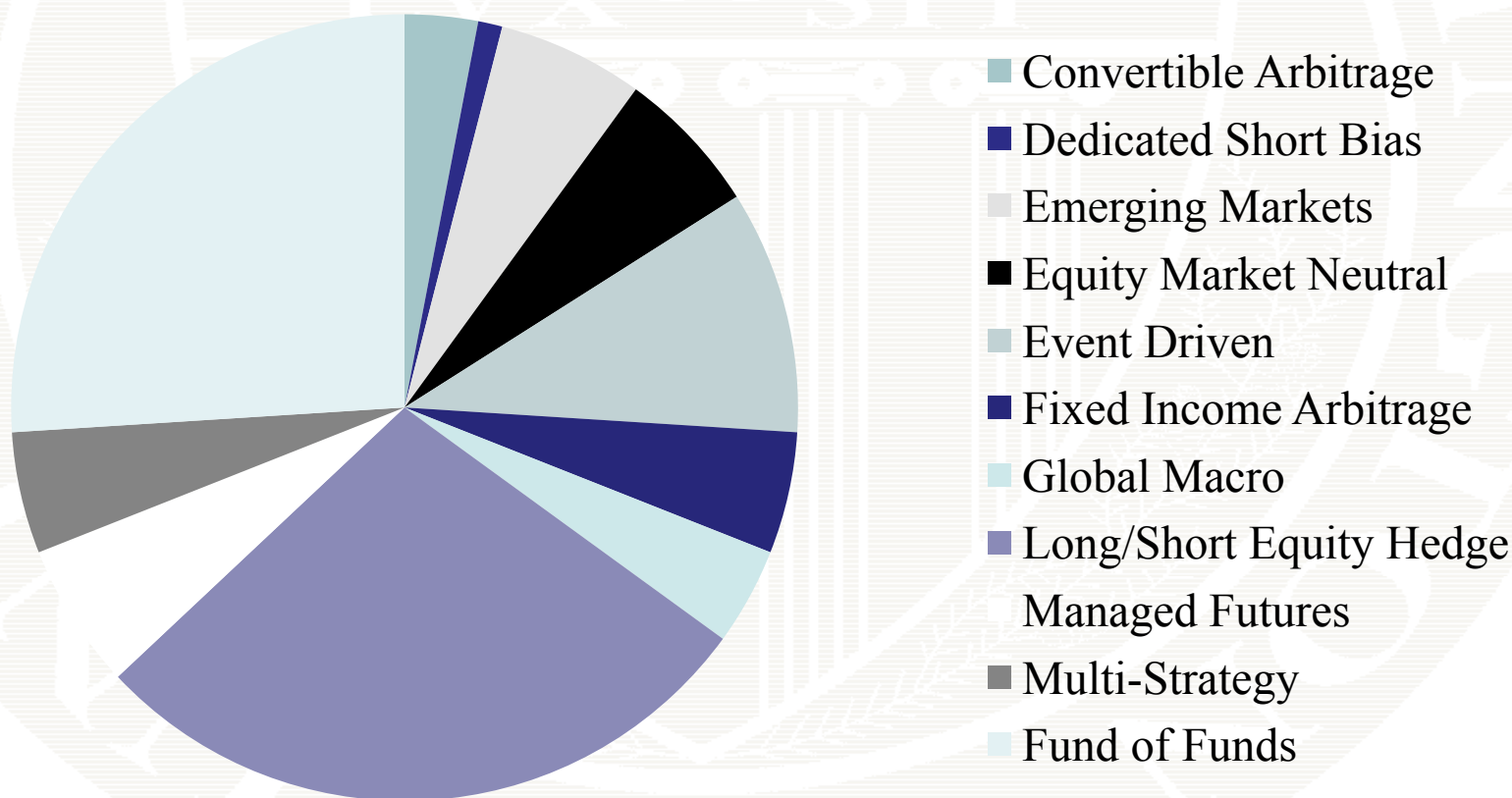
- Hedge fund of funds environment
- Factor model risk measurement
- R implementation in corporate environment
- Dealing with unequal data histories
- Some thoughts on S-PLUS and S+FinMetrics vs. R in Finance

# Hedge Fund of Funds Environment

- HFoFs are hedge funds that invest in other hedge funds
  - 20 to 30 portfolios of hedge funds
  - Typical portfolio size is 30 funds
- Hedge fund universe is large: 5000 live funds
  - Segmented into 10-15 distinct strategy types
- Hedge funds voluntarily report monthly performance to commercial databases
  - Altvest, CISDM, HedgeFund.net, Lipper TASS, CS/Tremont, HFR
- HFoFs often have partial position level data on invested funds

# Hedge Fund Universe

Live funds



# Characteristics of Monthly Returns

- Reporting biases
  - Survivorship, backfill
- Non-normal behavior
  - Asymmetry (skewness) and fat tails (excess kurtosis)
- Serial correlation
  - Performance smoothing, illiquid positions
- Unequal histories

# Characteristics of Hedge Fund Data

	fund1	fund2	fund3	fund4	fund5
Observations	122.0000	107.0000	135.0000	135.0000	135.0000
<b>NAs</b>	<b>13.0000</b>	<b>28.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Minimum	-0.0842	-0.3649	-0.0519	-0.1556	-0.2900
Quartile 1	-0.0016	-0.0051	0.0020	-0.0017	-0.0021
Median	0.0058	0.0046	0.0060	0.0073	0.0049
Arithmetic Mean	0.0038	-0.0017	0.0063	0.0059	0.0021
Geometric Mean	0.0037	-0.0029	0.0062	0.0055	0.0014
Quartile 3	0.0158	0.0129	0.0127	0.0157	0.0127
Maximum	0.0311	0.0861	0.0502	0.0762	0.0877
Variance	0.0003	0.0020	0.0002	0.0008	0.0013
Stdev	0.0176	0.0443	0.0152	0.0275	0.0357
<b>Skewness</b>	<b>-1.7753</b>	<b>-5.6202</b>	<b>-0.8810</b>	<b>-2.4839</b>	<b>-4.9948</b>
<b>Kurtosis</b>	<b>5.2887</b>	<b>40.9681</b>	<b>3.7960</b>	<b>13.8201</b>	<b>35.8623</b>
<b>Rho1</b>	<b>0.6060</b>	<b>0.3820</b>	<b>0.3590</b>	<b>0.4400</b>	<b>0.383</b>

Sample: January 1998 – March 2009

# Factor Model Risk Measurement in HFoFs Portfolio

- Quantify factor risk exposures
  - Equity, rates, credit, volatility, currency, commodity, etc.
- Quantify tail risk
  - VaR, ETL
- Risk budgeting
  - Component, incremental, marginal
- Stress testing and scenario analysis



# Commercial Products

www.riskdata.com

www.finanalytica.com

kdta — Financial Risk Management...

Portfolio Construction and Ri...

Index	Fat-Tailed VaR at 99%
Dow Jones Ind.	-4.1%
S&P 500	-5.0%
Russell 2000	-6.2%
NASDAQ Comp.	-4.6%
MSCI Germany	-4.9%
MSCI France	-4.3%
MSCI UK	-4.3%
MSCI Hong Kong	-4.2%
MSCI India	-5.3%
MSCI Japan	-3.7%
MSCI Russia	-9.5%
MSCI China	-5.3%
MSCI Energy	-5.4%

Very expensive! R is not!



# Factor Model: Methodology

$$\begin{aligned}R_{it} &= \alpha_i + \beta_{i1}F_{1t} + \cdots + \beta_{ik}F_{kt} + \varepsilon_{it}, \\ &= \alpha_i + \boldsymbol{\beta}'_i \mathbf{F}_t + \varepsilon_{it}\end{aligned}$$

$$i = 1, \dots, n; \quad t = t_i, \dots, T$$

$$\mathbf{F}_t \sim (\boldsymbol{\mu}_F, \boldsymbol{\Sigma}_F)$$

$$\varepsilon_{it} \sim (0, \sigma_{\varepsilon,i}^2)$$

$$\text{cov}(f_{jt}, \varepsilon_{it}) = 0 \text{ for all } j, i \text{ and } t$$

$$\text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0 \text{ for } i \neq j$$

# Practical Considerations

- Many potential risk factors ( $> 50$ )
- High collinearity among some factors
- Risk factors vary across discipline/strategy
- Nonlinear effects
- Dynamic effects
- Time varying coefficients
- Common histories for factors; unequal histories for fund performance

# Expected Return Decomposition

$$E[R_{it}] = \alpha_i + \beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to “beta” exposure

$$\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to manager specific “alpha”

$$\alpha_i = E[R_{it}] - (\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}])$$

# Variance Decomposition

$$\text{var}(R_{it}) = \underbrace{\beta_i' \text{var}(\mathbf{F}_t) \beta_i}_{\text{systematic}} + \underbrace{\text{var}(\varepsilon_{it})}_{\text{specific}} = \beta_i' \Sigma_{\mathbf{F}} \beta_i + \sigma_{\varepsilon,i}^2$$

Variance contribution due to factor exposures

$$\beta_1^2 \text{var}(F_{1t}) + \beta_2^2 \text{var}(F_{2t}) + \dots + \beta_k^2 \text{var}(F_{kt})$$

Variance contribution due to covariances between factors

$$2\beta_1\beta_2 \text{cov}(F_{1t}, F_{2t}) + \dots + 2\beta_{k-1}\beta_k \text{cov}(F_{k-1t}, F_{kt})$$

# Covariance

$$\mathbf{R}_t = \boldsymbol{\alpha} + \mathbf{B} \mathbf{F}_t + \boldsymbol{\varepsilon}_t$$

$n \times 1$        $n \times 1$        $n \times k$        $k \times 1$        $n \times 1$

$$\text{var}(\mathbf{R}_t) = \Sigma_{FM} = \mathbf{B} \Sigma_{\mathbf{F}} \mathbf{B}' + \mathbf{D}_{\varepsilon}$$

$$\mathbf{D}_{\varepsilon} = \text{diag}(\sigma_{\varepsilon,1}^2, \dots, \sigma_{\varepsilon,n}^2)$$

Note:  $\text{cov}(R_{it}, R_{jt}) = \boldsymbol{\beta}'_i \text{var}(\mathbf{F}_t) \boldsymbol{\beta}_j = \boldsymbol{\beta}'_i \Sigma_{\mathbf{F}} \boldsymbol{\beta}_j$

# Portfolio Analysis

$\mathbf{w} = (w_1, \dots, w_n)'$  = portfolio weights

$$\begin{aligned} R_{pt} &= \mathbf{w}'\mathbf{R}_t = \mathbf{w}'\boldsymbol{\alpha} + \mathbf{w}'\mathbf{B}\mathbf{F}_t + \mathbf{w}'\boldsymbol{\varepsilon}_t \\ &= \sum_{i=1}^n w_i R_{it} = \sum_{i=1}^n w_i \alpha_i + \sum_{i=1}^n w_i \boldsymbol{\beta}'_i \mathbf{F}_t + \sum_{i=1}^n w_i \varepsilon_{it} \\ &= \alpha_p + \boldsymbol{\beta}'_p \mathbf{F}_t + \varepsilon_{pt} \end{aligned}$$



# Portfolio Variance Decomposition

$$\sigma_p^2 = \text{var}(R_{pt}) = \mathbf{w}' \text{var}(\mathbf{R}_t) \mathbf{w} = \mathbf{w}' \mathbf{B} \Sigma_F \mathbf{B}' \mathbf{w} + \mathbf{w}' \mathbf{D} \mathbf{w}$$

$$\sigma_{p,\text{systematic}}^2 = \mathbf{w}' \mathbf{B} \Sigma_F \mathbf{B}' \mathbf{w}$$

$$\sigma_{p,\text{specific}}^2 = \mathbf{w}' \mathbf{D} \mathbf{w} = \sum_{i=1}^n w_i \sigma_{\varepsilon,i}^2 \quad R_p^2 = \frac{\sigma_{p,\text{systematic}}^2}{\sigma_p^2}$$

$$\sigma_{p,\text{systematic}}^2 = \boldsymbol{\beta}_p' \Sigma_F \boldsymbol{\beta}_p = \sum_{j=1}^k \beta_{p,j}^2 \sigma_{jj}^2 + \text{covariance terms}$$

$$\text{covariance terms} = \sigma_{p,\text{systematic}}^2 - \sum_{j=1}^k \beta_{p,j}^2 \sigma_{jj}^2$$

# Risk Budgeting: Volatility

$$\mathbf{MCR} = \frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\mathbf{B}\Sigma_F\mathbf{B}'\mathbf{w} + \mathbf{D}\mathbf{w}}{\sigma_p} \quad \text{Marginal contributions to risk}$$

$$\mathbf{MCR}_{\text{systematic}} = \frac{\mathbf{B}\Sigma_F\mathbf{B}'\mathbf{w}}{\sigma_p}$$

$$\mathbf{MCR}_{\text{specific}} = \frac{\mathbf{D}\mathbf{w}}{\sigma_p}$$

$$\mathbf{CR} = \mathbf{w} \odot \frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\mathbf{w} \odot (\mathbf{B}\Sigma_F\mathbf{B}'\mathbf{w} + \mathbf{D}\mathbf{w})}{\sigma_p} \quad \text{Components to risk}$$

$$\mathbf{1}'\mathbf{CR} = \sum_{i=1}^n CR_i = \sigma_p$$

# Tail Risk Measures

Value-at-Risk (VaR)

$$VaR_{\alpha} = -q_{\alpha} = -F^{-1}(\alpha)$$

$F$  = CDF of returns  $R$

Expected Shortfall (ES)

$$ES_{\alpha} = -E[R \mid R \leq VaR_{\alpha}]$$

## Tail Risk Measures: Normal Distribution

$$R_p \sim N(\mu_p, \sigma_p^2), \quad \sigma_p^2 = w' \Sigma_{FM} w$$

$$VaR_\alpha^N = -\mu_p - \sigma_p \times z_\alpha, \quad z_\alpha = \Phi^{-1}(\alpha)$$

$$ES_\alpha^N = \mu_p - \sigma_p \frac{1}{\alpha} \phi(z_\alpha)$$

See functions in [PerformanceAnalytics](#)

# Tail Risk Measures: Non-Normal Distributions

Use Cornish-Fisher expansion to account for asymmetry and fat tails

$$VaR_{\alpha}^{CF} = -\mu_i - \sigma_i \times z_{\alpha} + \sigma_i \left[ -\frac{1}{6} (z_{\alpha}^2 - 1) skew_i - \frac{1}{24} (z_{\alpha}^3 - 3z_{\alpha}) ekurt_i + \frac{1}{36} (2z_{\alpha}^3 - 5z_{\alpha}) skew_i^2 \right]$$

$ES_{\alpha}^{CF}$  : Formula given in Boudt, Peterson and Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk* and implementation in [PerformanceAnalytics](#)

# Risk Budgeting: Tail Risk

Value-at-Risk (VaR)

$$VaR_\alpha = \sum_{i=1}^n w_i \frac{\partial VaR_\alpha}{\partial w_i} = \sum_{i=1}^n \overbrace{w_i \times m VaR_{\alpha,i}}^{c VaR_{\alpha,i}},$$

$$m VaR_{\alpha,i} = \frac{\partial VaR_\alpha}{\partial w_i} = -E[R_i | R_p = VaR_\alpha]$$

Expected Shortfall (ES)

$$ES_\alpha = \sum_{i=1}^n w_i \frac{\partial ES_\alpha}{\partial w_i} = \sum_{i=1}^n \overbrace{w_i \times m ES_{\alpha,i}}^{c ES_{\alpha,i}},$$

$$m ES_{\alpha,i} = \frac{\partial ES_\alpha}{\partial w_i} = -E[R_i | R_p \leq VaR_\alpha]$$



# Risk Budgeting: Explicit Formulas

- Normal distribution
  - See Jorian (2007) or Dowd (2002)
- Non-normal distribution using Cornish-Fisher expansion
  - See Boudt, Peterson and Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk* and implementation in [PerformanceAnalytics](#)

# Risk Budgeting: Simulation

$\{R_{it}\}_{t=1}^M = M$  simulated returns

Method 1: Brute Force

$$mVaR_{\alpha,i} \approx \frac{\Delta VaR_{\alpha}}{\Delta w_i}, \quad mES_{\alpha,i} = \frac{\Delta ES_{\alpha}}{\Delta w_i}$$

Method 2: Average  $R_{it}$  around values for which  $R_{pt} = VaR_{\alpha}$

$$mVaR_{\alpha,i} \approx - \sum_{t: R_{pt} = VaR_{\alpha} \pm \varepsilon} R_{it}, \quad mES_{\alpha,i} \approx - \sum_{t: R_{pt} \leq VaR_{\alpha}} R_{it}$$

# R Functions for Factor Model Risk Analysis

Function	Function
factorModelCovariance	normalES
factorModelRiskDecomposition	normalPortfolioES
normalVaR	normalMarginalES
normalPortfolioVaR	normalComponentES
normalMarginalVaR	modifiedES
normalComponentVaR	modifiedPortfolioES
normalVaRreport	modifiedESreport
modifiedVaR	simulatedMarginalVaR
modifiedPortfolioVaR	simulatedComponentVaR
modifiedMarginalVaR	simulatedMarginalES
modifiedComponentVaR	simulatedComponentES

# Unequal Histories

Risk factors

$$\begin{array}{c} F_{1,T}, \dots, F_{k,T} \\ \vdots \\ F_{1,T-T_i}, \dots, F_{k,T-T_i} \\ \vdots \\ F_{1,1}, \dots, F_{k,1} \end{array}$$

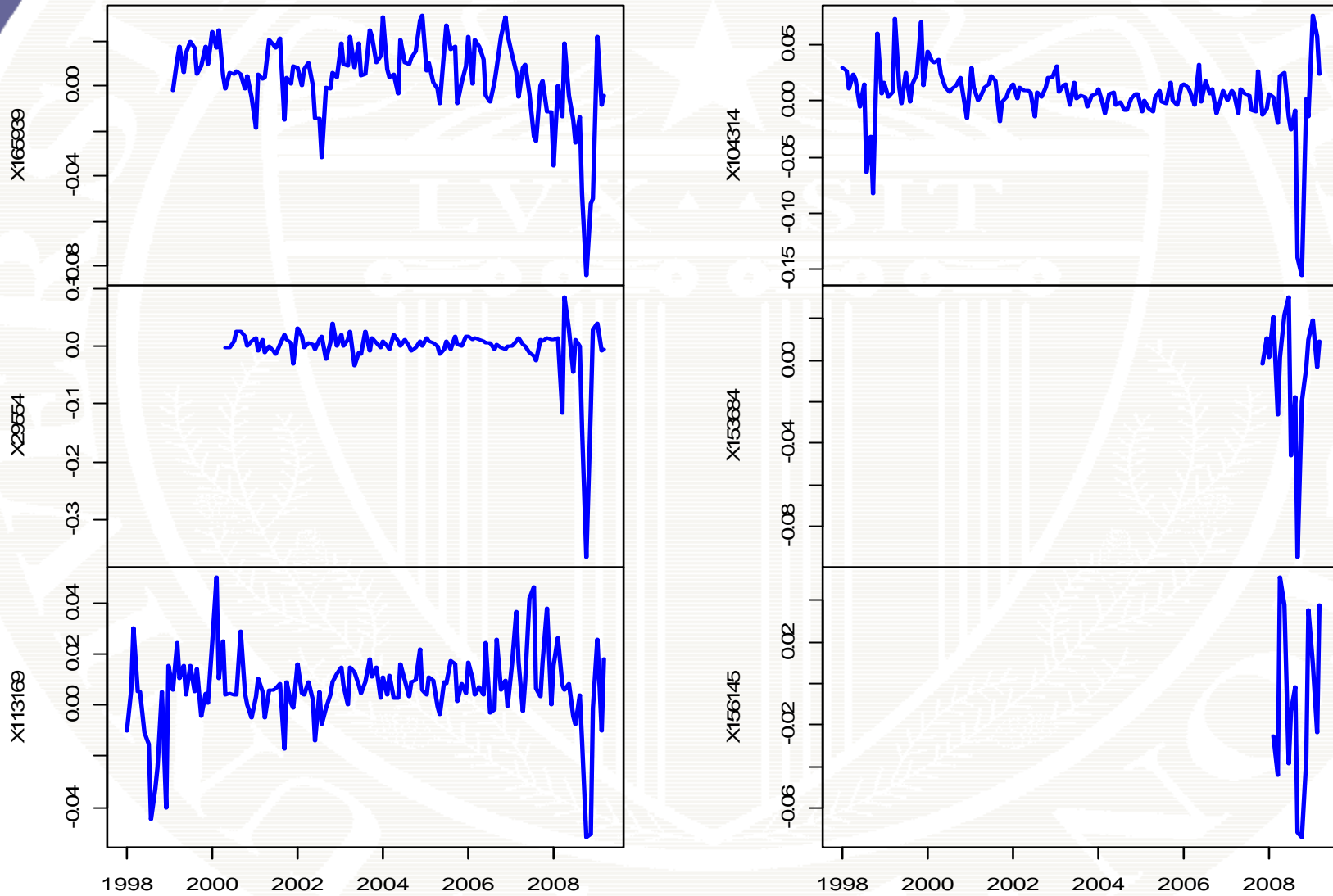
Fund performance

$$\begin{array}{cc} R_{1,T} & R_{n,T} \\ \vdots & \vdots \\ R_{1,T-T_1} & \vdots \\ & R_{n,T-T_n} \end{array}$$

Observe full history

Observe partial histories

# Example Portfolio: Unequal Histories of Individual Funds

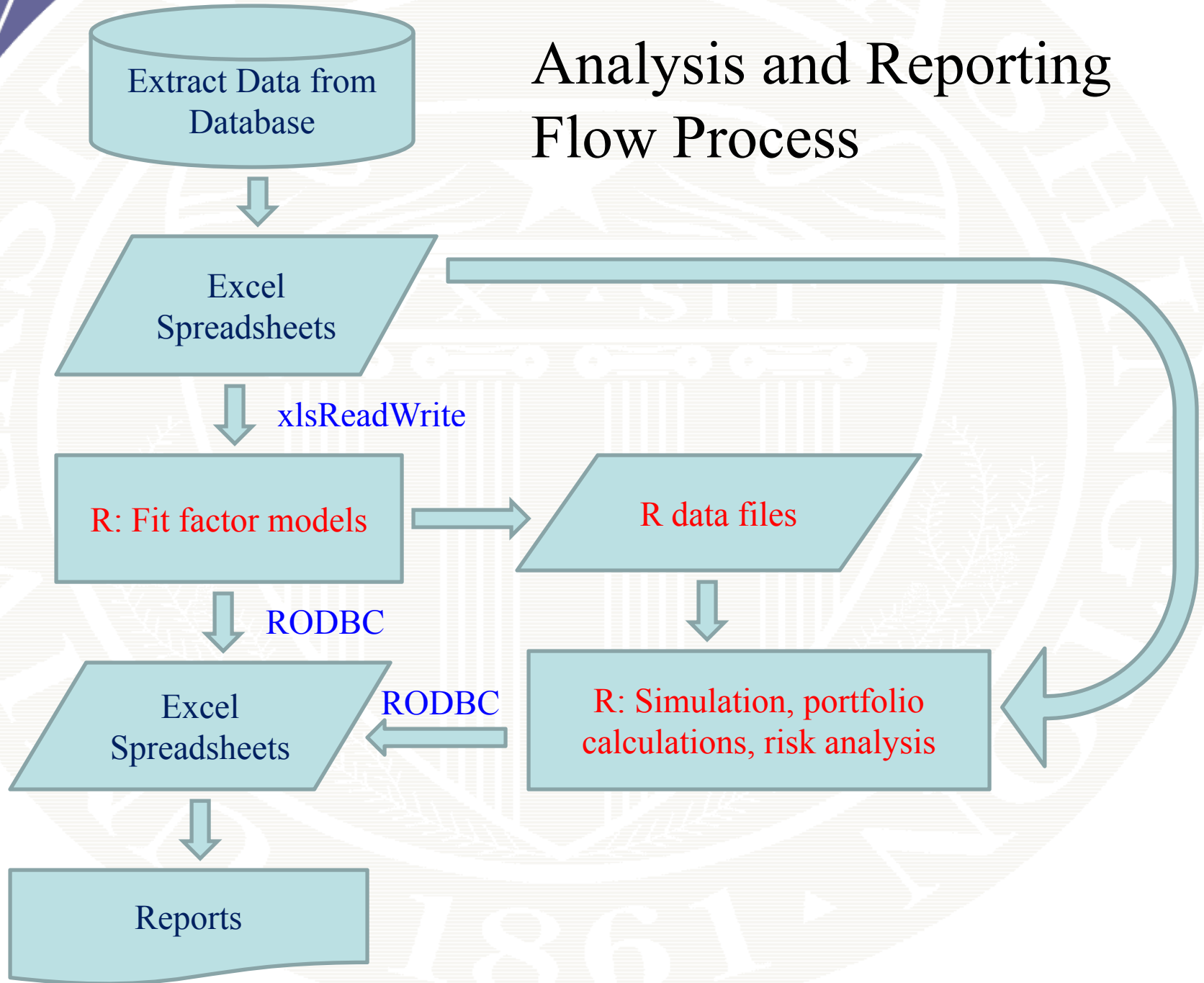


# Implication of Unequal Histories

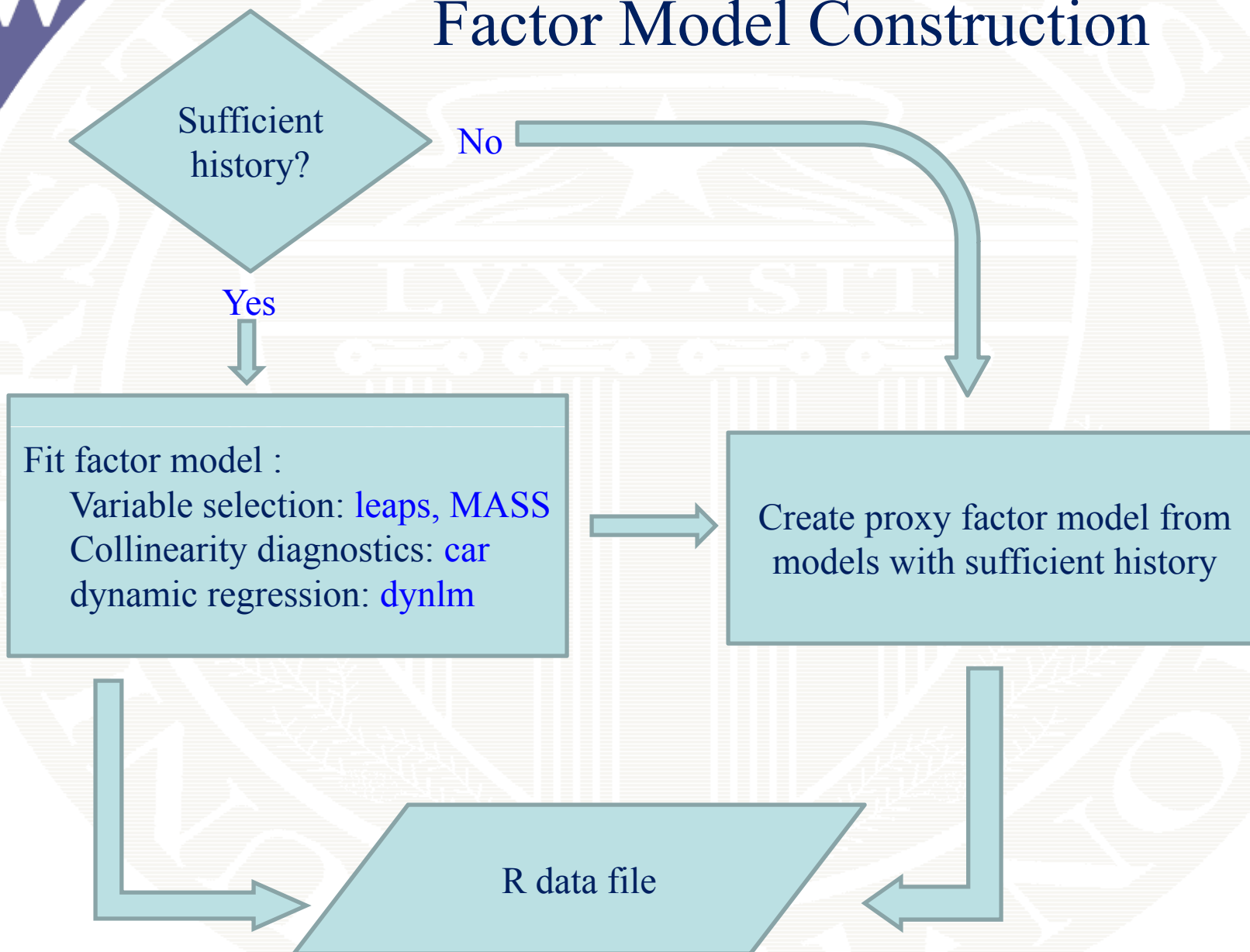
- Can't fit factor models to some funds
  - Need to create proxy factor model
- Statistics on common histories (truncated data) may be unreliable
- Difficult to compute non-normal tail risk measures



# Analysis and Reporting Flow Process



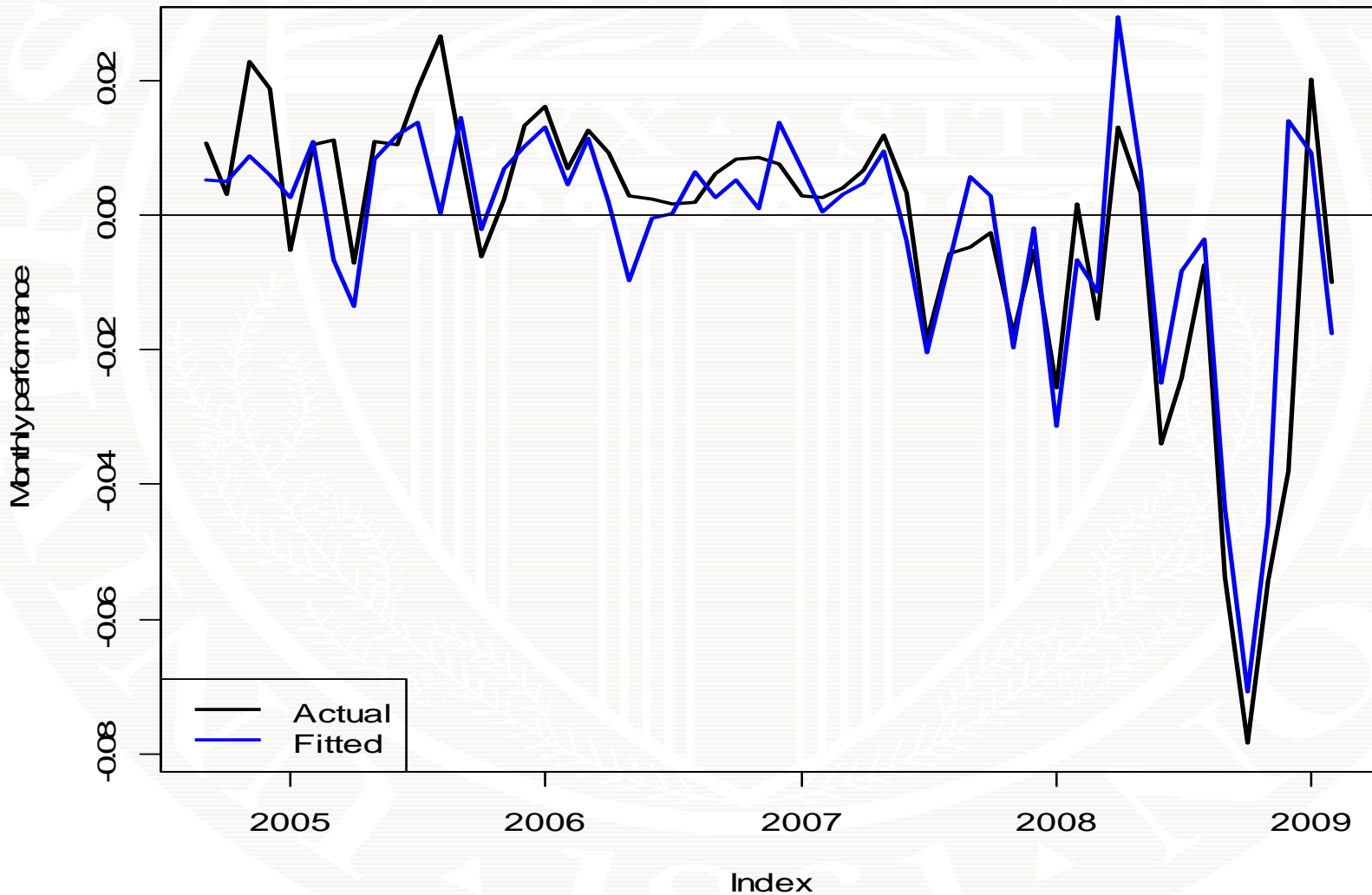
# Factor Model Construction



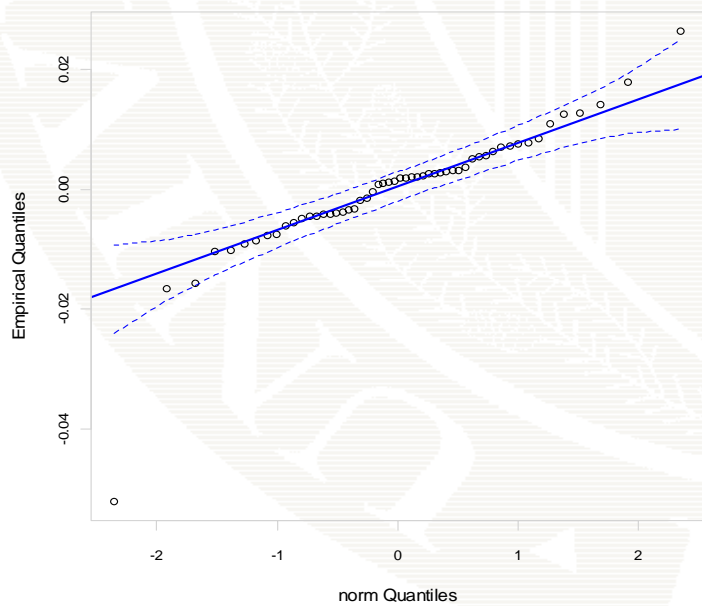
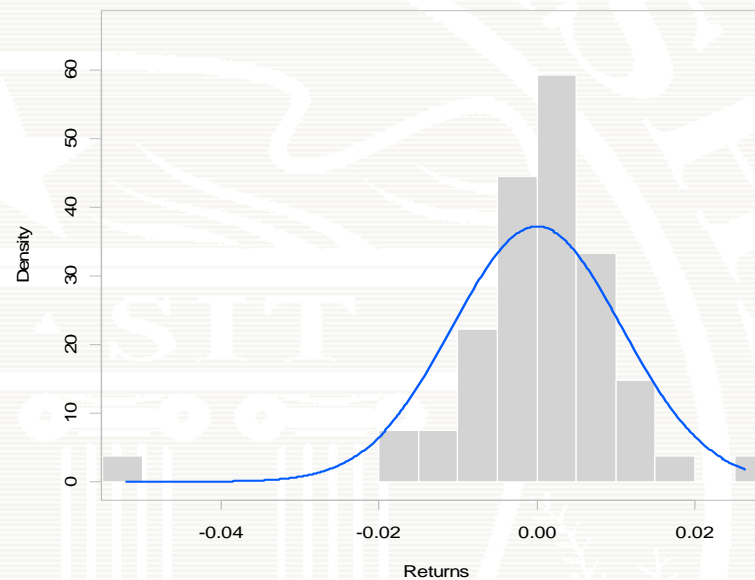
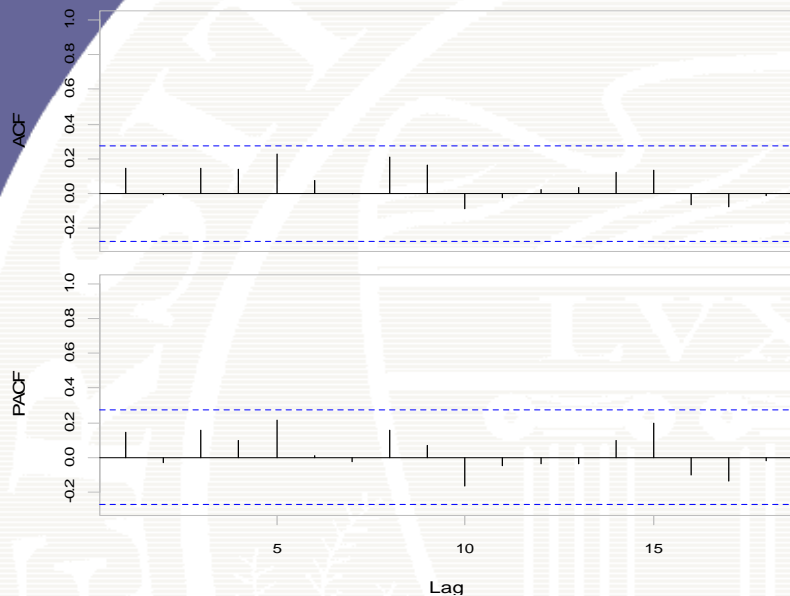
# Evaluation of Fitted Factor Models

- Graphical diagnostics
  - Created plot method appropriate for time series regression.
- Stability analysis
  - CUSUM etc: `strucchange`
  - Rolling analysis: `rollapply` (`zoo`)
  - Time varying parameters: `dlm`
- Dynamic effects
  - `dynlm`, `lmtest`

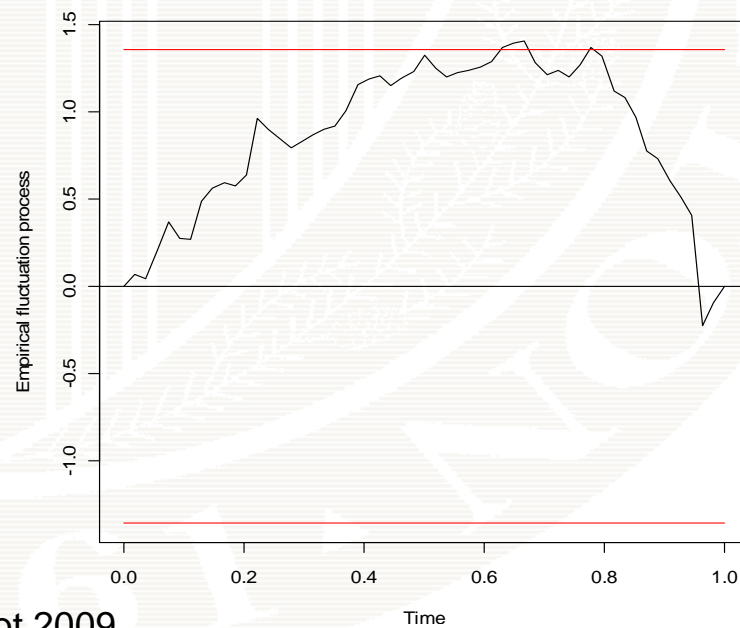
# Diagnostic Plots: Example Fund



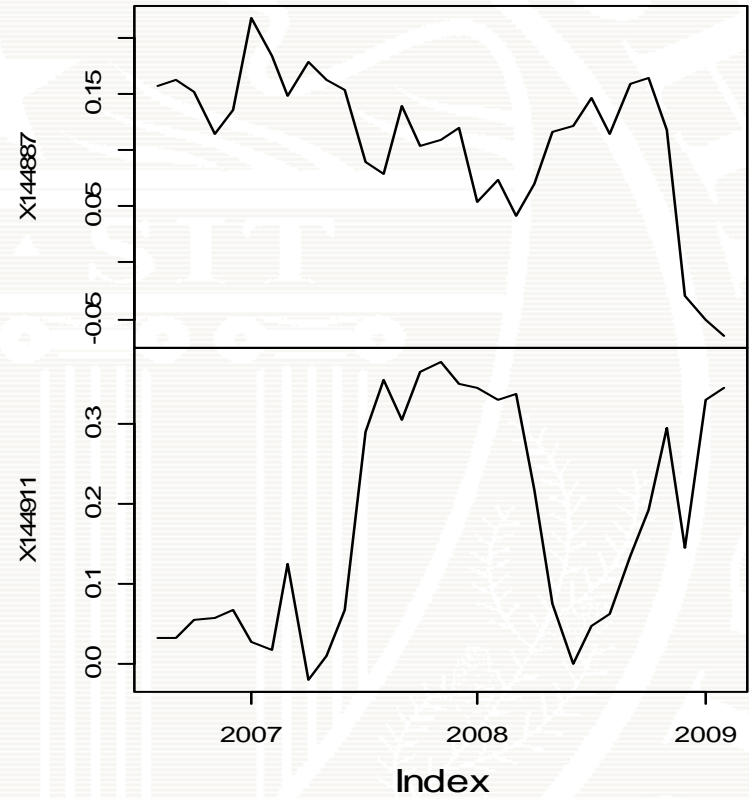
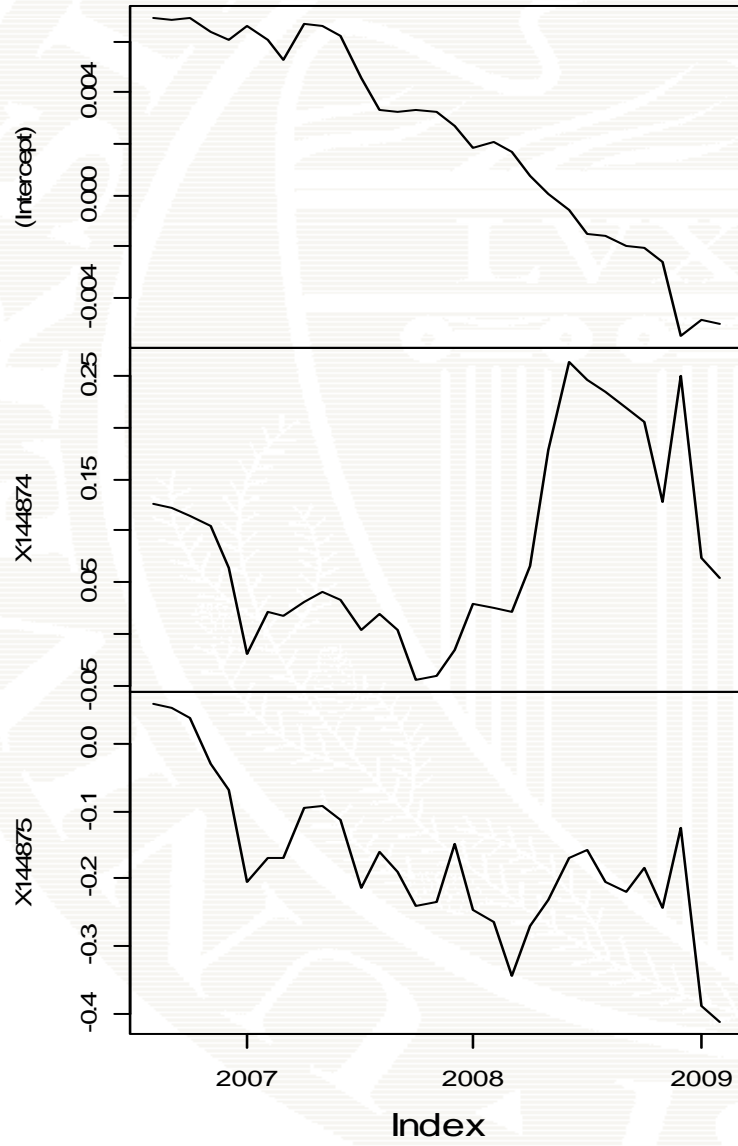
# Time Series Regression Residual diagnostics



**OLS-based CUSUM test**



# 24-month rolling estimates



# Dealing with Unequal Histories

- Estimate conditional distribution of  $R_i$  given  $F$ 
  - Fitted factor model or proxy factor model
- Estimate marginal distribution of  $F$ 
  - Empirical distribution, multivariate normal, copula
- Derive marginal distribution of  $R_i$  from  $p(R_i|F)$  and  $p(F)$
- Simulate  $R_i$  and Calculate functional of interest
  - Unobserved performance, Sharpe ratio, ETL etc



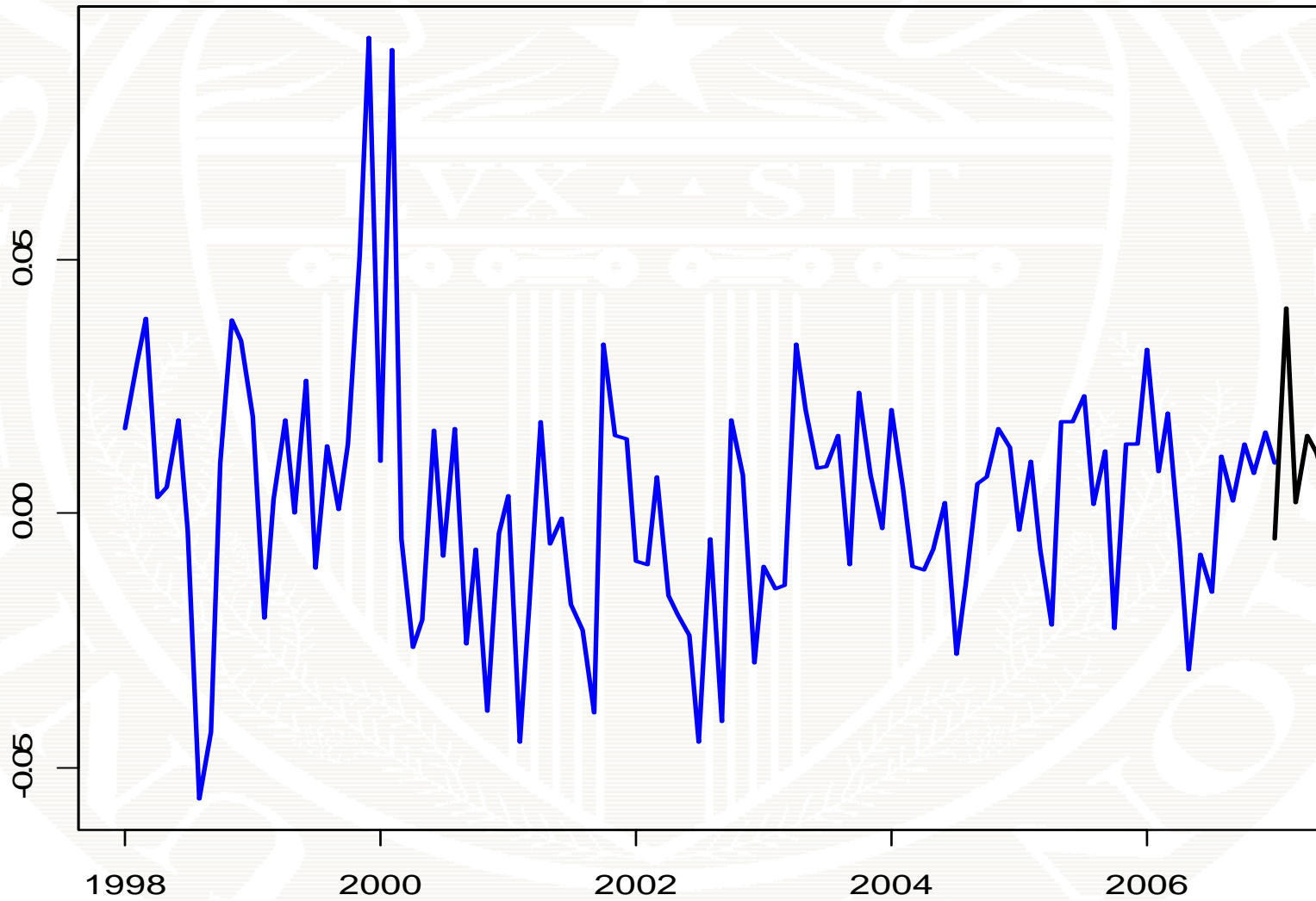
# Simulation Algorithm

- Draw  $\{\tilde{F}_1, \dots, \tilde{F}_M\}$  by resampling from the empirical distribution of  $F$ .
- For each  $\tilde{F}_u$  ( $u = 1, \dots, M$ ), draw a value  $\tilde{R}_{i,u}$  from the estimated conditional distribution of  $R_i$  given  $F = \tilde{F}_u$  (e.g., from fitted factor model assuming normal errors)
- $\{\tilde{R}_{i,u}\}_{u=1}^M$  is the desired sample for  $R_i$
- $M \approx 5000$

# What to do with $\{\tilde{R}_{i,u}\}_{u=1}^M$ ?

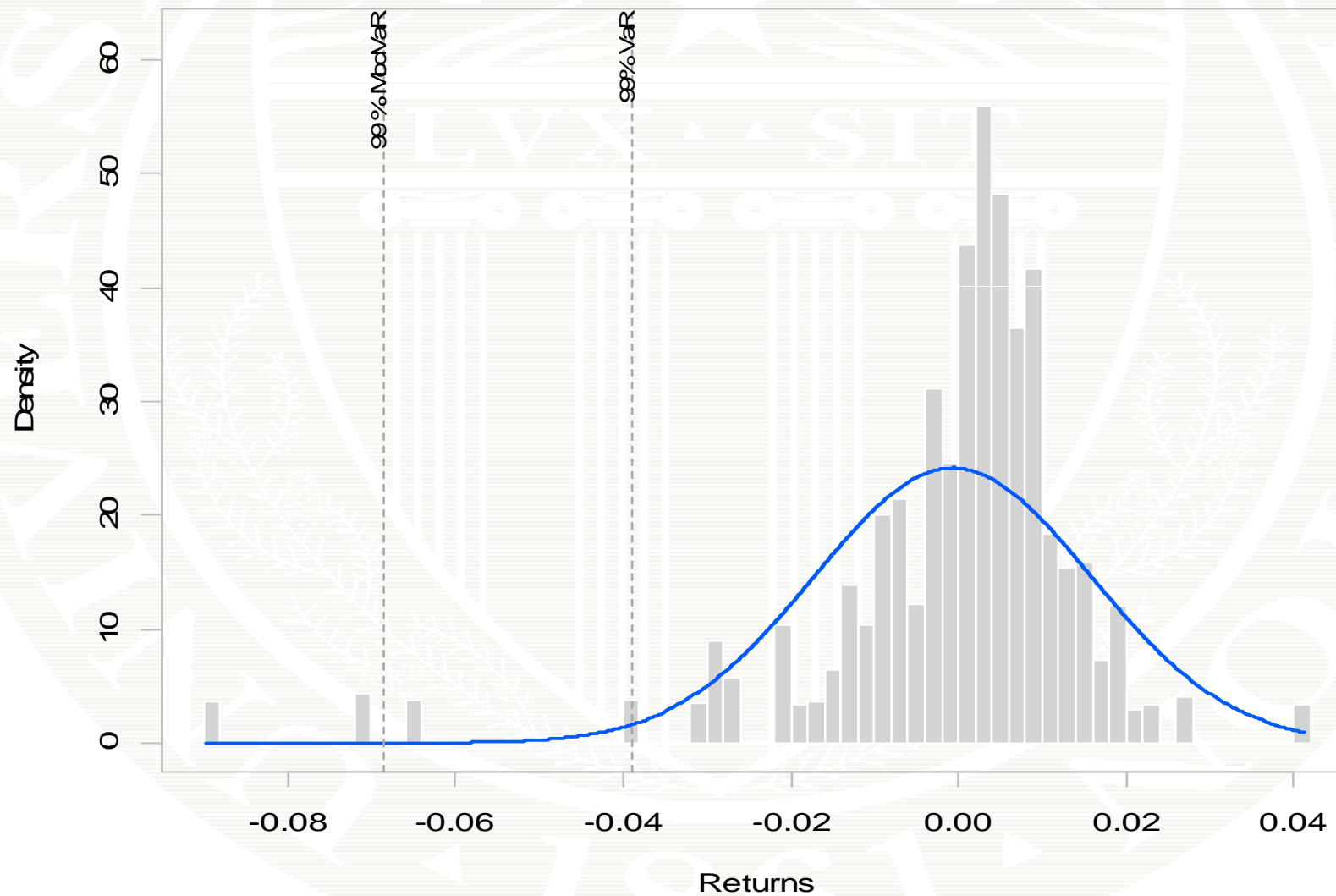
- Backfill missing fund performance
- Compute fund and portfolio performance measures
- Estimate non-parametric fund and portfolio tail risk measures
- Compute non-parametric risk budgeting measures
- Standard errors can be computed using a bootstrap procedure

# Example: Backfilled Fund Performance



Index

# Example: Simulated portfolio distribution



# S-PLUS and S+FinMetrics vs R

- Dealing with time series objects in R can be difficult and confusing
  - timeSeries, zoo, xts
- Time series regression in R is incompletely implemented
  - Diagnostic plots, prediction
- R packages give about 80% functional coverage to S+FinMetrics

## Some Thoughts About Using R in a Corporate Environment

- IT doesn't want to support it
- Firewalls block R downloads
- The world runs from an Excel spreadsheet
- Analysts with some programming experience learn R quickly
- Not good for the casual user

# References

- Boudt, K., B. Peterson and C. Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk*
- Dowd, K. (2002), *Measuring Market Risk*, John Wiley and Sons
- Goodworth, T. and C. Jones (2007). "Factor-based, Non-parametric Risk Measurement Framework for Hedge Funds and Fund-of-Funds," *The European Journal of Finance*.
- Hallerback, J. (2003). "Decomposing Portfolio Value-at-Risk: A General Analysis", *The Journal of Risk* 5/2.
- Jorion, P. (2007), *Value at Risk*, Third Edition, McGraw Hill



# References

- Jiang, Y. (2009). *Overcoming Data Challenges in Fund-of-Funds Portfolio Management*, PhD Thesis, Department of Statistics, University of Washington.
- Lo, A. (2007). *Hedge Funds: An Analytic Perspective*, Princeton.
- Yamai, Y. and T. Yoshiba (2002). “Comparative Analyses of Expected Shortfall and Value-at-Risk: Their Estimation Error, Decomposition, and Optimization, *Institute for Monetary and Economic Studies*, Bank of Japan.