

UW

Using R for Hedge Fund of Funds Risk Management

**R/Finance 2009: Applied Finance with R
University of Illinois Chicago, April 25, 2009**

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Outline

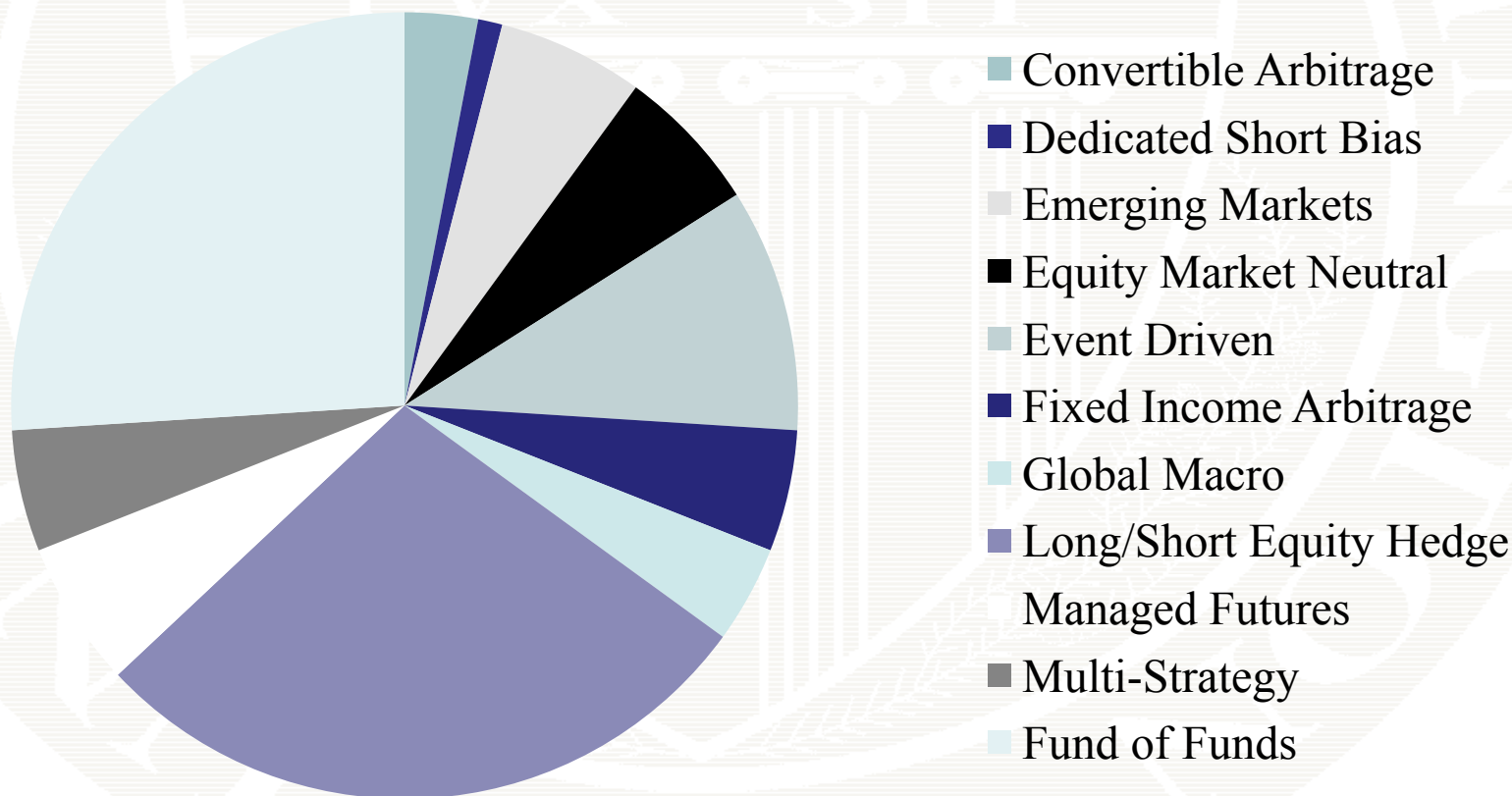
- Hedge fund of funds environment
- Factor model risk measurement
- R implementation in corporate environment
- Dealing with unequal data histories
- Some thoughts on S-PLUS and S+FinMetrics vs. R in Finance

Hedge Fund of Funds Environment

- HFoFs are hedge funds that invest in other hedge funds
 - 20 to 30 portfolios of hedge funds
 - Typical portfolio size is 30 funds
- Hedge fund universe is large: 5000 live funds
 - Segmented into 10-15 distinct strategy types
- Hedge funds voluntarily report monthly performance to commercial databases
 - Altvest, CISDM, HedgeFund.net, Lipper TASS, CS/Tremont, HFR
- HFoFs often have partial position level data on invested funds

Hedge Fund Universe

Live funds



Characteristics of Monthly Returns

- Reporting biases
 - Survivorship, backfill
- Non-normal behavior
 - Asymmetry (skewness) and fat tails (excess kurtosis)
- Serial correlation
 - Performance smoothing, illiquid positions
- Unequal histories

Characteristics of Hedge Fund Data

	fund1	fund2	fund3	fund4	fund5
Observations	122.0000	107.0000	135.0000	135.0000	135.0000
NAs	13.0000	28.0000	0.0000	0.0000	0.0000
Minimum	-0.0842	-0.3649	-0.0519	-0.1556	-0.2900
Quartile 1	-0.0016	-0.0051	0.0020	-0.0017	-0.0021
Median	0.0058	0.0046	0.0060	0.0073	0.0049
Arithmetic Mean	0.0038	-0.0017	0.0063	0.0059	0.0021
Geometric Mean	0.0037	-0.0029	0.0062	0.0055	0.0014
Quartile 3	0.0158	0.0129	0.0127	0.0157	0.0127
Maximum	0.0311	0.0861	0.0502	0.0762	0.0877
Variance	0.0003	0.0020	0.0002	0.0008	0.0013
Stdev	0.0176	0.0443	0.0152	0.0275	0.0357
Skewness	-1.7753	-5.6202	-0.8810	-2.4839	-4.9948
Kurtosis	5.2887	40.9681	3.7960	13.8201	35.8623
Rho1	0.6060	0.3820	0.3590	0.4400	0.383

Sample: January 1998 – March 2009

Factor Model Risk Measurement in HFoFs Portfolio

- Quantify factor risk exposures
 - Equity, rates, credit, volatility, currency, commodity, etc.
- Quantify tail risk
 - VaR, ETL
- Risk budgeting
 - Component, incremental, marginal
- Stress testing and scenario analysis

Commercial Products

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YOUR INVESTMENT INTEREST

- HEDGE FUNDS
- FUND OF FUNDS
- PENSION FUNDS

LATEST CASE STUDY

Antarctica Asset Management
The Challenge: Implement a risk management system developed specifically for Fund of Hedge Fund (FoHF) portfolios. VIEW ALL >>>

LATEST NEWS

March 16, 2009
Paris, France — RiskData, the leading risk management provider for pension funds and other institutional investors, today released a new comparative study which reveals that investors can hedge their portfolio against catastrophic "perfect storm" financial market events at no cost through quantitative risk management techniques. Entitled "Keeping the Devil in its Box," the paper looks at how... in the wake of a market meltdown... institutions can manage portfolios and cap potential losses by integrating extreme risk budgeting into their investment process. VIEW ALL >>>

February 9, 2009 Updated Study
(Initial Version posted on January 21 2009)
RiskData released its updated analysis of the Madoff case, demonstrating how quantitative

UPCOMING EVENTS

April 20-21, 2009
Dr. Raphael Douady will present a series of lectures at Shandong University, Jinan, China, entitled "New Developments in Risk management Techniques".

April 27-29, 2009
Dr. Douady will conduct a conference at Fudan University in Shanghai, China, entitled "Nonlinear Statistics and Hedge Fund Risks".

May 19, 2009
Dr. Douady will conduct a full day seminar "Continuous Time Finance" at the EDHEC Business School in Nice, France.

LATEST PUBLICATIONS

Keeping the Devil in Its Box
A new comparative study on how investors can hedge their portfolio against "perfect storm" financial market events through extreme risk budgeting technique. VIEW ALL >>>

Results of 2008: Navigating the Perfect Storm
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NASDAQ Comp.	-4.6%
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MSCI France	-4.3%
MSCI UK	-4.3%
MSCI Hong Kong	-4.2%
MSCI India	-5.3%
MSCI Japan	-3.7%
MSCI Russia	-9.5%
MSCI China	-5.3%
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Very expensive! R is not!

Factor Model: Methodology

$$\begin{aligned}R_{it} &= \alpha_i + \beta_{i1}F_{1t} + \cdots + \beta_{ik}F_{kt} + \varepsilon_{it}, \\ &= \alpha_i + \boldsymbol{\beta}'_i \mathbf{F}_t + \varepsilon_{it}\end{aligned}$$

$$i = 1, \dots, n; \quad t = t_i, \dots, T$$

$$\mathbf{F}_t \sim (\boldsymbol{\mu}_F, \boldsymbol{\Sigma}_F)$$

$$\varepsilon_{it} \sim (0, \sigma_{\varepsilon,i}^2)$$

$$\text{cov}(f_{jt}, \varepsilon_{it}) = 0 \text{ for all } j, i \text{ and } t$$

$$\text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0 \text{ for } i \neq j$$

Practical Considerations

- Many potential risk factors (> 50)
- High collinearity among some factors
- Risk factors vary across discipline/strategy
- Nonlinear effects
- Dynamic effects
- Time varying coefficients
- Common histories for factors; unequal histories for fund performance

Expected Return Decomposition

$$E[R_{it}] = \alpha_i + \beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to “beta” exposure

$$\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to manager specific “alpha”

$$\alpha_i = E[R_{it}] - (\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}])$$

Variance Decomposition

$$\text{var}(R_{it}) = \underbrace{\boldsymbol{\beta}'_i \text{var}(\mathbf{F}_t) \boldsymbol{\beta}_i}_{\text{systematic}} + \underbrace{\text{var}(\varepsilon_{it})}_{\text{specific}} = \boldsymbol{\beta}'_i \boldsymbol{\Sigma}_F \boldsymbol{\beta}_i + \sigma_{\varepsilon,i}^2$$

Variance contribution due to factor exposures

$$\beta_1^2 \text{var}(F_{1t}) + \beta_2^2 \text{var}(F_{2t}) + \dots + \beta_k^2 \text{var}(F_{kt})$$

Variance contribution due to covariances between factors

$$2\beta_1\beta_2 \text{cov}(F_{1t}, F_{2t}) + \dots + 2\beta_{k-1}\beta_k \text{cov}(F_{k-1t}, F_{kt})$$

Covariance

$$\mathbf{R}_t = \boldsymbol{\alpha} + \mathbf{B} \mathbf{F}_t + \boldsymbol{\varepsilon}_t$$

$n \times 1$ $n \times 1$ $n \times k$ $k \times 1$ $n \times 1$

$$\text{var}(\mathbf{R}_t) = \Sigma_{FM} = \mathbf{B} \Sigma_{\mathbf{F}} \mathbf{B}' + \mathbf{D}_{\varepsilon}$$

$$\mathbf{D}_{\varepsilon} = \text{diag}(\sigma_{\varepsilon,1}^2, \dots, \sigma_{\varepsilon,n}^2)$$

Note: $\text{cov}(R_{it}, R_{jt}) = \boldsymbol{\beta}'_i \text{var}(\mathbf{F}_t) \boldsymbol{\beta}_j = \boldsymbol{\beta}'_i \Sigma_{\mathbf{F}} \boldsymbol{\beta}_j$

Portfolio Analysis

$\mathbf{w} = (w_1, \dots, w_n)'$ = portfolio weights

$$R_{pt} = \mathbf{w}'\mathbf{R}_t = \mathbf{w}'\boldsymbol{\alpha} + \mathbf{w}'\mathbf{B}\mathbf{F}_t + \mathbf{w}'\boldsymbol{\varepsilon}_t$$

$$= \sum_{i=1}^n w_i R_{it} = \sum_{i=1}^n w_i \alpha_i + \sum_{i=1}^n w_i \boldsymbol{\beta}'_i \mathbf{F}_t + \sum_{i=1}^n w_i \varepsilon_{it}$$

$$= \alpha_p + \boldsymbol{\beta}'_p \mathbf{F}_t + \varepsilon_{pt}$$

Portfolio Variance Decomposition

$$\sigma_p^2 = \text{var}(R_{pt}) = \mathbf{w}' \text{var}(\mathbf{R}_t) \mathbf{w} = \mathbf{w}' \mathbf{B} \Sigma_F \mathbf{B}' \mathbf{w} + \mathbf{w}' \mathbf{D} \mathbf{w}$$

$$\sigma_{p,\text{systematic}}^2 = \mathbf{w}' \mathbf{B} \Sigma_F \mathbf{B}' \mathbf{w}$$

$$\sigma_{p,\text{specific}}^2 = \mathbf{w}' \mathbf{D} \mathbf{w} = \sum_{i=1}^n w_i \sigma_{\varepsilon,i}^2 \quad R_p^2 = \frac{\sigma_{p,\text{systematic}}^2}{\sigma_p^2}$$

$$\sigma_{p,\text{systematic}}^2 = \boldsymbol{\beta}_p' \Sigma_F \boldsymbol{\beta}_p = \sum_{j=1}^k \beta_{p,j}^2 \sigma_{jj}^2 + \text{covariance terms}$$

$$\text{covariance terms} = \sigma_{p,\text{systematic}}^2 - \sum_{j=1}^k \beta_{p,j}^2 \sigma_{jj}^2$$

Risk Budgeting: Volatility

$$\mathbf{MCR} = \frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\mathbf{B}\Sigma_F\mathbf{B}'\mathbf{w} + \mathbf{D}\mathbf{w}}{\sigma_p} \quad \text{Marginal contributions to risk}$$

$$\mathbf{MCR}_{\text{systematic}} = \frac{\mathbf{B}\Sigma_F\mathbf{B}'\mathbf{w}}{\sigma_p}$$

$$\mathbf{MCR}_{\text{specific}} = \frac{\mathbf{D}\mathbf{w}}{\sigma_p}$$

$$\mathbf{CR} = \mathbf{w} \odot \frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\mathbf{w} \odot (\mathbf{B}\Sigma_F\mathbf{B}'\mathbf{w} + \mathbf{D}\mathbf{w})}{\sigma_p} \quad \text{Components to risk}$$

$$\mathbf{1}'\mathbf{CR} = \sum_{i=1}^n CR_i = \sigma_p$$

Tail Risk Measures

Value-at-Risk (VaR)

$$VaR_{\alpha} = -q_{\alpha} = -F^{-1}(\alpha)$$

F = CDF of returns R

Expected Shortfall (ES)

$$ES_{\alpha} = -E[R \mid R \leq VaR_{\alpha}]$$

Tail Risk Measures: Normal Distribution

$$R_p \sim N(\mu_p, \sigma_p^2), \quad \sigma_p^2 = w' \Sigma_{FM} w$$

$$VaR_\alpha^N = -\mu_p - \sigma_p \times z_\alpha, \quad z_\alpha = \Phi^{-1}(\alpha)$$

$$ES_\alpha^N = \mu_p - \sigma_p \frac{1}{\alpha} \phi(z_\alpha)$$

See functions in [PerformanceAnalytics](#)

Tail Risk Measures: Non-Normal Distributions

Use Cornish-Fisher expansion to account for asymmetry and fat tails

$$VaR_{\alpha}^{CF} = -\mu_i - \sigma_i \times z_{\alpha} + \sigma_i \left[-\frac{1}{6} (z_{\alpha}^2 - 1) skew_i - \frac{1}{24} (z_{\alpha}^3 - 3z_{\alpha}) ekurt_i + \frac{1}{36} (2z_{\alpha}^3 - 5z_{\alpha}) skew_i^2 \right]$$

ES_{α}^{CF} : Formula given in Boudt, Peterson and Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk* and implementation in [PerformanceAnalytics](#)

Risk Budgeting: Tail Risk

Value-at-Risk (VaR)

$$VaR_\alpha = \sum_{i=1}^n w_i \frac{\partial VaR_\alpha}{\partial w_i} = \sum_{i=1}^n \overbrace{w_i \times mVaR_{\alpha,i}}^{cVaR_{\alpha,i}},$$

$$mVaR_{\alpha,i} = \frac{\partial VaR_\alpha}{\partial w_i} = -E[R_i | R_p = VaR_\alpha]$$

Expected Shortfall (ES)

$$ES_\alpha = \sum_{i=1}^n w_i \frac{\partial ES_\alpha}{\partial w_i} = \sum_{i=1}^n \overbrace{w_i \times mES_{\alpha,i}}^{cES_{\alpha,i}},$$

$$mES_{\alpha,i} = \frac{\partial ES_\alpha}{\partial w_i} = -E[R_i | R_p \leq VaR_\alpha]$$

Risk Budgeting: Explicit Formulas

- Normal distribution
 - See Jorion (2007) or Dowd (2002)
- Non-normal distribution using Cornish-Fisher expansion
 - See Boudt, Peterson and Croux (2008) "Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns," *Journal of Risk* and implementation in [PerformanceAnalytics](#)

Risk Budgeting: Simulation

$\{R_{it}\}_{t=1}^M = M$ simulated returns

Method 1: Brute Force

$$mVaR_{\alpha,i} \approx \frac{\Delta VaR_{\alpha}}{\Delta w_i}, \quad mES_{\alpha,i} = \frac{\Delta ES_{\alpha}}{\Delta w_i}$$

Method 2: Average R_{it} around values for which $R_{pt} = VaR_{\alpha}$

$$mVaR_{\alpha,i} \approx - \sum_{t: R_{pt} = VaR_{\alpha} \pm \varepsilon} R_{it}, \quad mES_{\alpha,i} \approx - \sum_{t: R_{pt} \leq VaR_{\alpha}} R_{it}$$

R Functions for Factor Model Risk Analysis

Function	Function
factorModelCovariance	normalES
factorModelRiskDecomposition	normalPortfolioES
normalVaR	normalMarginalES
normalPortfolioVaR	normalComponentES
normalMarginalVaR	modifiedES
normalComponentVaR	modifiedPortfolioES
normalVaRreport	modifiedESreport
modifiedVaR	simulatedMarginalVaR
modifiedPortfolioVaR	simulatedComponentVaR
modifiedMarginalVaR	simulatedMarginalES
modifiedComponentVaR	simulatedComponentES

Unequal Histories

Risk factors

$$\begin{array}{c}
 F_{1,T}, \dots, F_{k,T} \\
 \vdots \\
 F_{1,T-T_i}, \dots, F_{k,T-T_i} \\
 \vdots \\
 F_{1,1}, \dots, F_{k,1}
 \end{array}$$

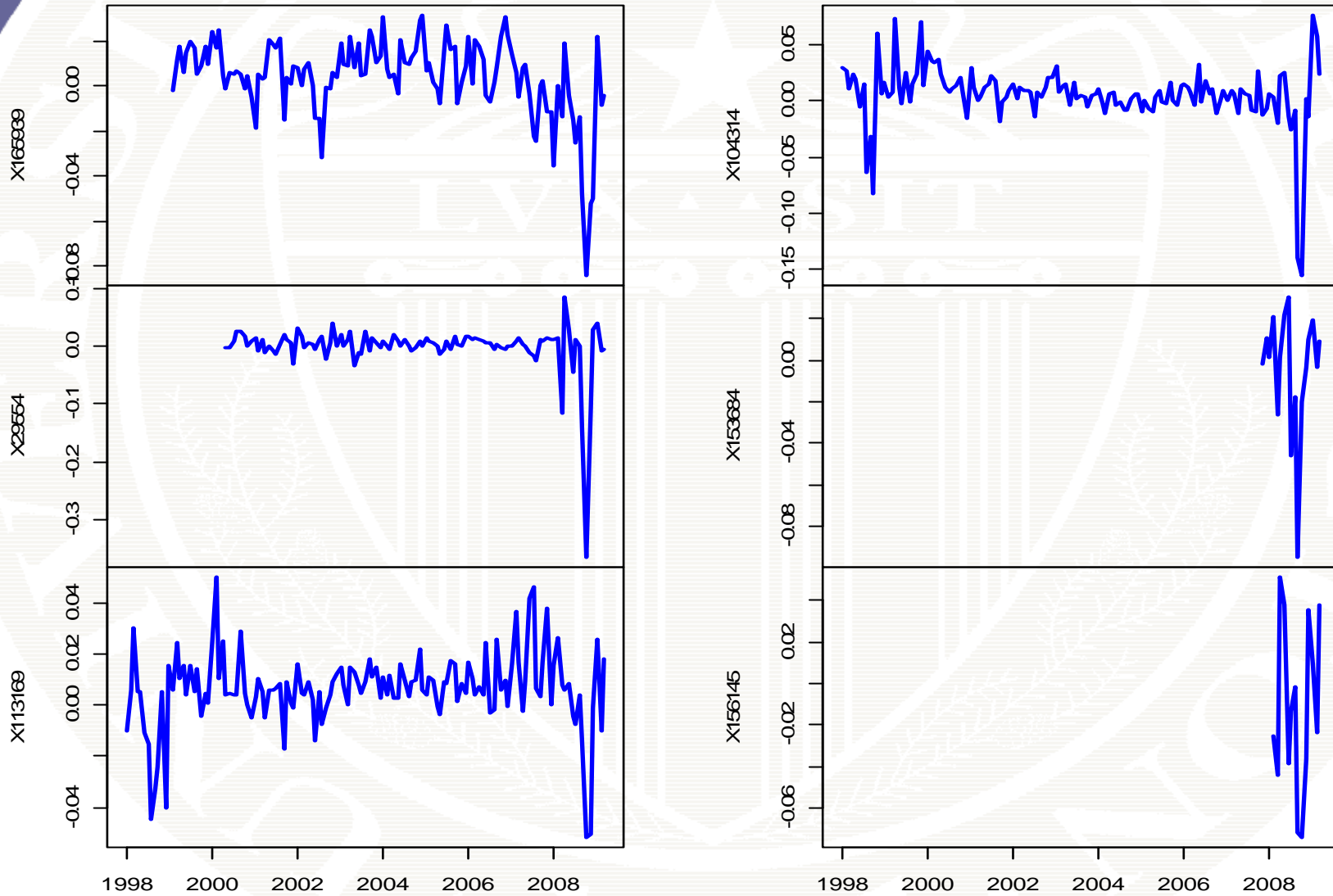
Fund performance

$$\begin{array}{cc}
 R_{1,T} & R_{n,T} \\
 \vdots & \vdots \\
 R_{1,T-T_1} & \vdots \\
 & R_{n,T-T_n}
 \end{array}$$

Observe full history

Observe partial histories

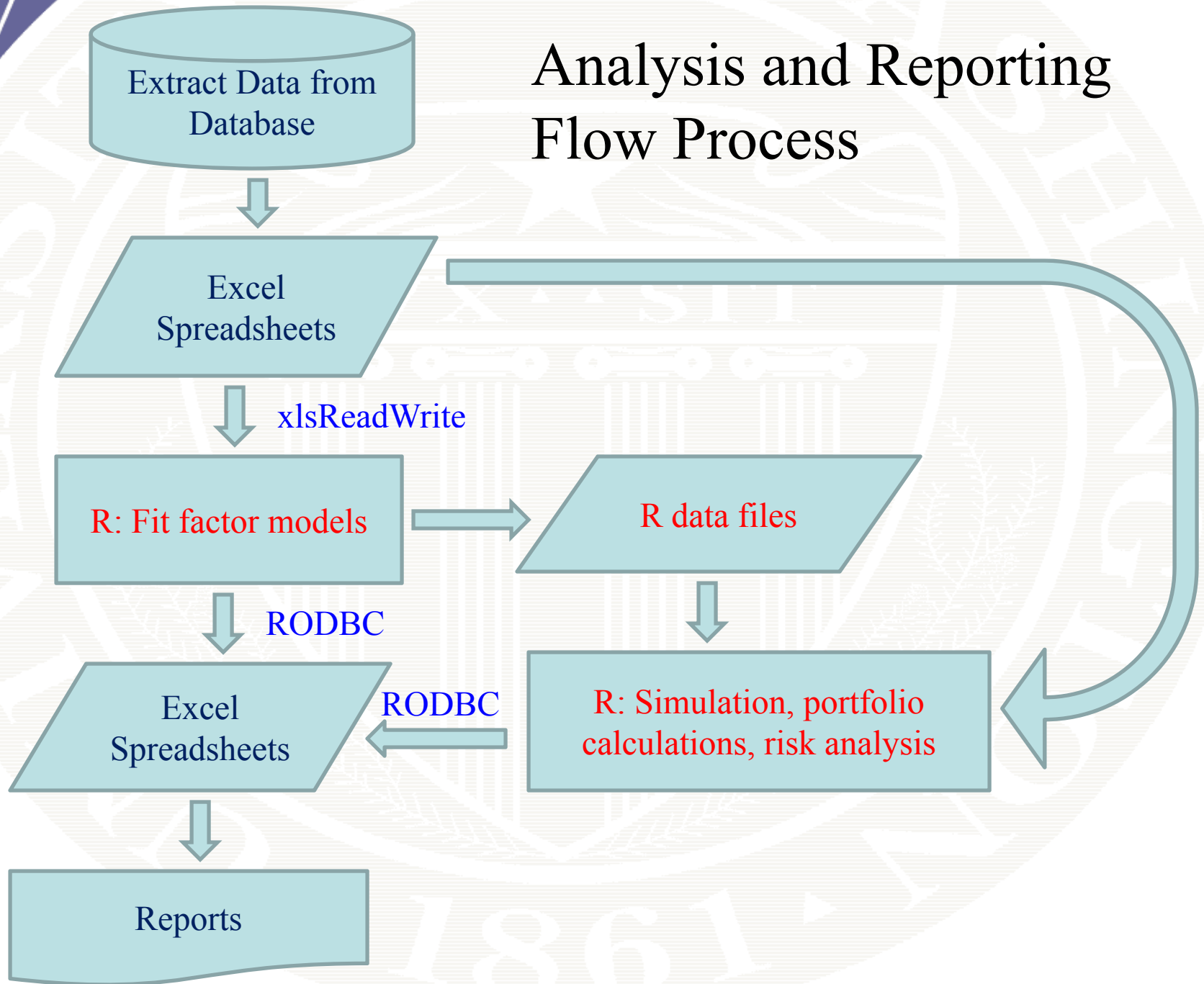
Example Portfolio: Unequal Histories of Individual Funds



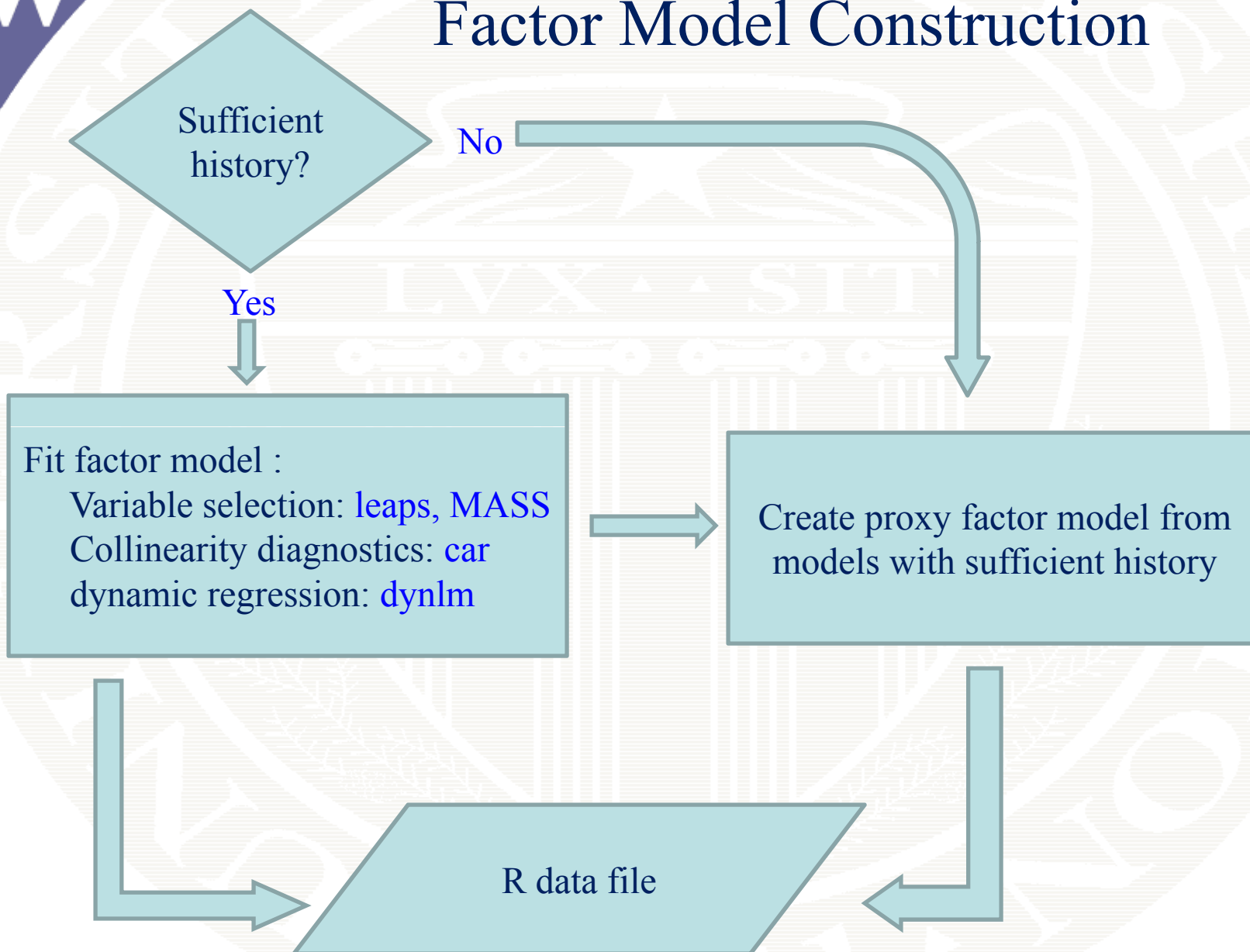
Implication of Unequal Histories

- Can't fit factor models to some funds
 - Need to create proxy factor model
- Statistics on common histories (truncated data) may be unreliable
- Difficult to compute non-normal tail risk measures

Analysis and Reporting Flow Process



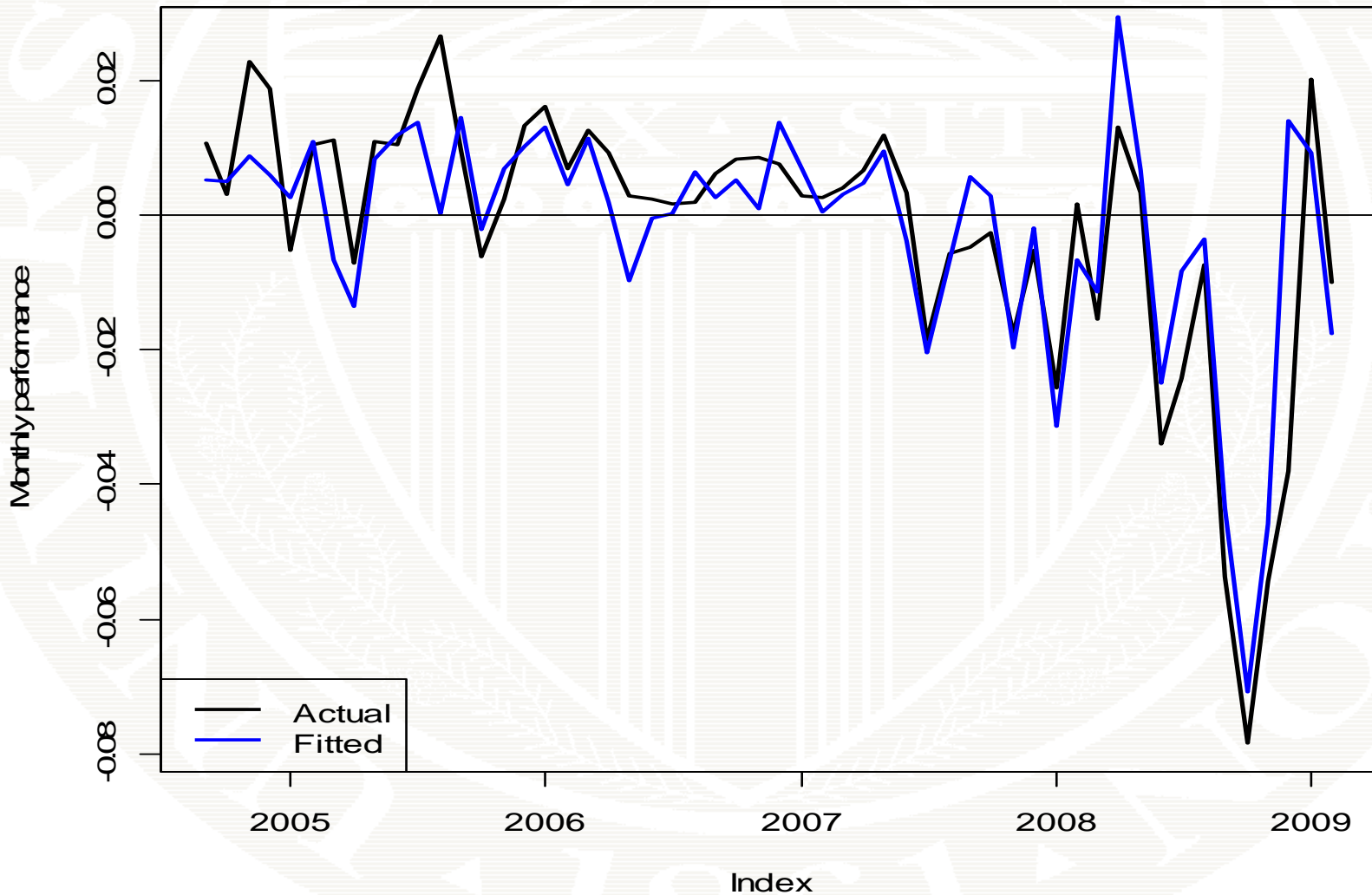
Factor Model Construction



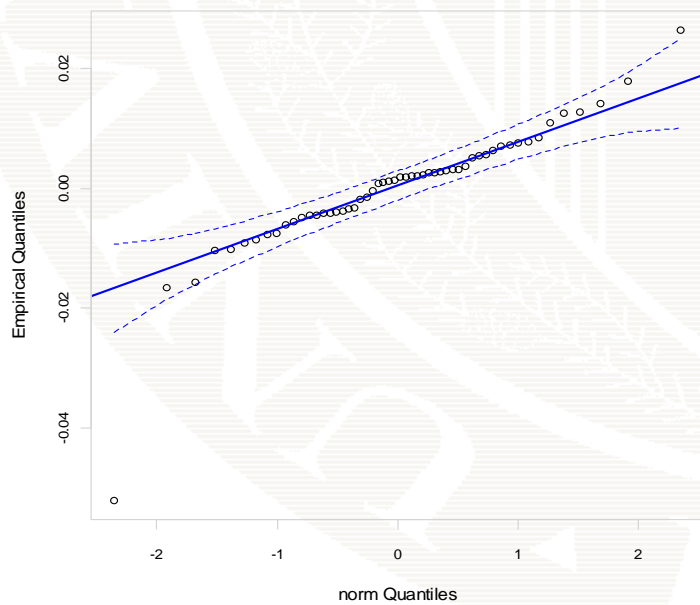
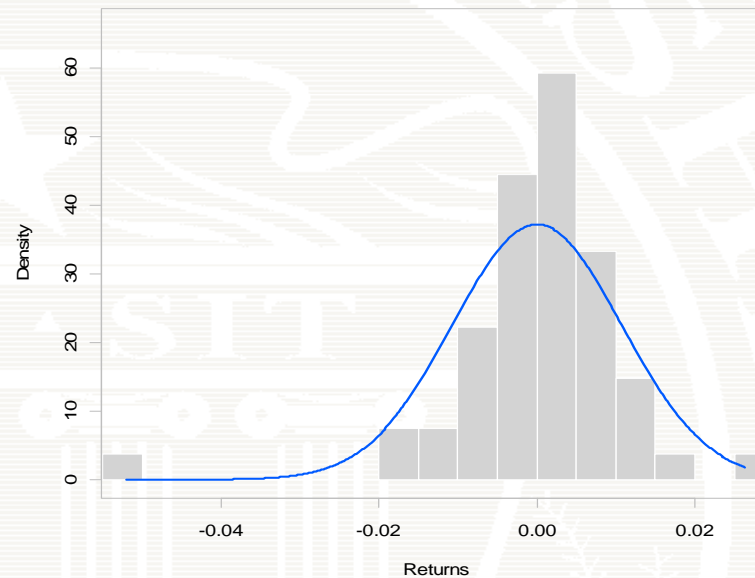
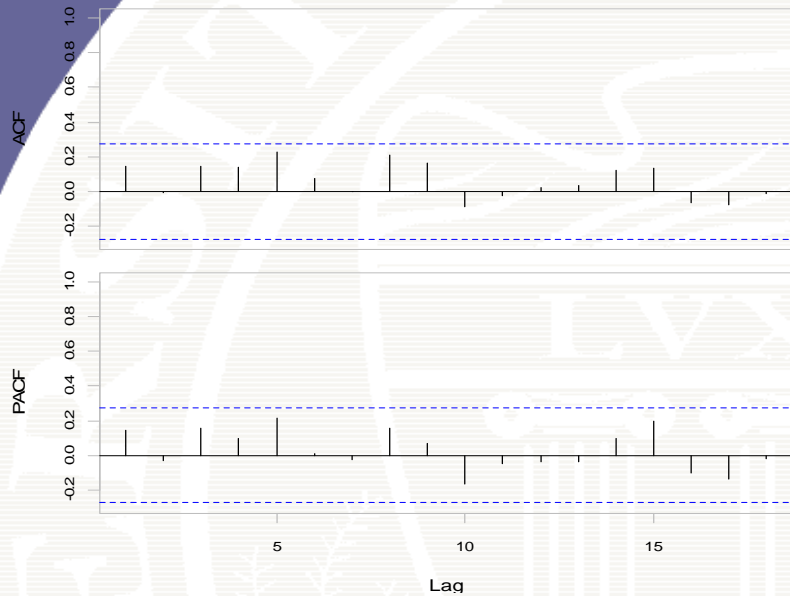
Evaluation of Fitted Factor Models

- Graphical diagnostics
 - Created plot method appropriate for time series regression.
- Stability analysis
 - CUSUM etc: `strucchange`
 - Rolling analysis: `rollapply` (`zoo`)
 - Time varying parameters: `dlm`
- Dynamic effects
 - `dynlm`, `lmtest`

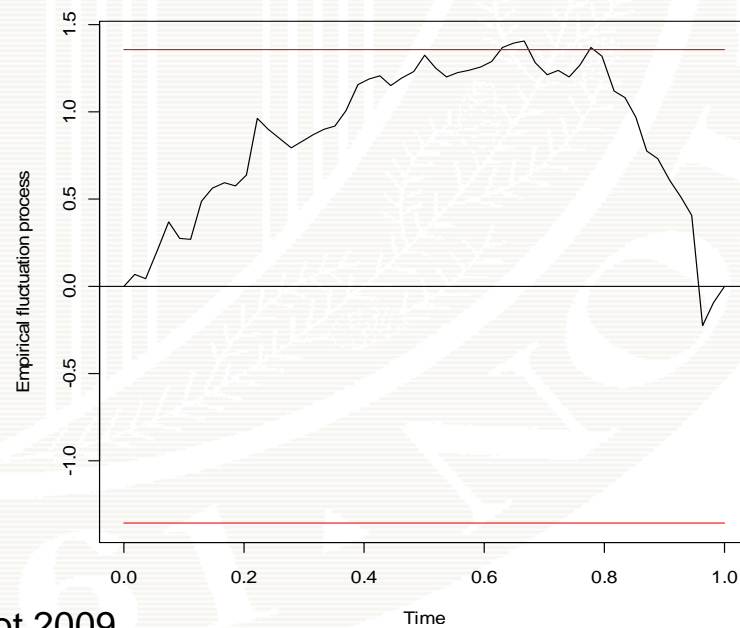
Diagnostic Plots: Example Fund



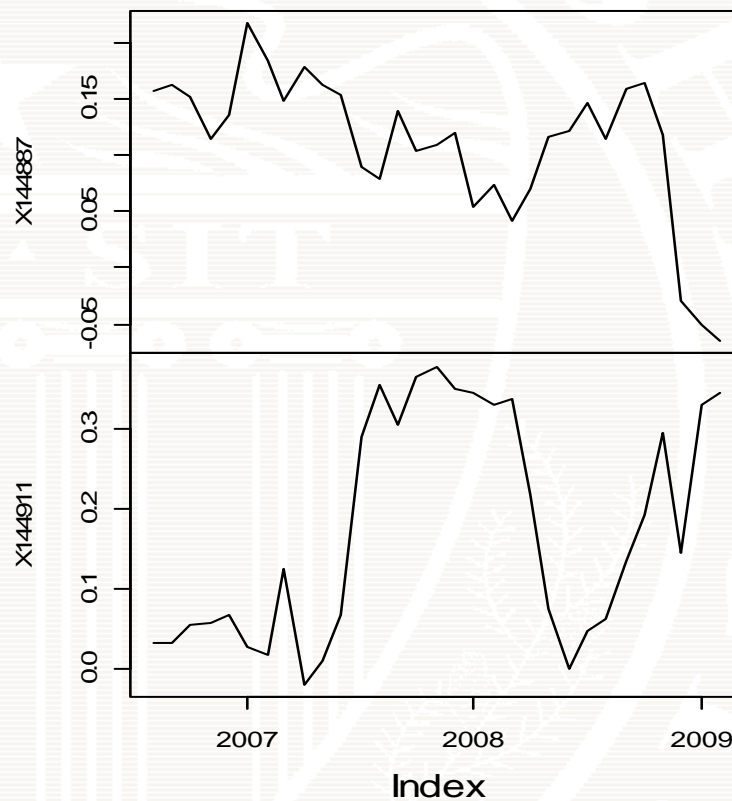
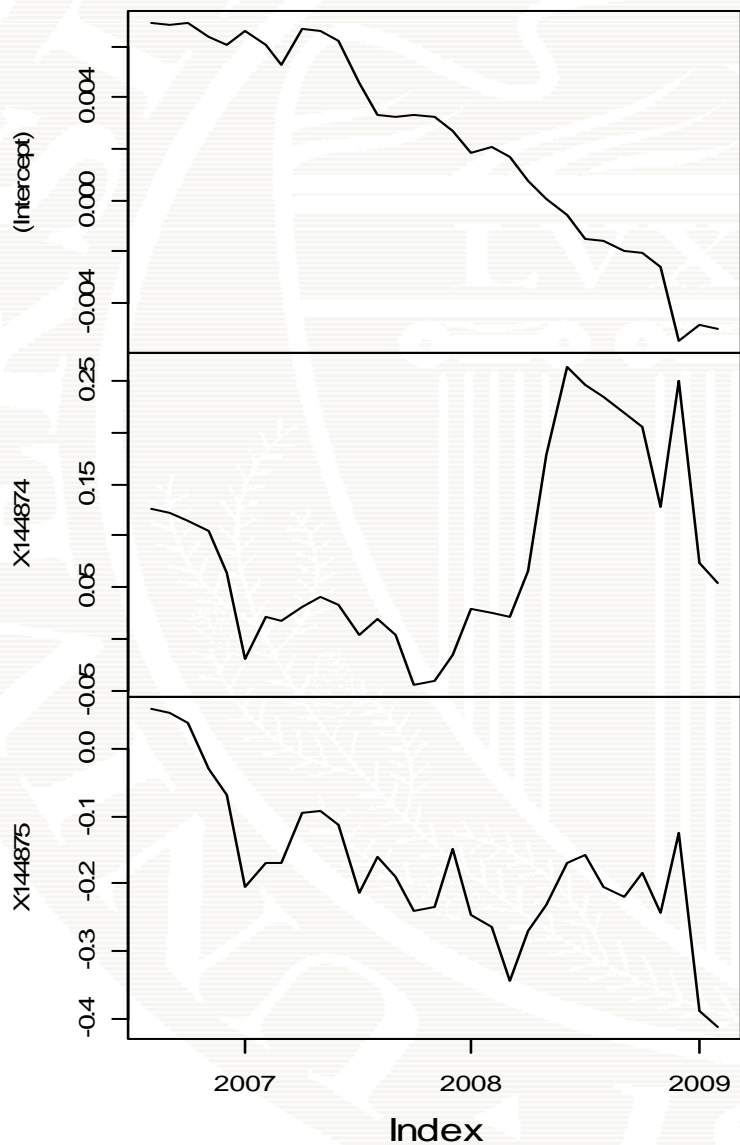
Time Series Regression Residual diagnostics



OLS-based CUSUM test



24-month rolling estimates



Dealing with Unequal Histories

- Estimate conditional distribution of R_i given F
 - Fitted factor model or proxy factor model
- Estimate marginal distribution of F
 - Empirical distribution, multivariate normal, copula
- Derive marginal distribution of R_i from $p(R_i|F)$ and $p(F)$
- Simulate R_i and Calculate functional of interest
 - Unobserved performance, Sharpe ratio, ETL etc

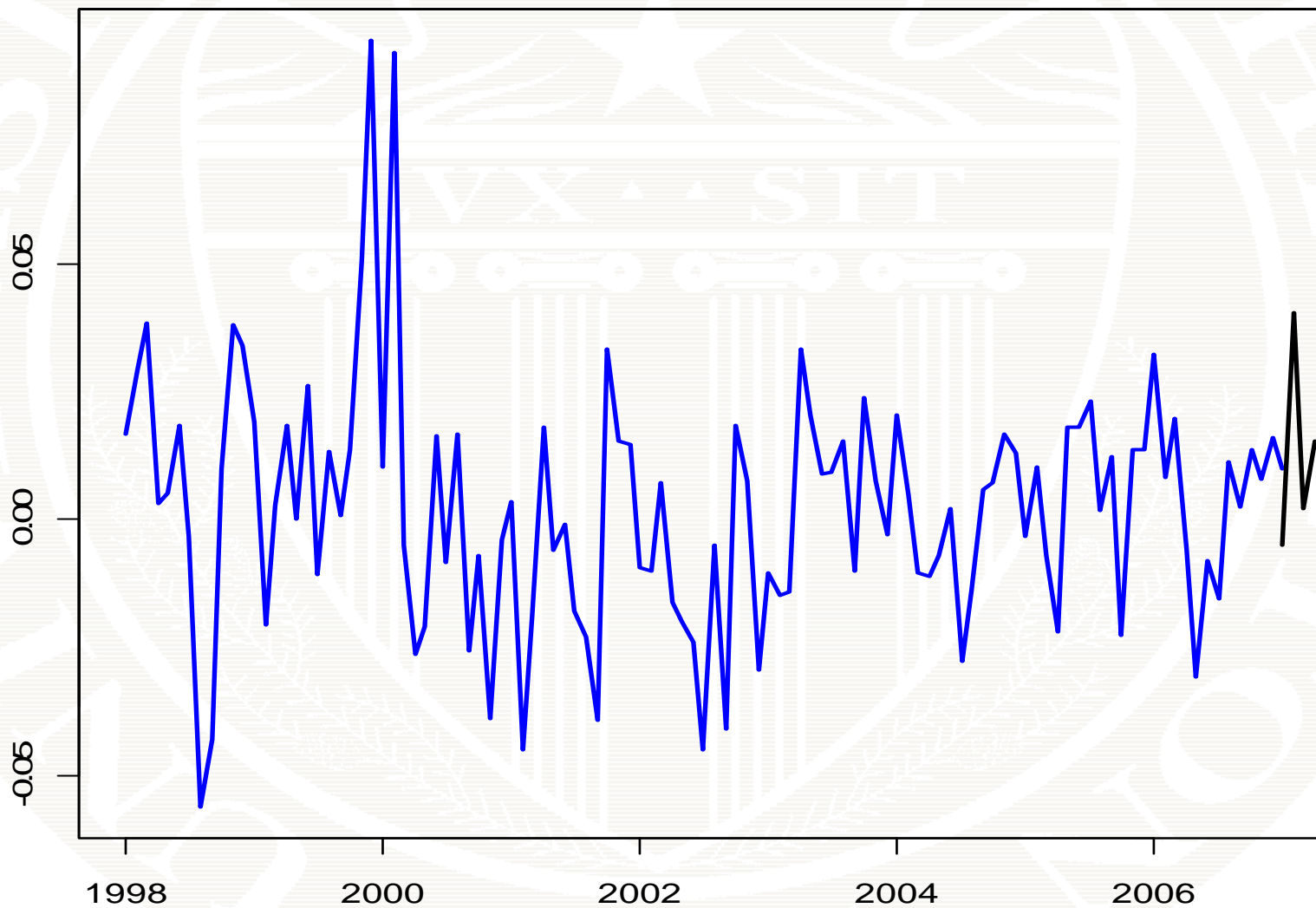
Simulation Algorithm

- Draw $\{\tilde{F}_1, \dots, \tilde{F}_M\}$ by resampling from the empirical distribution of F .
- For each \tilde{F}_u ($u = 1, \dots, M$), draw a value $\tilde{R}_{i,u}$ from the estimated conditional distribution of R_i given $F = \tilde{F}_u$ (e.g., from fitted factor model assuming normal errors)
- $\{\tilde{R}_{i,u}\}_{u=1}^M$ is the desired sample for R_i
- $M \approx 5000$

What to do with $\{\tilde{R}_{i,u}\}_{u=1}^M$?

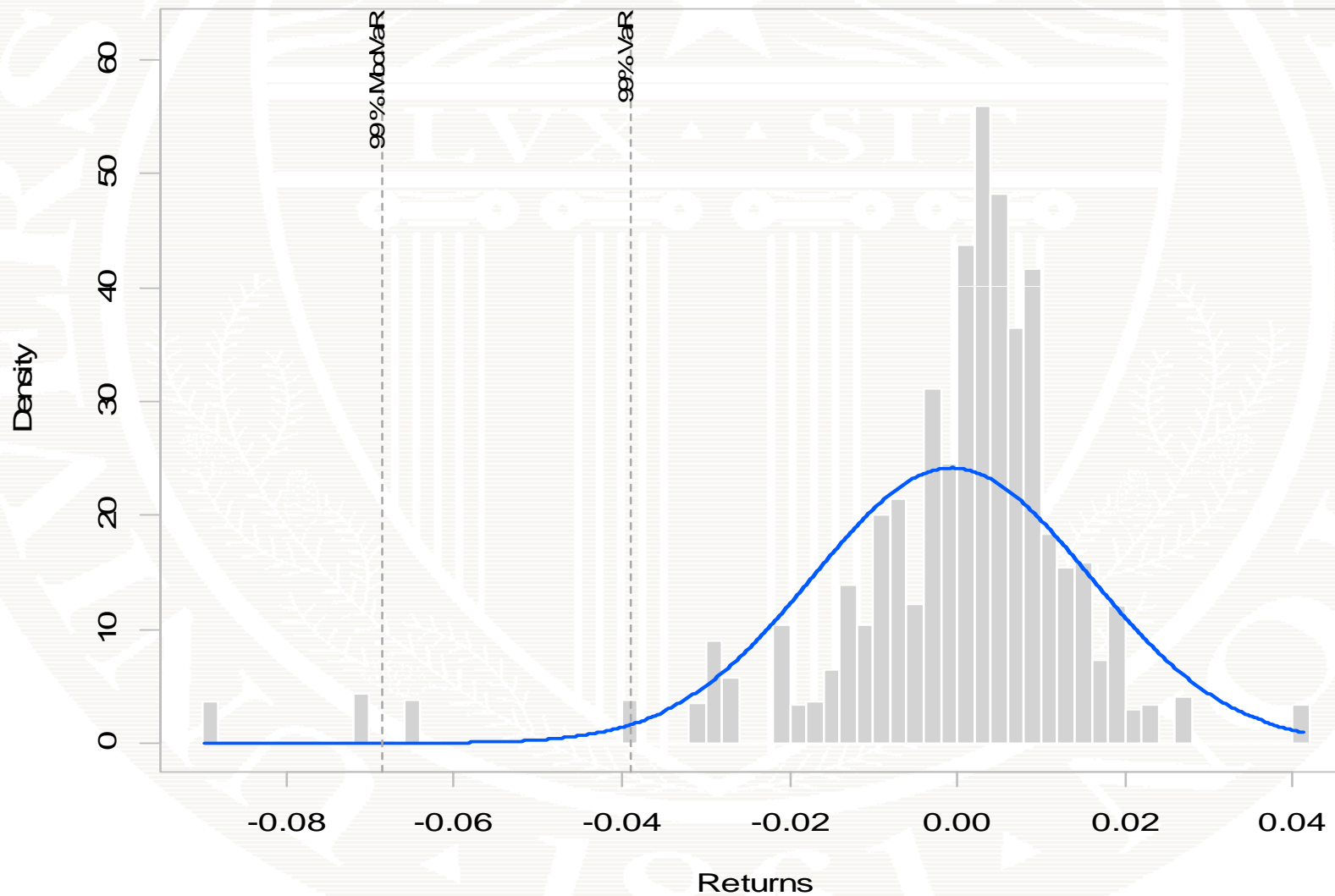
- Backfill missing fund performance
- Compute fund and portfolio performance measures
- Estimate non-parametric fund and portfolio tail risk measures
- Compute non-parametric risk budgeting measures
- Standard errors can be computed using a bootstrap procedure

Example: Backfilled Fund Performance



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Example: Simulated portfolio distribution



S-PLUS and S+FinMetrics vs R

- Dealing with time series objects in R can be difficult and confusing
 - timeSeries, zoo, xts
- Time series regression in R is incompletely implemented
 - Diagnostic plots, prediction
- R packages give about 80% functional coverage to S+FinMetrics

Some Thoughts About Using R in a Corporate Environment

- IT doesn't want to support it
- Firewalls block R downloads
- The world runs from an Excel spreadsheet
- Analysts with some programming experience learn R quickly
- Not good for the casual user

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