

A Latent Variable Approach to Validate Credit Rating Systems using R

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Motivation I

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Static Model

Empirical Example I

Dynamic Model

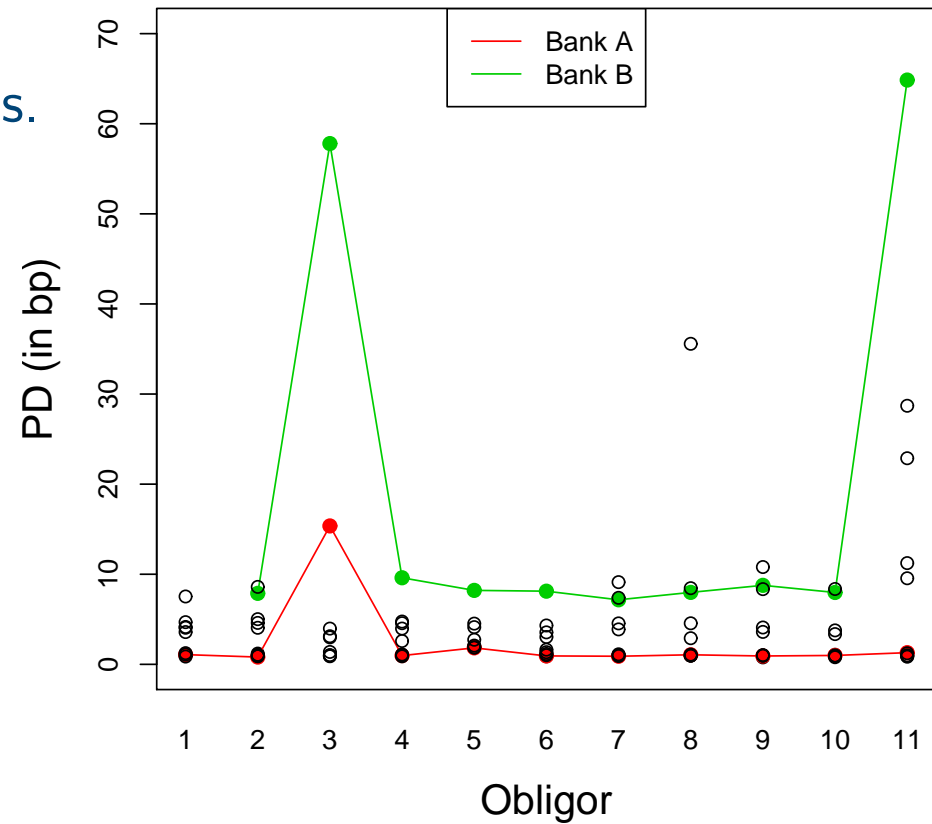
Estimation

Empirical Example II

Summary

Multi-rater panel provides information about rating heterogeneity

- ✓ across obligors,
- ✓ across raters,
- ✓ across time points.



Motivation II

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Summary

- ✓ Rating validation:
 - ✗ How accurate (or precise) are PD estimates?
 - ✗ Measures of rating bias and “error variance”.
- ✓ Consensus PD:
 - ✗ What is the consensus PD estimate when ratings differ across raters?
- ✓ Explaining rating heterogeneity:
 - ✗ Opaqueness of industries (Morgan 2002),
 - ✗ Regional differences (Hornik et al. 2009).



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Summary

- ✓ Need for a probabilistic model.
- ✓ Our Approach: PD treated as latent variable
 - ✗ Ratings are noisy observations of PD.
 - ✗ Provides a probabilistic model to directly estimate the rating error of a rater.
 - ✗ Explore patterns of heterogeneity.
 - ✗ Can obtain the “consensus rating” of the obligor.
- ✓ Requirement: Multi rater panel with rating information on the same metric scale.
 - ✗ In practice: “PD ratings” (PD estimates by different raters for all obligors).



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- ✓ Merton (1974): Distance to default of firm i

$$DD_i = a_i + b_i V_i$$

where V_i is the log asset value of firm i .

- ✓ The probability of default is given as $PD_i = \Phi(-DD_i)$ where

$$\Phi : (-\infty, +\infty) \rightarrow [0, 1]$$

is the distribution function of the standard normal distribution.

- ✓ Probit of the PD is linearly related to the log asset value.



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- ✓ The “estimate” PD_{ij} as derived by rater j for the true PD of firm i is

$$PD_{ij} = \Phi(-DD_i + \epsilon_{ij})$$

where ϵ_{ij} is the corresponding error (which depends on j in particular).

- ✓ Motivates a multi-rater model where the rater effects (“errors”) are additive on the probit (Φ^{-1}, DD_i) scale, i.e.,

$$\Phi^{-1}(PD_{ij}) = -DD_i + \epsilon_{ij} = \Phi^{-1}(PD_i) + \epsilon_{ij}$$

- ✓ Similar considerations apply for PD models based on, e.g., logistic regression.



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Summary

- ✓ Let $S_{ij} = \Phi^{-1}(PD_{ij})$ denote the *score* of the observed PD and $S_i = \Phi^{-1}(PD_i)$ the latent true PD score.
- ✓ Write μ_{ij} and σ_{ij} for the mean and standard deviation of ϵ_{ij} , respectively.
- ✓ The latent variable model can be written as

$$S_{ij} = S_i + \mu_{ij} + \sigma_{ij}Z_{ij}$$

where the standardized rating errors $Z_{ij} = (\epsilon_{ij} - \mu_{ij})/\sigma_{ij}$ have mean zero and unit variance.



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Summary



- ✓ Simple model:

$$S_{ij} = S_i + \mu_j + \sigma_j Z_{i,j}$$

- ✗ Rating errors are independent of obligors and their characteristics (e.g., creditworthiness).
- ✗ Parameters depend on raters only: $\mu_{ij} = \mu_j, \sigma_{ij} = \sigma_j$.

- ✓ Extended model:

$$S_{ij} = S_i + \mu_{g(i),j} + \sigma_{g(i),j} Z_{ij}$$

- ✗ Groups of obligors with homogeneous rating error characteristics.
- ✗ Parameters depend on rater and obligor groups:
 $\mu_{ij} = \mu_{g(i),j}, \sigma_{ij} = \sigma_{g(i),j}$.

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Summary

- ✓ Ensuring identifiability:

Average rating bias within each obligor group is zero.

$$\mathbb{E}(S_{ij}) = \mathbb{E}(S_i) + \mu_{g(i),j} \text{ with } \sum_j \mu_{g,j} = 0.$$

- ✓ Modelling marginal group effects:

$$\mathbb{E}(S_i) = \nu_{g(i)}$$

$$\text{var}(S_i) = \tau^2 \text{ or } \text{var}(S_i) = \tau_{g(i)}^2.$$



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- ✓ Mixed-effects model for the observed S_{ij} ,
 - ✗ latent true PD scores S_i enter as random effects.
- ✓ Given parametric models for the μ_{ij} and σ_{ij} and the distributions of true PD scores and standardized rating errors.
- ✓ Estimation using R:
 - ✗ Marginal maximum likelihood: nlme (ratermodel), or
 - ✗ Bayesian techniques: rjags and coda



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Summary

- ✓ Assessment of rater effects (rater heterogeneity) in terms of their μ/σ^2 bias/variance characteristics.
- ✓ Estimated random effects \hat{S}_i can be interpreted as the consensus rating for obligor i .
- ✓ Residuals: $S_{ij} - \hat{S}_i - \hat{\mu}_{ij}$, the observed scores minus the estimated random and fixed effects,
 - ✗ are analyzed using some covariates (e.g., country, industry, legal form),
 - ✗ are analyzed for single obligors as well as obligor groups of interest.



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Summary

- ✓ Data from the Austrian Credit Register from September 2007.
- ✓ Multi-rater panel containing:
 - ✗ obligor specific information (country, industry, ...),
 - ✗ rating information (original rating, PD).
- ✓ Include 13 banks which report PDs and have sufficiently many co-ratings (every bank has at least 4 co-ratings with every other bank).
 - ✗ 5432 co-ratings of 13 banks for 2090 obligors.



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Summary

✓ Variety of models analyzed using AIC and BIC.

✗ Industry affiliation as grouping variable.

✗ Constant variance τ^2 for PD score.

$$S_{ij} = S_i + \mu_{g(i),j} + \sigma_{g(i),j} Z_{ij}, \quad S_i \sim N(\nu_{g(i)}, \tau^2),$$

$$\mathbb{E}(S_{ij}) = \mathbb{E}(S_i) + \mu_{g(i),j}, \quad \sum_j \mu_{g,j} = 0,$$

where

$\mu_{g,j}$ rating bias to the mean PD score of bank j for obligors in industry g ,

$\sigma_{g,j}$ standard deviation of the rating error of bank j for obligors in industry g , and

ν_g mean PD score in industry g .



Parameter Estimates I

Industry specific means:

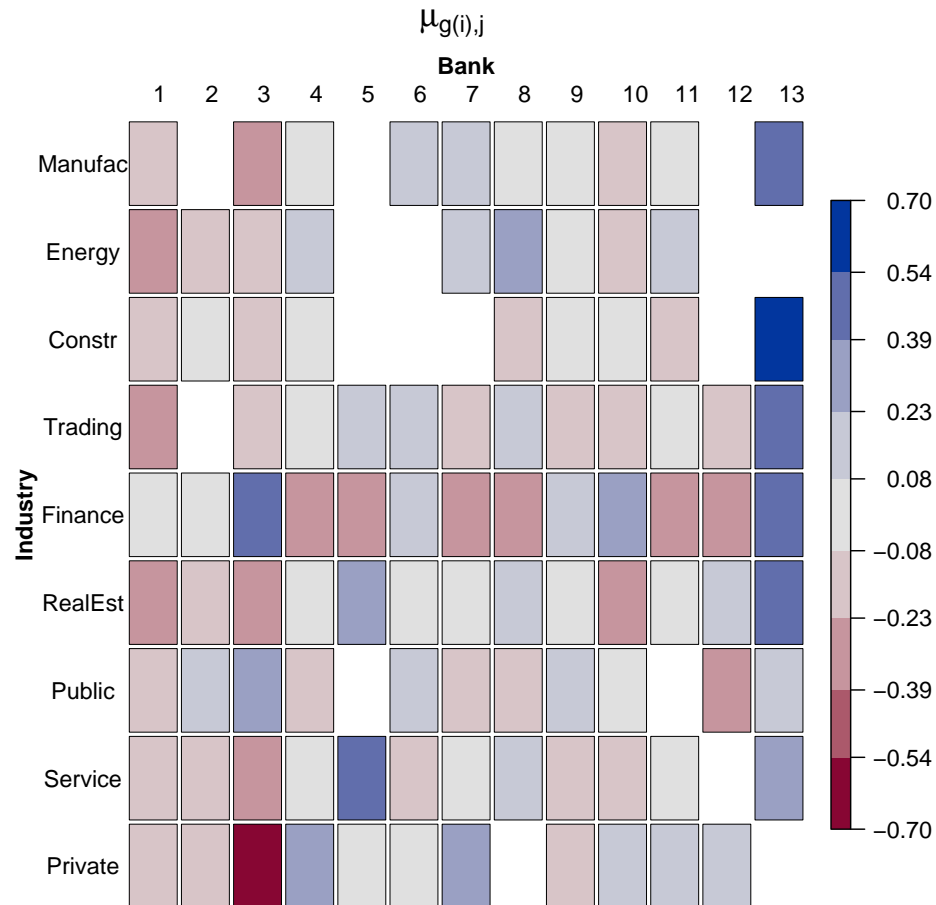
Industry	ν_g	$\Phi(\nu_g)$	$\Phi(\nu_g - \tau)$	$\Phi(\nu_g + \tau)$
Manufac	-2.542	55.1	17.7	151.1
Energy	-2.993	13.8	3.8	44.2
Constr	-2.448	71.8	23.8	190.9
Trading	-2.375	87.7	29.8	227.5
Finance	-3.256	5.6	1.5	19.2
RealEst	-2.474	66.8	22.6	174.7
Public	-3.330	4.3	1.1	15.1
Service	-2.517	59.2	19.8	157.0
Private	-2.296	108.4	40.0	267.4

The corresponding standard deviation τ is 0.357 and the PD intervals are measured in basis points (10^{-4}).



Parameters Estimates II

Rating bias for rater/industry combinations:



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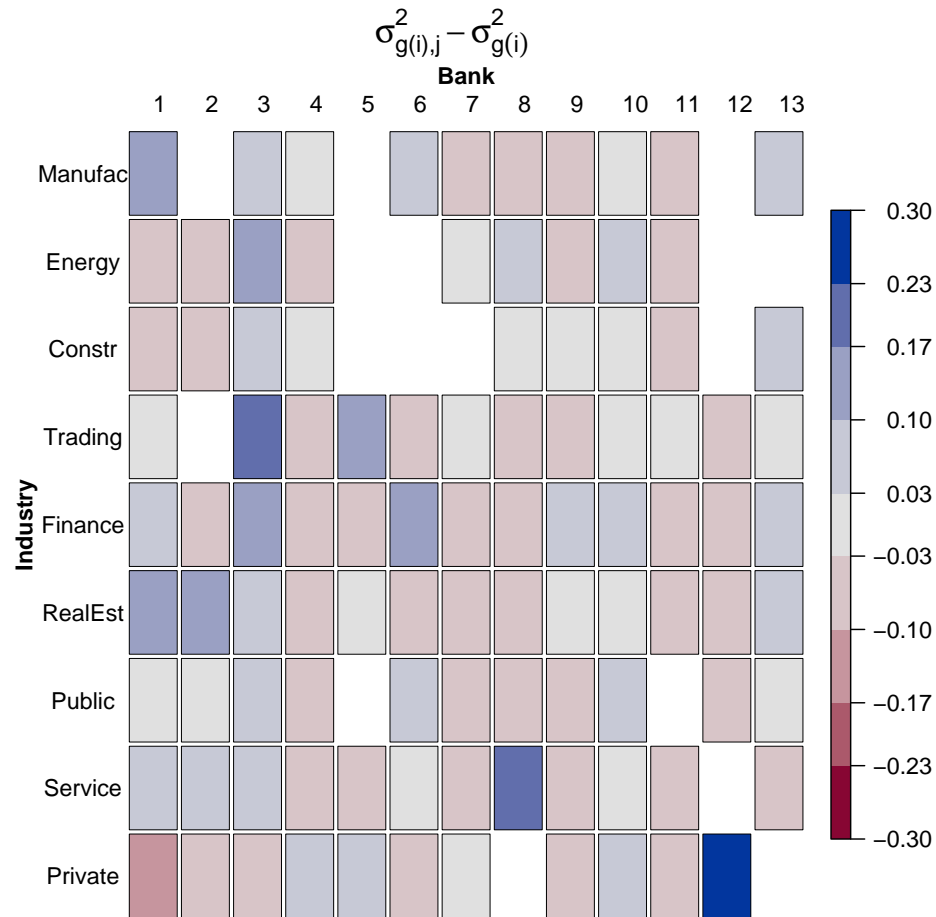
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Parameters Estimates III

Rating error variance for rater/industry combinations corrected for industry effects:



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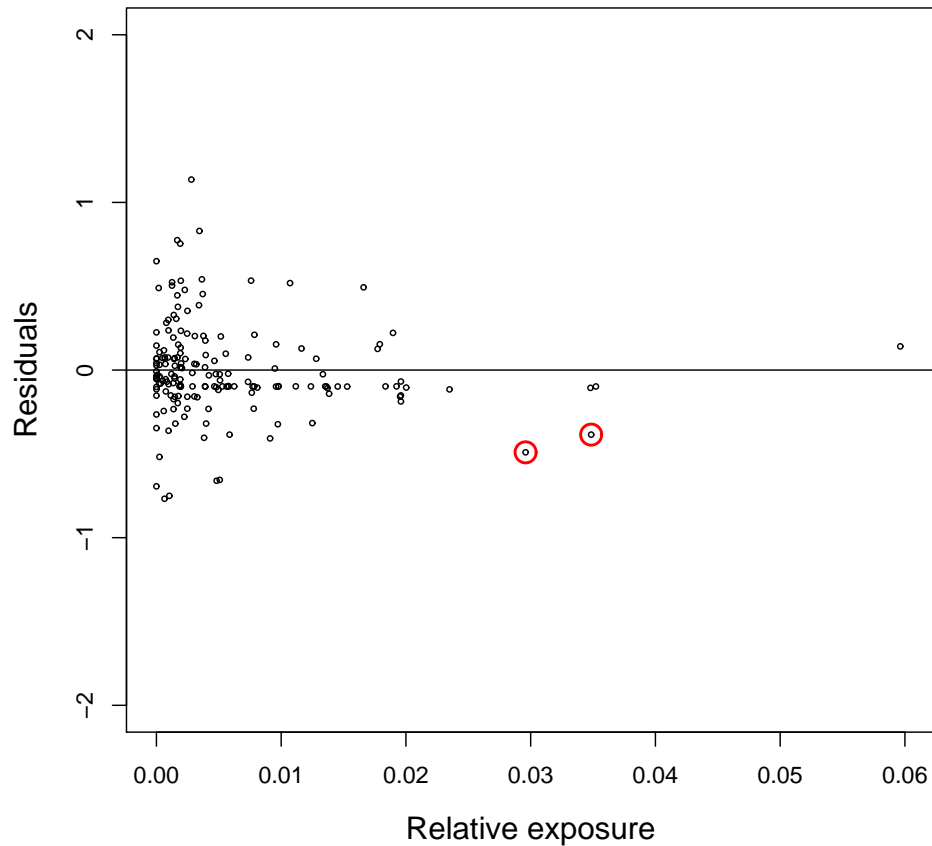
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Across the relative exposure:



This Bank rates two obligors with high relative exposure rather too favorably.



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Summary

- ✓ Formulate a statistical framework for a dynamic rating process:
 - ✗ Explaining the rating heterogeneity not only cross-sectionally but also longitudinally.
 - ✗ Intertemporal change of consensus ratings.
 - ✗ Effects of macroeconomic shocks – *systematic risk* (McNeil and Wendin, 2007).
 - ✗ Correlations in rating migrations (Stefanescu et al., 2009)
 - ✗ Evaluation of raters.

- ✓ Requirement: Multi-rater panel data with rating information on the same PD-scale.



Simple Model

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Summary

✓ Simple Model:

$$S_{ij}(t) = S_i(t) + \mu_j + \sigma_j Z_{ij}(t),$$

- ✗ Rating errors are independent of obligors and their characteristics (e.g., creditworthiness).
- ✗ Parameters depend on raters only: $\mu_{ij} = \mu_j, \sigma_{ij} = \sigma_j$.
- ✗ Latent score $S_i(t)$ is assumed to follow an AR(1) process:

$$S_i(t) = \alpha_i S_i(t-1) + \nu_i(1 - \alpha_i) + \xi_i(t),$$

where $S_i(0) = \nu_i + \xi_i(0)$, $\nu_i \sim N(\nu, \tau^2)$ and $\xi_i(t) \sim N(0, \phi^2)$.

- ✗ AR(1) process induces auto-correlation in the latent credit worthiness of obligor i .
- ✗ Latent scores $S_i(t)$ are uncorrelated across obligors.



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Summary

- ✓ Let $f(t)$ denote a market factor, latent score $S_i(t)$ can then be written as

$$S_i(t) = \alpha_i S_i(t-1) + \nu_i(1 - \alpha_i) + \beta_{1,i} f_1(t) + \beta_{2,i} f_2(t) + \xi_i(t),$$

- ✗ $S_i(0) = \nu_i + \xi_i(0) + \beta_{1,i} f_1(0)$.
- ✗ $\beta_{j,i} \sim N(\mu_\beta, \sigma_\beta^2)$, ($j = 1, 2$) denotes cross sectional correlation coefficient of scores $S_i(t)$.
- ✗ Assume $f_1(t)$ follows AR(1) process
 $f_1(t) := \gamma f_1(t-1) + \zeta(t)$, $f_1(0) = \zeta(0) / \sqrt{1 - \gamma^2}$, and
 $f_2(t) \sim N(0, 1) \quad \forall t$ to simulate market volatility shocks.



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Summary

- ✓ Assessment of rater effects (rater heterogeneity) in terms of their μ/σ^2 bias/variance characteristics.
- ✓ Estimated random effects $\hat{S}_i(t)$ can be interpreted as the consensus rating for obligor i over time.
- ✓ Market correlation structure.
 - ✗ Can the market explain the correlation structure between the obligors?



Bayesian Approach

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Bayesian Approach

rjags

Bayesian Inference
using R

model.bug

Output diagnostic

Empirical Example II

Summary

- ✓ Both, *prior* and *likelihood* are used to derive the *posterior*
- ✗ $p(\theta|y) \propto p(y|\theta)p(\theta)$.
- ✓ *Informative* and *non-informative* priors $p(\theta)$
 - ✗ Prior denotes beliefs about parameter before observing data.
 - ✗ Allows for expert opinions through the use of informative priors → informative prior.
 - ✗ All information resulting in a prior arose from data → noninformative prior.
- ✓ Gibbs sampling is used to derive posterior distribution.



rjags

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Summary

- ✓ The R package, rjags permits a direct interface from R to the main JAGS library (Plummer, 2008).
- ✗ `coda.samples(model, var, n.iter, thin = 5)` generates posterior sample in `mcmc.list` object
- ✗ Function to extract random samples of the penalized deviance from jags model (`dic.samples(model, n.iter, thin =5, type)`).
- ✗ includes functions for model selection (`diffdic(model1,model2)`).



Bayesian Inference using R

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Summary

- ✓ A JAGS model is defined by
 1. A model description (in a file `model.bug`).
 2. The data (a list of vectors/matrices or arrays).
 3. Optional, a set of initial values for the chain(s).

- ✓ `jags.model()` is used to create an Bayesian graphical model

```
> modeldyn <- jags.model(file = "model.bug", data = ratings,  
                          inits =ratingsinit, n.chains = 4)
```

```
Compiling model graph  
  Declaring variables  
  Resolving undeclared variables  
  Allocating nodes  
Graph Size: 23498
```



model.bug

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Summary

```
model
{
  f[1] <- 0;
  sf[1] <- 0;
  for (t in 2:T) {
    f[t] ~ dnorm(gamma*f[t-1], 1); sf[t] ~ dnorm(0, 1);
  }
  for (i in 1:N) {
    v[i] ~ dnorm(nu, tau);
    m[i,1] ~ dnorm(v[i] + beta[i] * f[1]+ sfbeta[i] * sf[1], sigma.i);
    for (t in 2:T) {
      m[i,t] ~ dnorm(alpha[i] * m[i,t-1] + beta[i] * f[t] + sfbeta[i]
        * sf[t]+ (1-alpha[i]) * v[i], sigma.i);
    }
    for (j in 1:J) {
      for (t in 1:T) {
        y[i,j,t] ~ dnorm(m[i,t] + mu[j], sigma[j]);
      }
    }
  }
  alpha[i] ~ dnorm(a, A); beta[i] ~ dnorm(b, B); sfbeta[i] ~ dnorm(c,C);
}
sigma.i ~ dgamma(1.0E-3, 1.0E-3);
a ~ dnorm(0, 1.0E-6)T(-1.0,1.0);      A ~ dgamma(1.0E-3, 1.0E-3);
b ~ dnorm(0, 1.0E-6);                B ~ dgamma(1.0E-3, 1.0E-3);
c ~ dnorm(0, 1.0E-6)T(-1.0,1.0);     C ~ dgamma(1.0E-3, 1.0E-3);
nu ~ dnorm(0, 1.0E-6);                tau ~ dgamma(1.0E-3, 1.0E-3);
gamma ~ dnorm(0, 1.0E-6)T(-1.0,1.0);
for (j in 1:J) {
  mu[j] ~ dnorm(0, 1.0E-6);
  sigma[j] ~ dgamma(1.0E-3,1.0E-3);
}
}
```



Output diagnostic

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Summary

- ✓ Output diagnostic can be done with R package coda (Convergence Diagnosis and Output Analysis, Plummer et al., 2008).
- ✗ coda package includes functions for convergence tests of MCMC chains.
 - ✓ `autocorr(x, lags = c() ,)` calculates the autocorrelation for the MCMC chain (`x = mcmc.obj`) at the lags `lags`.
 - ✓ `gelman.diag(x,)` calculates the scale reduction factor for a variable `x`.
 - ✓ `geweke.diag(x,)` can be used for Geweke diagnostic.



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- ✓ External monthly rating data from the Big Three rating agencies Standard & Poor's, Fitch and Moody's.
- ✓ Time period: February 2007 to January 2009
- ✓ This study: 87 corporates out of the 125 corporates included in the iTraxx Europe index (Series 10)
 - ✗ iTraxx: most-liquid CDS referencing European investment-grade entities.
- ✓ Source: Historical rating announcements taken from Reuters Credit Views.

Mapping of ordinal ratings

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- ✓ Rating agencies provide ratings only on a ordinal scale
- ✗ Unclear whether PD or EL is measured.
- ✗ Unclear which time horizon.
- ✓ Long history of default rates available.
- ✓ However, agency ratings are widely used as PD-equivalents by the industry (Erlenmaier, 2006; Tasche, 2008).
- ✓ This study: Mapping of ordinal ratings to PD provided by OENB.
- ✓ **Note:** All results of this study are conditional on used mapping procedure.
- ✗ Quality of rating system can not be separated from mapping (in particular bias).

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Summary



- ✓ Variety of models analyzed using deviance information criteria.
- ✗ Simple dynamic model.
- ✗ Dynamic market model including the EUROSTOXX50 as an exogenous market factor.
- ✗ Dynamic market model with a market factor following an AR(1) process.
- ✗ Dynamic market model with $f(t) = (f_1(t), f_2(t))$ with $f_1(t)$ as an AR(1) process and $f_2(t) \sim N(0, 1)$
- ✓ Dynamic market model with two market factors performed best.

Convergence Diagnostic – Trace Plot

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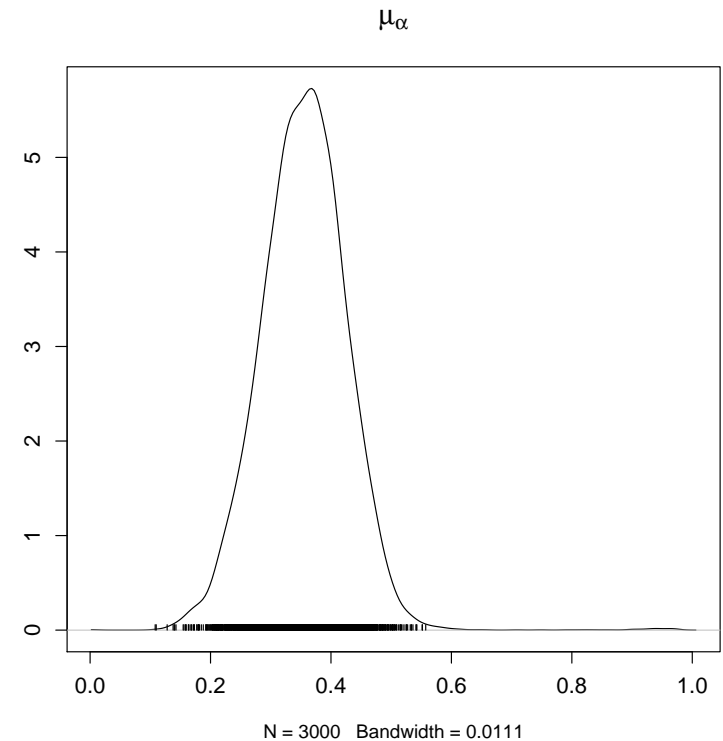
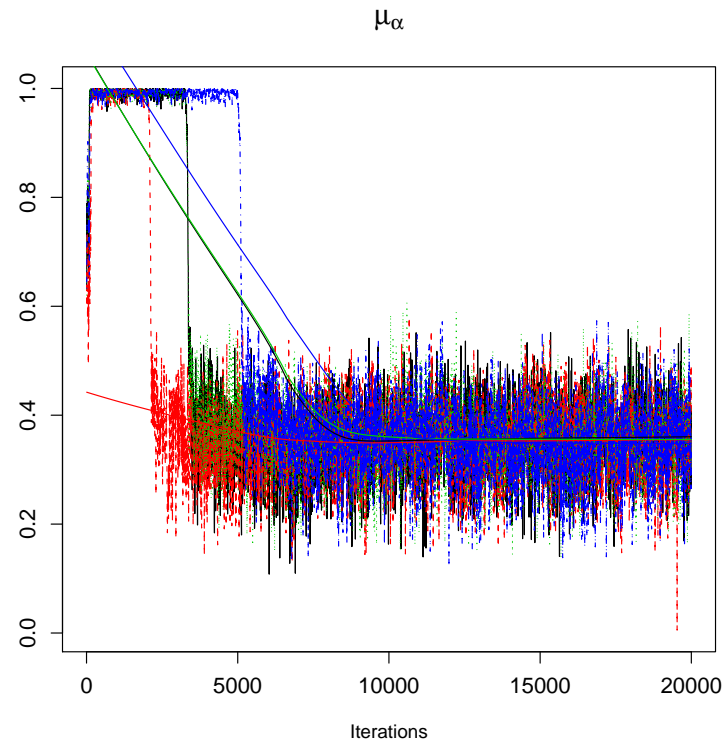
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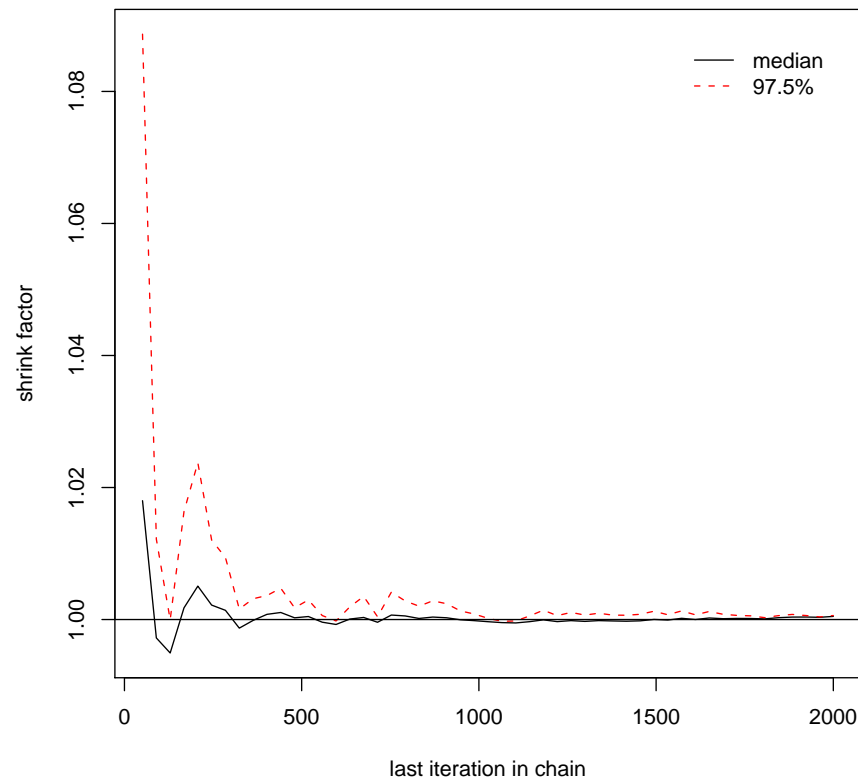
✓ **Trace plot** for parameter μ_α .

- ✗ Trace plot is plot of the iteration number against the value of drawn of parameter for each iteration
- ✗ **Burn-in** period, about 5000 iterations



Convergence Diagnostic – Gelman Rubin Diagnosis

- ✓ Calculate the estimated variance of a parameter as a weighted sum of the within-chain W and between-chain variance B .
- ✗ **Gelman Rubin** factor R is a ratio of W and B .



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Rating Errors

Rating bias and rating error variance for the three raters:

μ	Fitch	Moody's	S&P
mean	0.0298 (0.55 bp)	-0.1085 (-1.59 bp)	0.0787 (1.57 bp)
SD	0.0031	0.0044	0.0024

Bps relative to the grand mean ν (-3.295 , in bps: 5.01).

σ	Fitch	Moody's	S&P
mean	0.1443	0.1987	0.0156
SD	0.0025	0.0045	6e-04

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Consensus I

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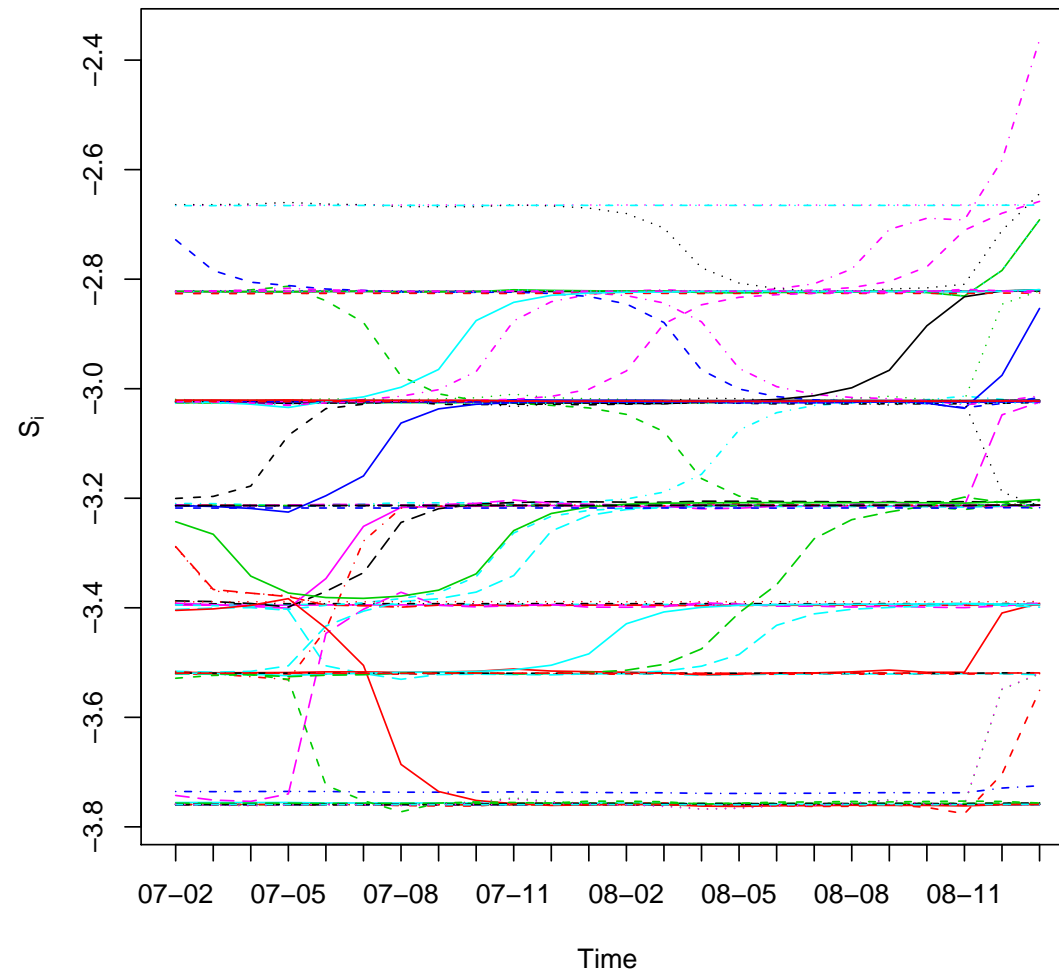
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Consensus II

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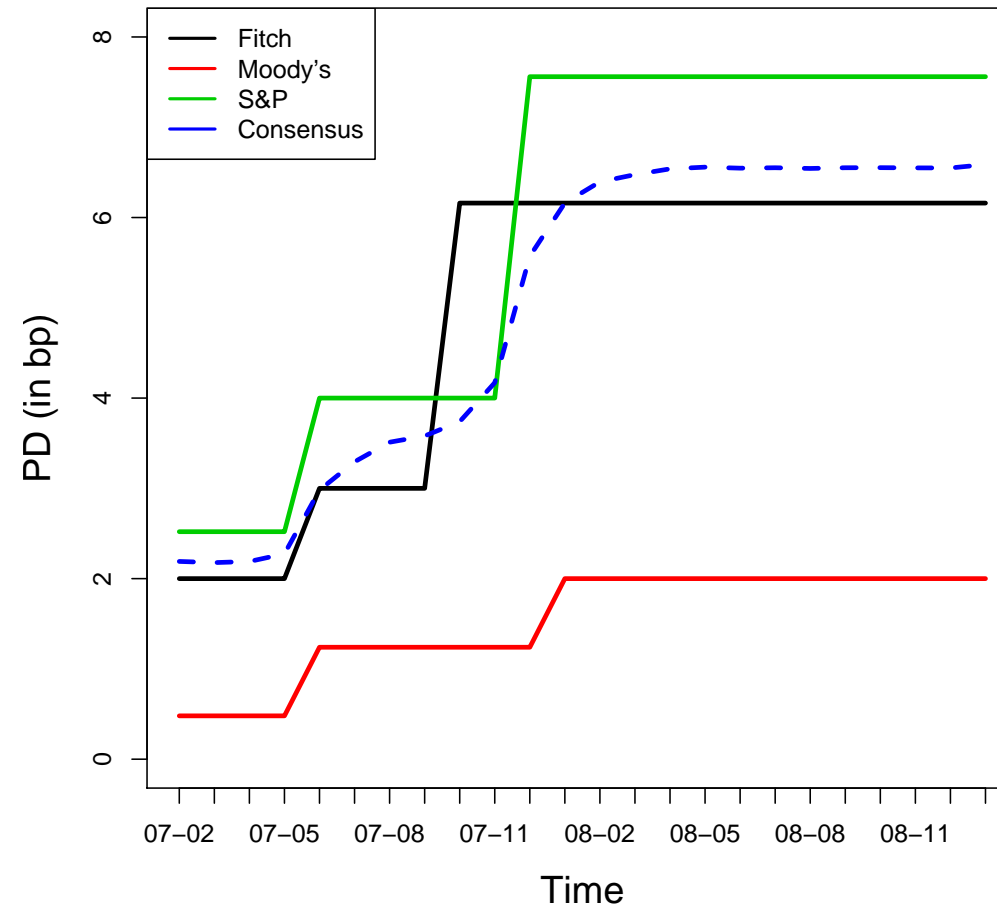
Consensus II

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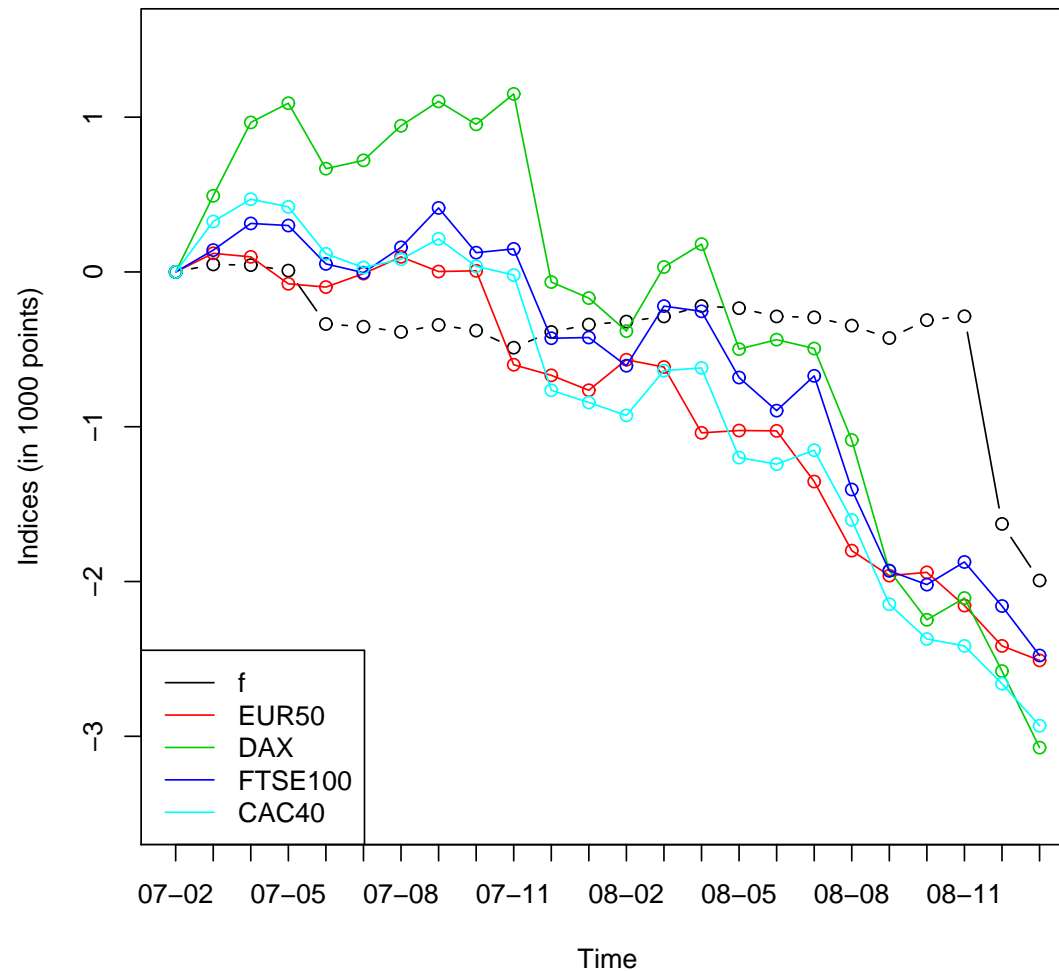


Enel SpA



Market Factor

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- ✓ This methodology can be used to make fundamental statements about the heterogeneity of raters of a multi-rater panel.
- ✓ The “consensus rating” of obligors can be estimated.
- ✓ Using the consensus rating a residual analysis can be performed for single obligors as well as obligor groups of interest.
- ✓ The effect of a market factor on the consensus can be estimated.

Requirement: The number of co-ratings within a subgroup must be sufficiently large.

