Risk Capital for Interacting Market and Credit Risk: VEC and GVAR Models using R

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Outline

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Regulation of Market and Credit Risk: the traditional view

- In the work of the Basel Commitee there has been a tradition to distinguish market from credit risk and to treat both categories independently in the calculation of risk capital.
- Leaving aside operational risk, Pillar 1 of Basle II requires separate regulatory capital for market and credit risk:

$$RC = RC_c + RC_m$$

- In our paper[1] we argue that this approach is problematic and that it can lead to significant underestimation of risk.

[1] Breuer, Jandacka, Rheinberger, Summer: *Does Adding Up of Economic Capital for Market- and Credit Risk amount to Conservative Risk Assessment?*, Journal of Banking and Finance, in print, 2009

Integrated vs. separate analysis of market- and credit risk

- Market risk is defined as the risk that a financial position changes its value due to changes of underlying market risk factors, like a stock price, an exchange rate or an interest rate.
- Credit risk is defined as the risk of not receiving the promised payment on an outstanding claim, i. e., due to changes of underlying credit risk factors, like default probabilities.

However

Some risk factors may influence both market and credit risk, e. g., interest rates are market prices which influence default probabilities.

A formalization of separated and integrated risk analysis

- Assume market risk factors are described by a vector $e \in E$ and credit risk factors by a vector $a \in A$.
- The value of the portfolio in dependence of a and e is given by a function $v : A \times E \longrightarrow \mathbb{R}$.
- Market risk is the value change due to moves in market risk factors, assuming that credit risk factors are constant:

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0).$$

- Credit risk is defined analogously as:

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0).$$

- Integrated risk is the value change caused by simultaneous moves of market and credit risk factors:

$$\Delta v(a, e) := v(a, e) - v(a_0, e_0).$$

Approximation of regulatory capital by adding up categories

In a separate analysis of market and credit risk regulatory capital for market and credit risk are simply added up. This amounts to assuming that integrated risk is approximately the sum of market risk plus credit risk:

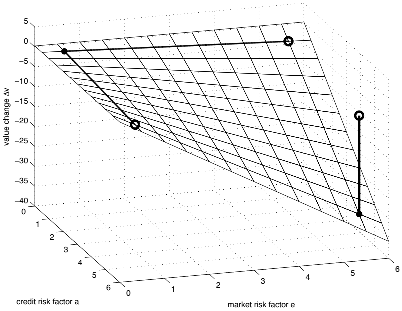


Figure: $V(a,e) = -a \cdot e$

$$
\begin{aligned}
\Delta v(a,e) &\approx \Delta c(a) + \Delta m(e) \\
v(a,e) &\approx v(a_0, e_0) + \Delta c(a) + \Delta m(e) \\
&=: \hat{v}(a,e)
\end{aligned}
$$

Approximation errors can go both ways

Our main message is that the approximation $\Delta c(a) + \Delta m(e)$ not always overestimates but sometimes underestimates the true integrated Δv. If in some scenario (a, e) the approximation error

$$\begin{aligned} d(a,e) &:= \Delta v(a,e) - \Delta c(a) - \Delta m(e) \\ &= v(a,e) - \hat{v}(a,e) \end{aligned}$$

is negative, we have a malign risk interaction. If d is non-negative in all scenarios, we have a benign interaction effect.

A classification of portfolios where the approximation is exact

Proposition

The approximation is exact, that is

$$\Delta v(a, e) = \Delta c(a) + \Delta m(e),$$

if and only if v has the form

$$v(a, e) = v_1(a) + v_2(e). \tag{1}$$

for some functions v_1, v_2. In this case the portfolio is separable into two subportfolios, one depending only on credit risk factors, the other depending only on market risk factors.

In particular **linear** portfolio value functions fulfil condition (1).

Measuring the effects of integrated risk analysis

- Given a probability measure on the space $A \times E$ of scenarios the value function v can be assigned a risk capital by

$$\mathrm{RC}_\alpha(X) := \mathrm{E}(X) - \mathrm{ES}_\alpha(X),$$

where ES_α is Expected Shortfall at some confidence level α

- We measure the integration effect by the index:

$$\mathrm{I}_{rel} := \frac{\mathrm{RC}_\alpha(\Delta v)}{\mathrm{RC}_\alpha(\Delta c) + \mathrm{RC}_\alpha(\Delta m)}.$$

I_{rel} measures the percentage amount by which total risk exceeds the sum of market and credit risk.

A real world example

foreign currency loan portfolio

This form of mortgage financing has been especially popular in Austria and in Central and Eastern Europe and raised the concern of supervisory authorities.

- PD may depend on market risk factors
  Interest rate increase drives up PD.
  FX rate increase drives up PD
- EAD depends on market risk factors
- LGD depends on market risk factors
  Increased defaults trigger fire sales of collateral
  and market prices of collateral fall

Outline

Time series models for risk factors

To model the probability law of risk factors we use the GVAR time series model due to (Pesaran, Schuermann, Weiner 2006).

- The GVAR model is an **error correction model** that allows a parsimonious modelling of economic interdependence between countries or regions.
- We estimate a GVAR model for **Switzerland**, **Austria** and include their three most important trading partners **Germany**, **Italy** and **France** as well as the most important trading partner of Germany, the **US**.
- The variables we consider for each country are **real GDP**, the **three month LIBOR interest rate**, and the **exchange rate** to the US dollar.
- historic data from 1986 Q3 to 2005 Q4, quarterly.

Modelling of Non-Stationary Multivariate Time Series

- VAR(k) ... **V**ector **A**uto-**R**egressive model of order k:

$$Y_t = A_1 Y_{t-1} + \ldots + A_k Y_{t-k} + \Phi D_t + \epsilon_t,$$

  where the deterministic terms ΦD_t can include constant and linear parts and $\epsilon_t \sim N_p(0, \Omega)$. It can be rewritten as a

- VECM(k-1) ... **V**ector **E**rror **C**orrection **M**odel of order $k-1$

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{k-1} \Delta Y_{t-k+1} + \Phi D_t + \epsilon_t \quad (2)$$

If all components of Y_t are integrated of order ≤ 1 then all terms in (2) are stationary.

Global VAR (GVAR) Modelling

GOAL: Model *m* variables in each of *N* countries including interactions.

- With a VAR(k) or a VECM(k-1) model approx. $(Nm)^2(k+1)$ parameters would have to be estimated – too many for short historic data.
- **GVAR**: For each country estimate a VECM with *m* endogenous (national) and *m* weakly-exogenous (foreign, trade-weighted) variables.
- The *N* VEC models can then be combined to a VAR model including all countries.

VECMs for each country

- We consider for each of N countries an n-vector of random domestic variables $y_t^i, t \in \mathbb{Z}, i \in \{1, \ldots, N\}$.
- The strengths of trade relationships between a country i and all other countries $j \neq i$ are used as weights $\omega_{ij} \geq 0, \omega_{ii} = 0, \sum_{j=1}^N \omega_{ij} = 1$ to build the m foreign variables:

$$x_t^i := \sum_{j \neq i} \omega_{ij} y_t^j.$$

- For each country i a weakly exogenous VECM is estimated for

$$z_t^i := (y_t^i, x_t)^T,$$

where the foreign variables x_t^i are treated as weakly exogenous and the domestic variables y_t^i are treated as endogenous.

Trade Weigths for our GVAR model

Trade Weigths

	US	AT	FR	DE	IT	CH
US	0	0.03	0.24	0.46	0.18	0.09
AT	0.07	0	0.07	0.65	0.13	0.08
FR	0.19	0.03	0	0.46	0.24	0.08
DE	0.25	0.14	0.29	0	0.2	0.11
IT	0.16	0.06	0.29	0.39	0	0.09
CH	0.15	0.06	0.17	0.45	0.16	0

the GVAR model for all countries

Stacking all country variables y_t^i upon each other

$$y_t = (y_t^1, \dots, y_t^N)^T,$$

the corresponding VECMs can be written as

$$Gy_t = c_0 + c_1 t + \sum_{k=0}^{r} H_k y_{t-k} + u_t, \tag{3}$$

The square matrix G is non-singular, such that equation (3) can be multiplied by G^{-1} from the left to yield the GVAR model

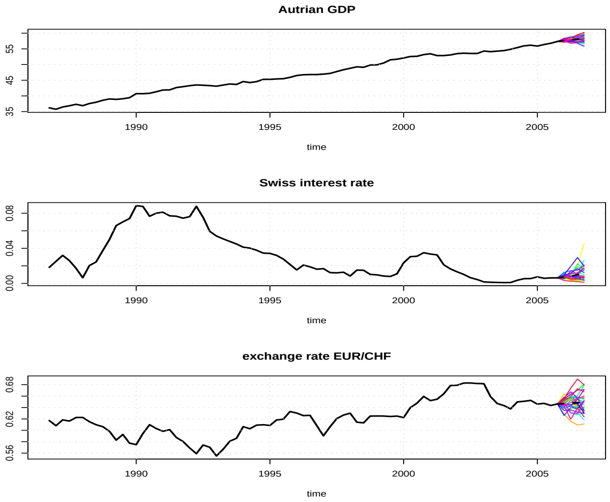
$$y_t = \tilde{c}_0 + \tilde{c}_1 t + \sum_{k=0}^{r} \tilde{H}_k y_{t-k} + \tilde{u}_t.$$

forthcoming VEC and GVAR models R-package

The R code is maintained by Rainer Puhr (Oesterreichische Nationalbank, Austria) and will finally include:

- tests for identifying **unit roots**, **cointegration ranks**, **model characteristics** and **weak exogeneity** of variables
- functions for **scenario generation** in VECMs with or without exogenous I(1) variables and GVAR models
- functions to identify, imply and test **restrictions on the model parameters**
- functions for **impulse response analysis**

Scenarios of risk factors



Outline

Modelling the profit function of foreign currency loans

- N obligors indexed by $i = 1, \ldots, N$; time horizon of 1 year.
- Customer's payment obligation to the bank at time 1 in home currency is

$$o_i = l_i\,(1+r)\frac{f(1)}{f(0)} + l_i\ s\ \frac{f(1)}{f(0)}.$$

- The payment ability of obligor i is distributed according to

$$a_i(1) = a_i(0)\cdot \frac{GDP(1)}{GDP(0)}\cdot \epsilon_i, \quad \text{with } \log(\epsilon_i) \sim N(\mu, \sigma)$$

  where $a(0)$ is a constant, $E(\epsilon_i) = 1$, and the ϵ_i are independent.
- The profit which the bank makes with obligor i is

$$v_i := \min(a_i, o_i) - l_i(1+r)\frac{f(1)}{f(0)}.$$

Calibrating the integrated risk model to a rating system

- Let p_i be the annual default probability

$$p_i = P[a_i(\sigma) < o_i(s)]$$

- Spreads are set to achieve some target expected profit for each loan:

$$\mathrm{E}(v_i(\sigma, s)) = \mathrm{EP}_{\mathrm{target}},$$

  where v_i is the profit with obligor i and $\mathrm{EP}_{\mathrm{target}}$ is some target expected profit.
- The two free parameters σ and s are determined from these two conditions.

Risk factors and countries

For our foreign currency loan portfolio model

- the market risk factors are $e := (r_f, f(1))$,
- the credit risk factors are $a := (GDP, (\epsilon_i)_{i=1,\ldots N})$,
- the portfolio value function is $v = \sum_{i=1}^{N} v_i$, and
- the countries are Austria (GDP) and Switzerland (r_f and $f(1)$).

Portfolio and Monte Carlo Simulation

Setting

- $N = 100$ loans of $l_i = 10\,000$ EUR taken out in CHF by customers in the rating class B+ ($p_i = 2\%$), or in rating class BBB+ ($p_i = 0.1\%$).
- Bank extends loans only to customers with $a_i(0)$ equal to 1.2 times the loan amount.
- Profit distribution calculated by a Monte Carlo simulation of 100 000 draws from the distribution of market and macro risk factors $f(1)$, $GDP(1)$, and r_f. In each macro scenario defaults of the customers' payment abilities were determined by draws from the distribution of the payment ability process.

Risc capital of different risks

rating	α	RC(Δm)	RC(Δc)	RC(Δv)	I_{rel}
BBB+	10%	1 059	0	1 193	1.13
BBB+	5%	1 234	0	1 522	1.23
BBB+	1%	1 576	0	3 056	1.94
BBB+	0.5%	1 698	1	4 641	2.73
BBB+	0.1%	1 951	3	16 076	8.22
B+	10%	1 102	795	2 711	1.43
B+	5%	1 285	1 022	4 420	1.92
B+	1%	1 641	1 523	11 201	3.54
B+	0.5%	1 768	1 730	15 658	4.48
B+	0.1%	2 032	2 257	32 568	7.59

Conclusions

- The traditional approach of treating market and credit risk separately in current regulation is problematic because many portfolios are not separable along these categories.
- Current practice of determining regulatory capital is conservative only if separable subportfolios can be constructed.
- If portfolio positions depend simultaneously on both market and credit risk factors a separation into subportfolios of market and credit risk leads to wrong valuation and as a consequence to wrong assessment of the true portfolio risk.
- A proper model of credit risk has to take into account all risk factors which have an effect on default losses.