R Tools for Portfolio Optimization

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Backgrounder

- Rotella Capital Management
  - Quantitative Research Analyst
    - Systematic CTA hedge fund trading 80+ global futures and foreign exchange markets
- Insightful Corporation
  - Director of Financial Engineering
    - Developers of S-PLUS®, S+FinMetrics®, and S+NuOPT®
- J.E. Moody, LLC
  - Financial Engineer
    - Futures Trading, Risk Management, Business Development
- OGI School of Engineering at Oregon Health & Science University
  - Adjunct Instructor
    - Statistical Computing & Financial Time Series Analysis
- Electro Scientific Industries, Inc
  - Director of Engineering, Vision Products Division
    - Machine Vision and Pattern Recognition
- **Started Using R in 1999**

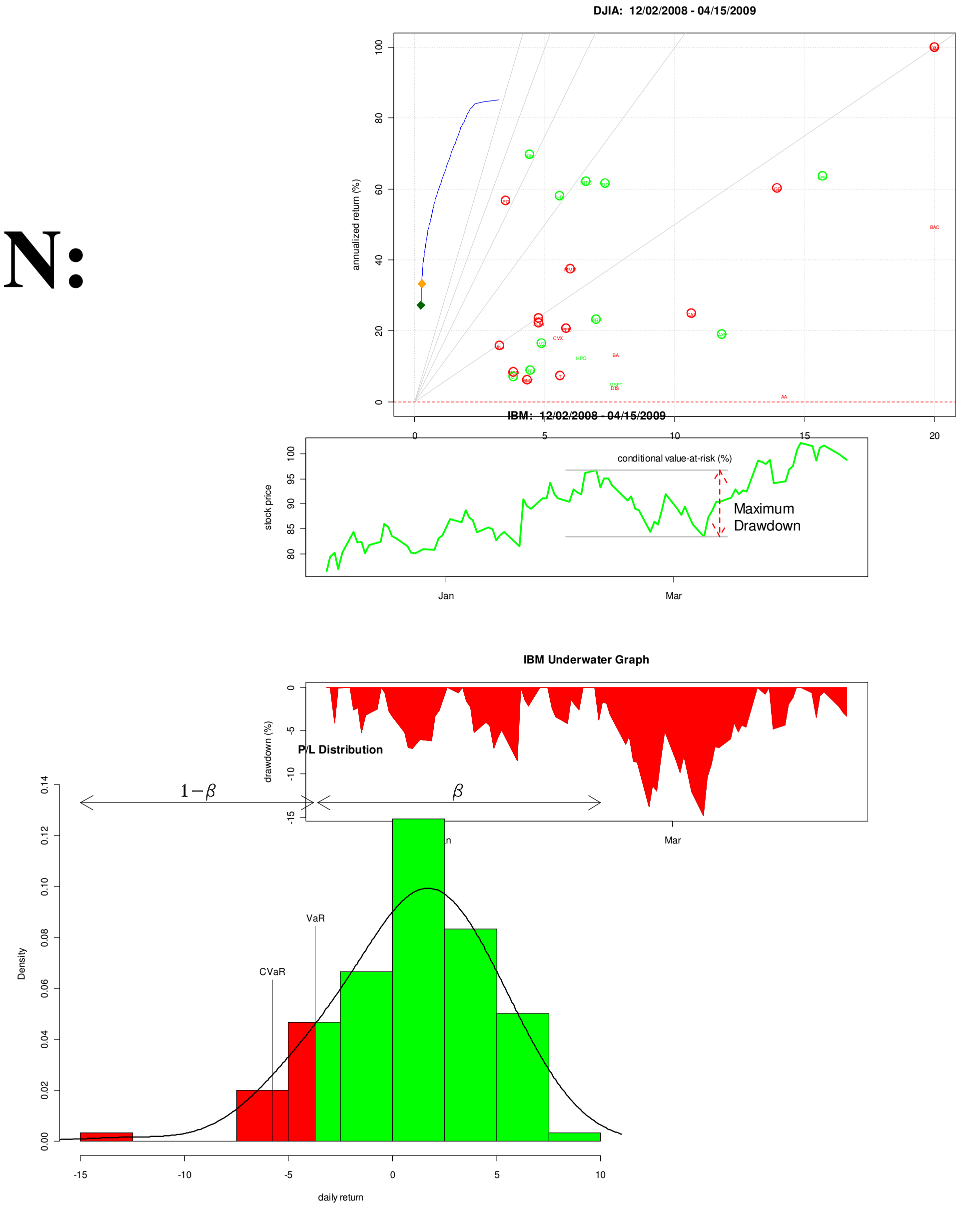
Introduction

R-SIG-FINANCE QUESTION:

Can I do < fill in the blank > portfolio optimization in R?

ANSWER:

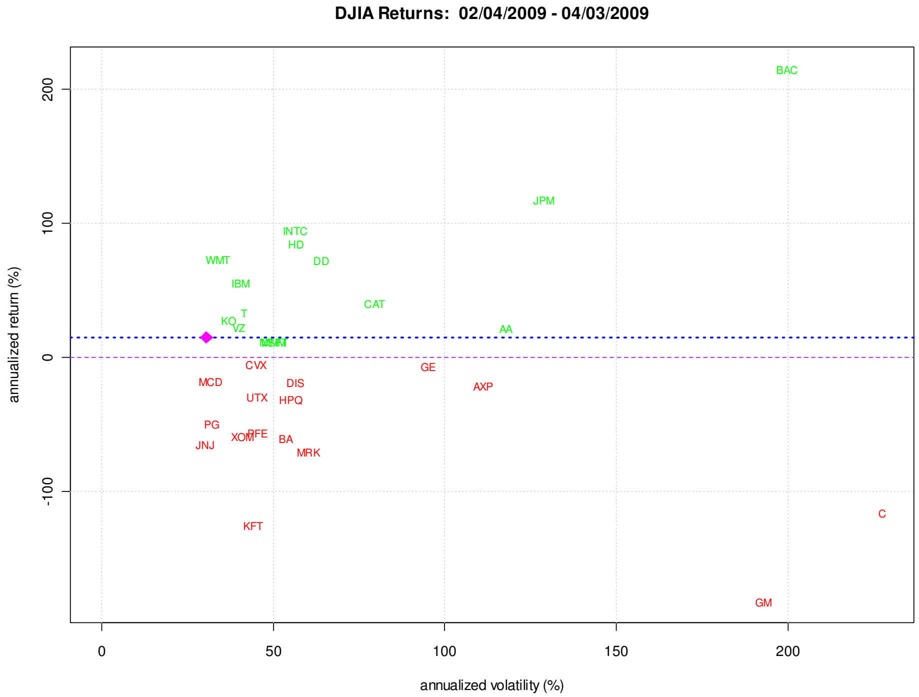
Yes! (98% confidence level)

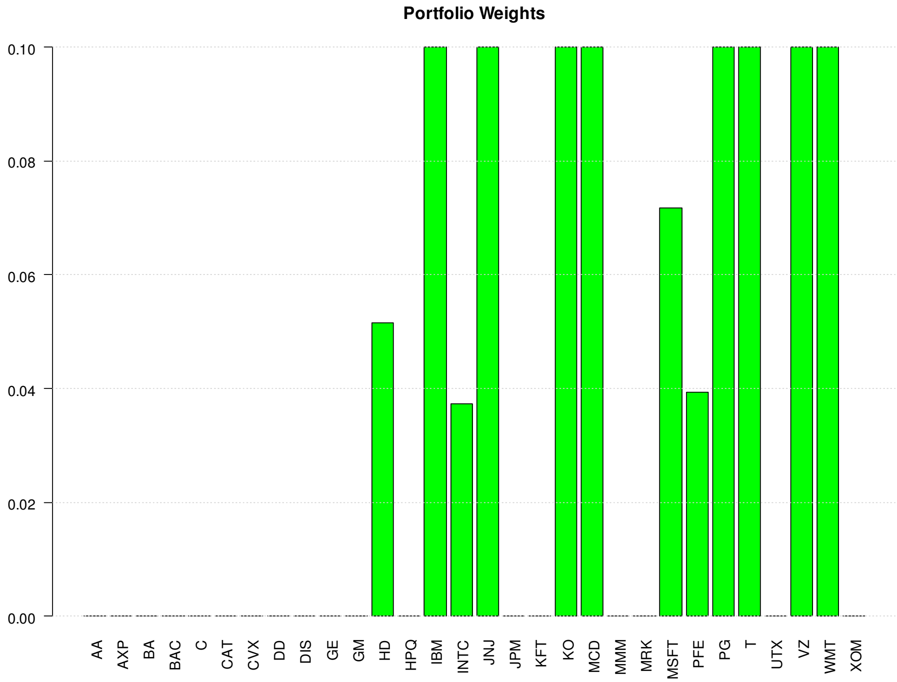


Outline

- Mean-Variance Portfolio Optimization
  - quadratic programming
    - tseries, quadprog
- Conditional Value-at-Risk Optimization
  - linear programming
    - Rglpk_solve_LP package
- General Nonlinear Optimization
  - Differential Evolution Algorithm
    - DEoptim package
      - Omega Optimization
      - Adding Constraints
      - Maximum Drawdown Optimization
      - R-Ratio Optimization
- Wrap-Up

Efficient Portfolio Solution





Mean-Variance Portfolio Optimization

- Function
  - portfolio.optim {tseries}
- Description
  - computer mean-variance efficient portfolio
- Usage

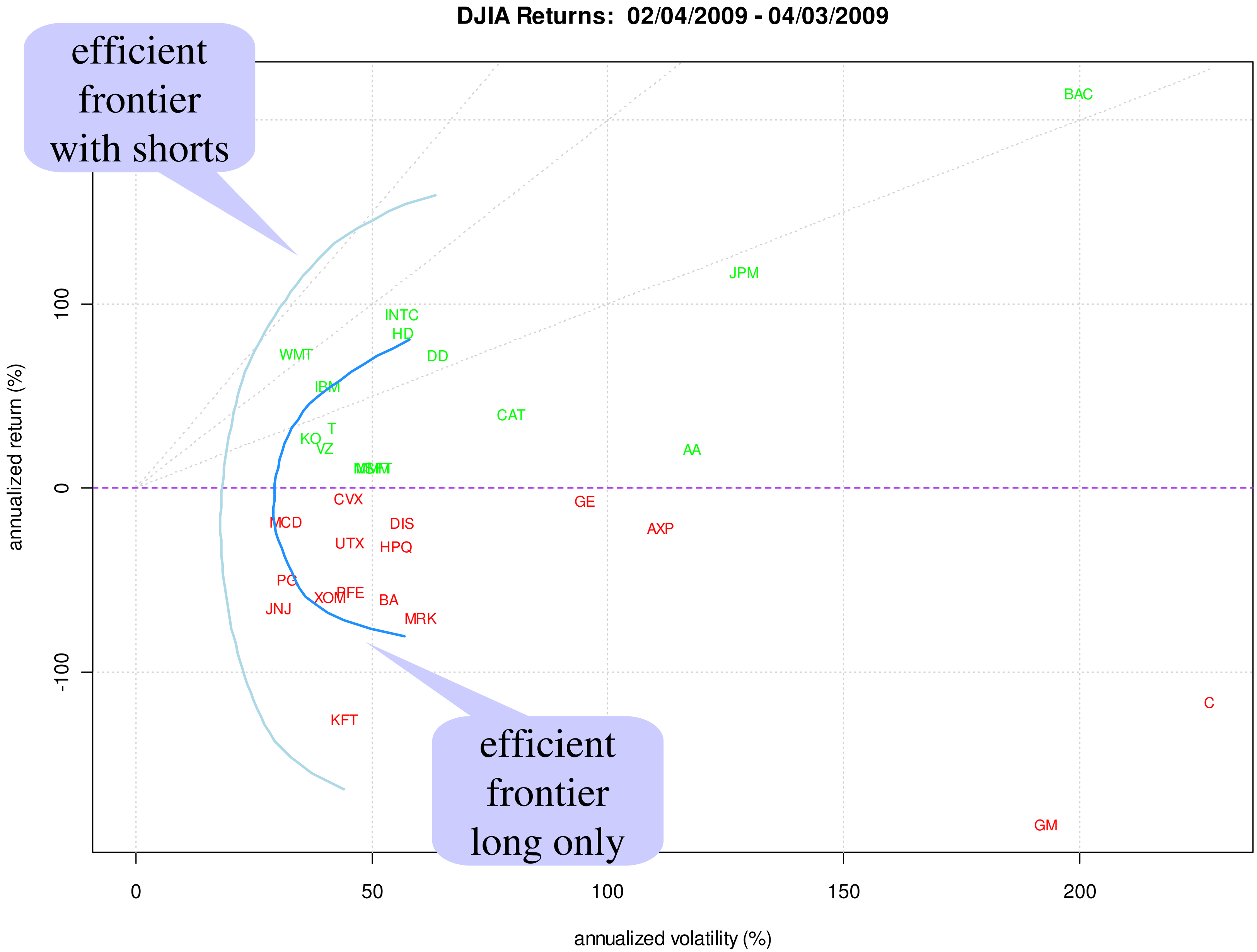
```
portfolio.optim(x, pm = mean(x), riskless = FALSE, shorts = FALSE,
  rf = 0.0, reslow = NULL, reshigh = NULL, covmat = cov(x), ...)
```

- Example

```
> averet = matrix(colMeans(r),nrow=1)
> rcov = cov(r)
> target.return = 15/250
> port.sol = portfolio.optim(x = averet, pm = target.return,
    covmat = rcov, shorts = F, reslow = rep(0,30), reshigh = rep(0.1,30))
> w = cleanWeights(port.sol$pw,syms)
> w[w!=0]
  HD  IBM INTC  JNJ   KO  MCD MSFT  PFE   PG    T   VZ  WMT
0.05 0.10 0.04 0.10 0.10 0.10 0.07 0.04 0.10 0.10 0.10 0.10
```

1 - returns vector
2 - covariance matrix
3 - minimum return

Efficient Frontier

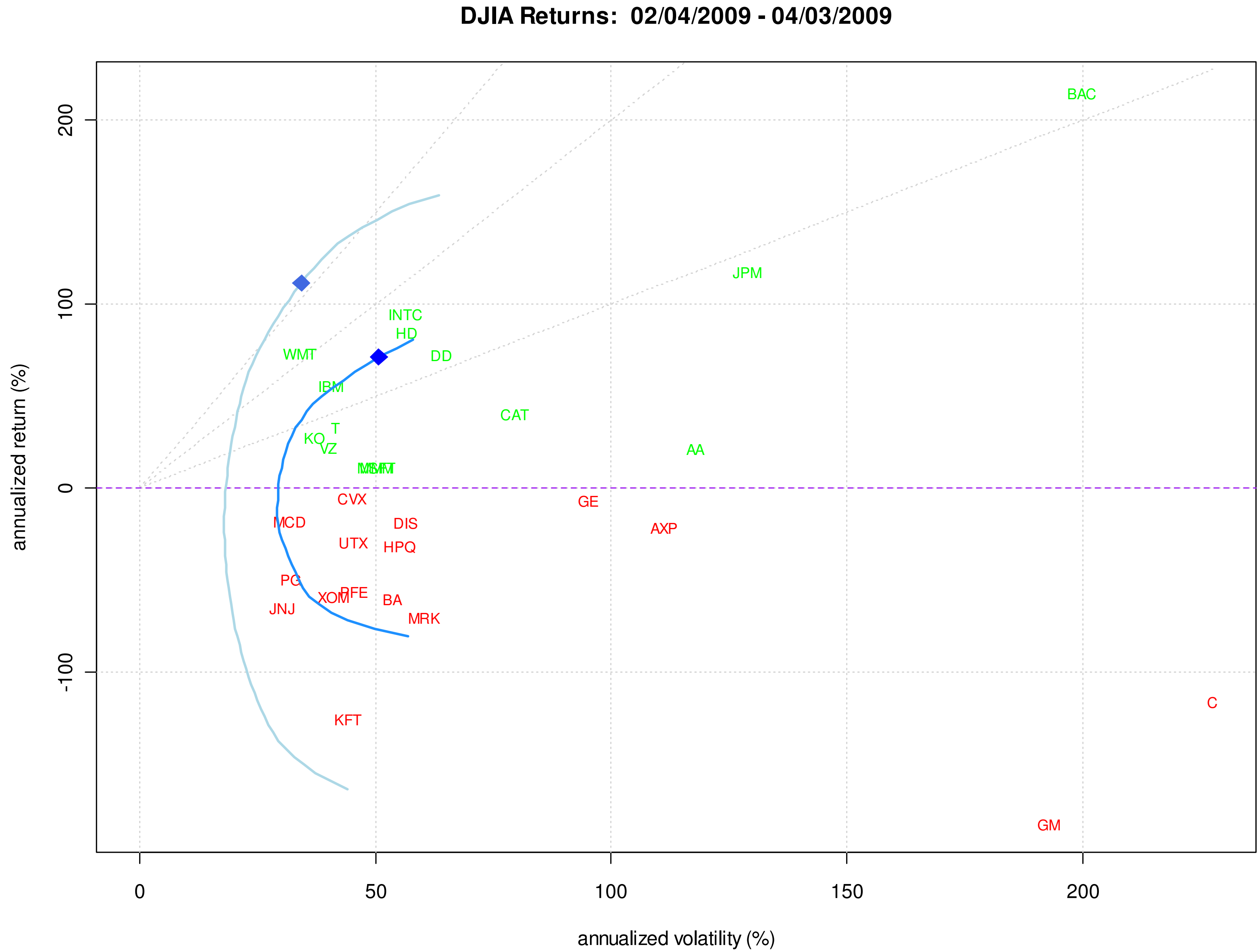


Efficient Frontier Calculation

```
effFrontier = function (averet, rcov, nports = 20, shorts=T, wmax=1)
{
    mxret = max(abs(averet))
    mnret = -mxret
    n.assets = ncol(averet)
    reshigh = rep(wmax,n.assets)
    if( shorts )
    {
      reslow = rep(-wmax,n.assets)
    } else {
      reslow = rep(0,n.assets)
    }
    min.rets = seq(mnret, mxret, len = nports)
    vol = rep(NA, nports)
    ret = rep(NA, nports)
    for (k in 1:nports)
    {
        port.sol = NULL
        try(port.sol <- portfolio.optim(x=averet, pm=min.rets[k], covmat=rcov,
          reshigh=reshigh, reslow=reslow,shorts=shorts),silent=T)
        if ( !is.null(port.sol) )
        {
          vol[k] = sqrt(as.vector(port.sol$pw %*% rcov %*% port.sol$pw))
          ret[k] = averet %*% port.sol$pw
        }
    }
    return(list(vol = vol, ret = ret))
}
```

wrapped in try to handle unfeasible optimizations

Maximum Sharpe Ratio



Maximum Sharpe Ratio

```
maxSharpe = function (averet, rcov, shorts=T, wmax = 1)
{
  optim.callback = function(param,averet,rcov,reshigh,reslow,shorts)
  {
    port.sol = NULL
    try(port.sol <- portfolio.optim(x=averet, pm=param, covmat=rcov,
      reshigh=reshigh, reslow=reslow, shorts=shorts), silent = T)
    if (is.null(port.sol)) {
      ratio = 10^9
    } else {
      m.return = averet %*% port.sol$pw
      m.risk = sqrt(as.vector(port.sol$pw %*% rcov %*% port.sol$pw))
      ratio = -m.return/m.risk
      assign("w",port.sol$pw,inherits=T)
    }
    return(ratio)
  }

  ef = effFrontier(averet=averet, rcov=rcov, shorts=shorts, wmax=wmax, nports = 100)

  n = ncol(averet)
  reshigh = rep(wmax,n)
  if( shorts ) {
    reslow = -reshigh
  } else {
    reslow = rep(0,n)
  }

  max.sh = which.max(ef$ret/ef$vol)
  w = rep(0,ncol(averet))
  xmin = optimize(f=optim.callback, interval=c(ef$ret[max.sh-1], upper=ef$ret[max.sh+1]),
    averet=averet,rcov=rcov,reshigh=reshigh,reslow=reslow,shorts=shorts)
  return(w)
}
```

callback function calls portfolio.optim()

use optimize() to find return level with maximum Sharpe ratio

Solving Quadratic Programs

- Function
  - solve.QP {quadprog}
- Description
  - solve quadratic program

general quadratic program

$$\text{Minimize:} \quad -d^T b + \frac{1}{2} b^T D b$$
$$\text{Subject to:} \quad A^T b \geq b_0$$

$$\Longrightarrow$$

mean-variance portfolio optimization

$$\text{Minimize:} \quad w^T \Omega w$$
$$\text{Subject to:} \quad \sum_i \overline{r}_i w_i = r_{min}$$
$$\sum_i w_i = 1$$
$$w_i^{min} \leq w_i \leq w_i^{max}$$

- Usage

```
solve.QP(Dmat, dvec, Amat, bvec, meq=0, factorized=FALSE)
```

Extending portfolio.optim

- Modify portfolio.optim
  - Market neutral (weights sum to zero)

```
if (!is.null(reslow) & !is.null(reshigh)) {
  a3 <- matrix(0, k, k)
  diag(a3) <- 1
  Amat <- t(rbind(a1, a2, a3, -a3))
  b0 <- c(1, pm, reslow, -reshigh)
} else {
  Amat <- t(rbind(a1, a2))
  b0 <- c(1, pm)
}
```

```
if (!is.null(reslow) & !is.null(reshigh)) {
  a3 <- matrix(0, k, k)
  diag(a3) <- 1
  Amat <- t(rbind(a1, a2, a3, -a3))
  b0 <- c(weight.sum, pm, reslow, -reshigh)
} else {
  Amat <- t(rbind(a1, a2))
  b0 <- c(weight.sum, pm)
}
```

- Call solve.QP directory
  - add group constraints
  - add linear transaction cost constraints
  - etc.

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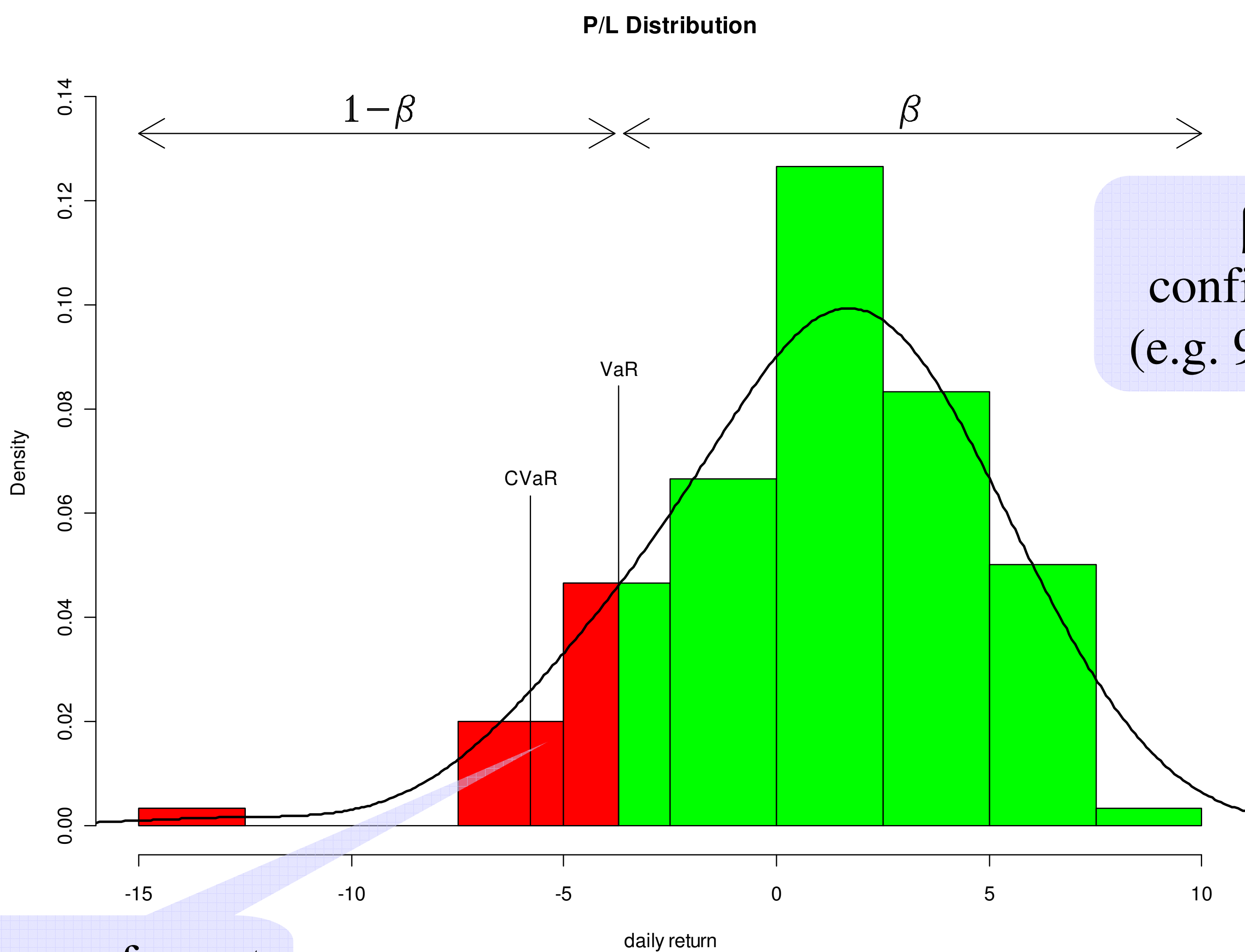
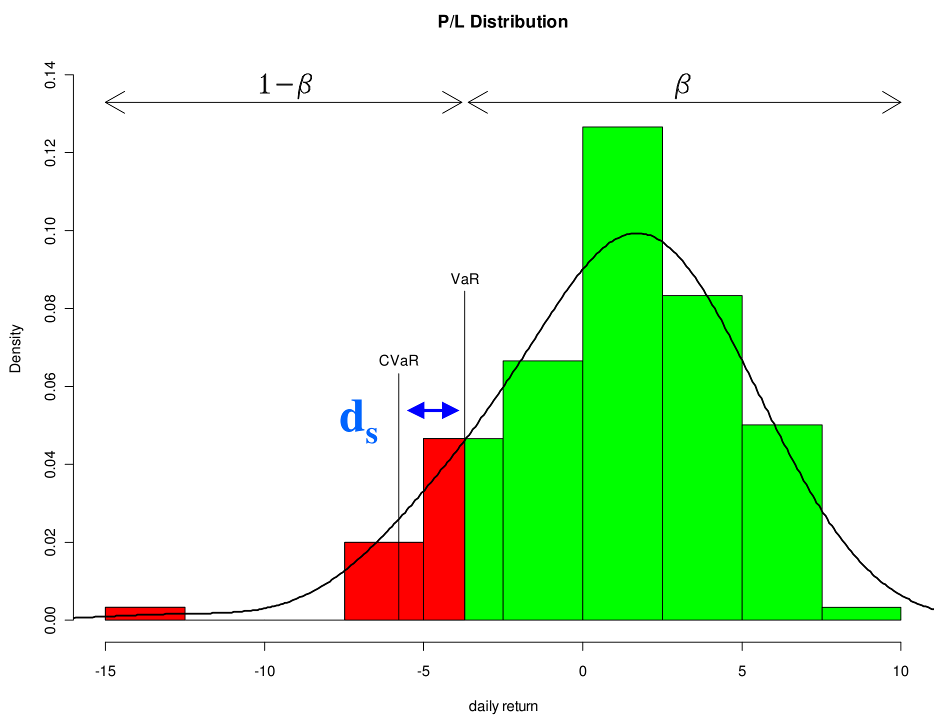
Conditional Value-at-Risk



β is our confidence level (e.g. 95% or 90%)

mean of worst 1-β losses

CVaR Optimization as a Linear Program



- CVaR

$$R_{CVAR}(\mathbf{w},\beta) = \underbrace{R_{VAR} + \underbrace{\frac{\overbrace{\frac{1}{S}\sum_{s=1}^{S}\max(R_{VAR} - \mathbf{w}'\mathbf{R_s}, 0)}^{\text{average excess loss over all scenarios}}}{\underbrace{1-\beta}_{\text{probability of excess loss}}}}_{\text{conditional excess loss}}}_{\text{average loss if loss occurs}}$$

- CVaR Optimization

$$\begin{aligned} \text{Minimize:} \quad & R_{VAR} + \frac{1}{S}\frac{1}{1-\beta}\sum_{s=1}^{S} d_s \\ \text{Subject to:} \quad & d_s \geq R_{VAR} + \mathbf{w}'\mathbf{R_s} \\ & d_s \geq 0 \\ & \mathbf{w}'\mathbf{R} \geq R_{min} \\ & \sum_i w_i = 1 \end{aligned}$$

see B. Sherer 2003

Solving Linear Programs

- Function
  - Rglpk_solve_LP {Rglpk}
- Description
  - solves linear and MILP programs (via GNU Linear Programming Kit)

general linear program

$$\begin{aligned} &\text{Minimize:} && c^T x \\ &\text{Subject to:} && Ax \geq b_0 \end{aligned} \Longrightarrow$$

CVaR portfolio optimization

$$\mathbf{c^T} = \begin{bmatrix} 0 & 0 & \dots & 0 & \frac{-1}{(1-\beta)S} & \frac{-1}{(1-\beta)S} & \dots & \frac{-1}{(1-\beta)S} & -1 \end{bmatrix}$$

$$\mathbf{x^T} = \begin{bmatrix} w_1 & w_2 & \dots & w_n & d_1 & d_2 & \dots & d_S & R_{VaR} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 & \dots & 0 & 0 \\ \bar{r}_1 & \bar{r}_2 & \dots & \bar{r}_n & 0 & \dots & 0 & 0 \\ r_{11} & r_{12} & \dots & r_{1n} & 1 & 0 & \dots & 1 \\ r_{21} & r_{22} & \dots & r_{2n} & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1 \\ r_{s1} & r_{s2} & \dots & r_{sn} & 0 & \dots & 1 & 1 \end{bmatrix} \quad \mathbf{b_0} = \begin{bmatrix} 1 \\ rmin \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- Usage

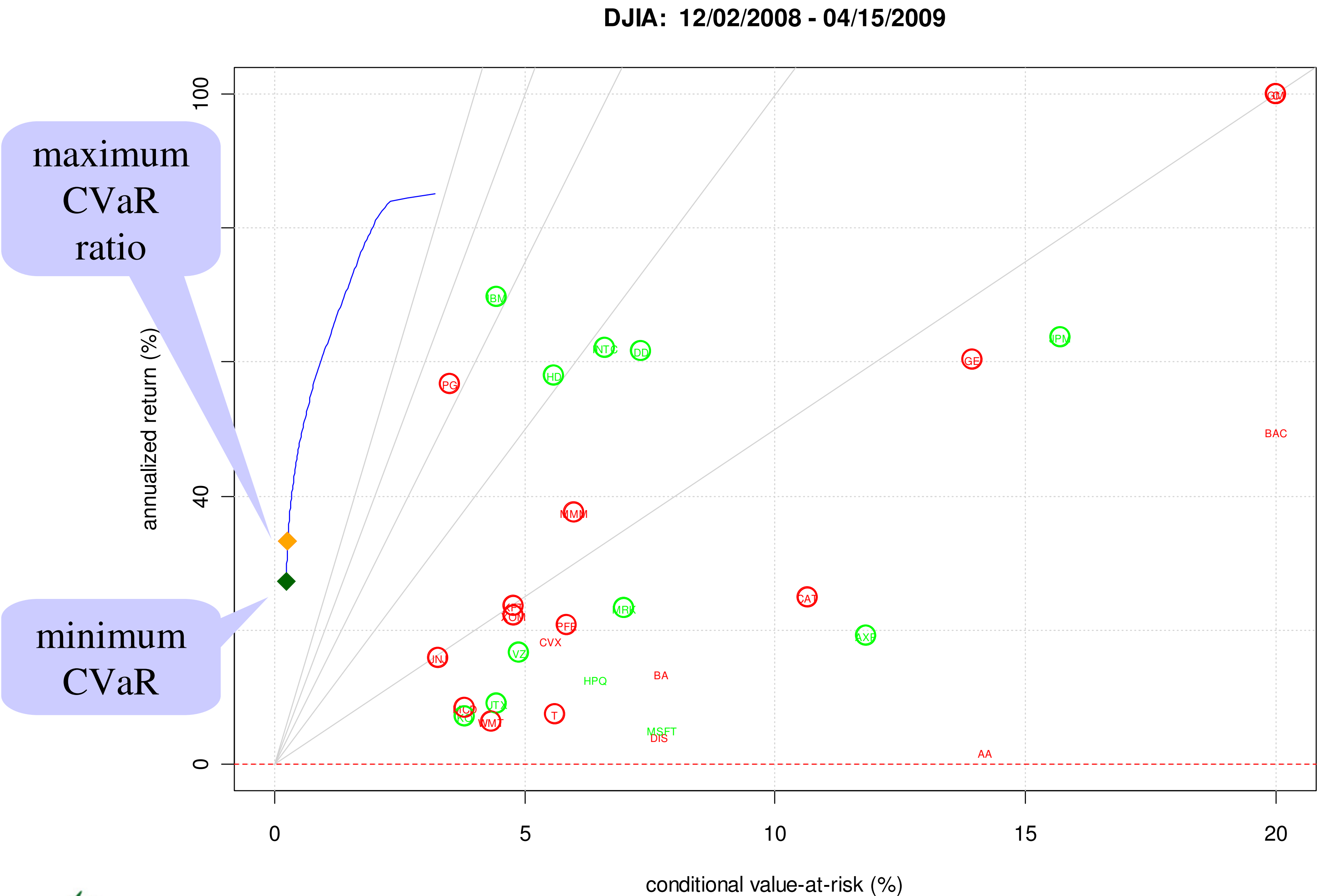
```
Rglpk_solve_LP(obj, mat, dir, rhs, types = NULL, max = FALSE,
  bounds = NULL, verbose = FALSE)
```

CVaR Optimization

```
cvarOpt = function(rmat, alpha=0.05, rmin=0, wmin=0, wmax=1, weight.sum=1)
{
  require(Rglpk)
  n = ncol(rmat) # number of assets
  s = nrow(rmat) # number of scenarios i.e. periods
  averet = colMeans(rmat)

  # creat objective vector, constraint matrix, constraint rhs
  Amat = rbind(cbind(rbind(1,averet),matrix(data=0,nrow=2,ncol=s+1)),
    cbind(rmat,diag(s),1))
  objL = c(rep(0,n), rep(-1/(alpha*s), s), -1)
  bvec = c(weight.sum,rmin,rep(0,s))

  # direction vector
  dir.vec = c("==",">=",rep(">=",s))

  # bounds on weights
  bounds = list(lower = list(ind = 1:n, val = rep(wmin,n)),
                upper = list(ind = 1:n, val = rep(wmax,n)))

  res = Rglpk_solve_LP(obj=objL, mat=Amat, dir=dir.vec, rhs=bvec,
    types=rep("C",length(objL)), max=T, bounds=bounds)

  w = as.numeric(res$solution[1:n])
  return(list(w=w,status=res$status))
}
```

supports general equality/inequality constraints

CVaR Efficient Frontier



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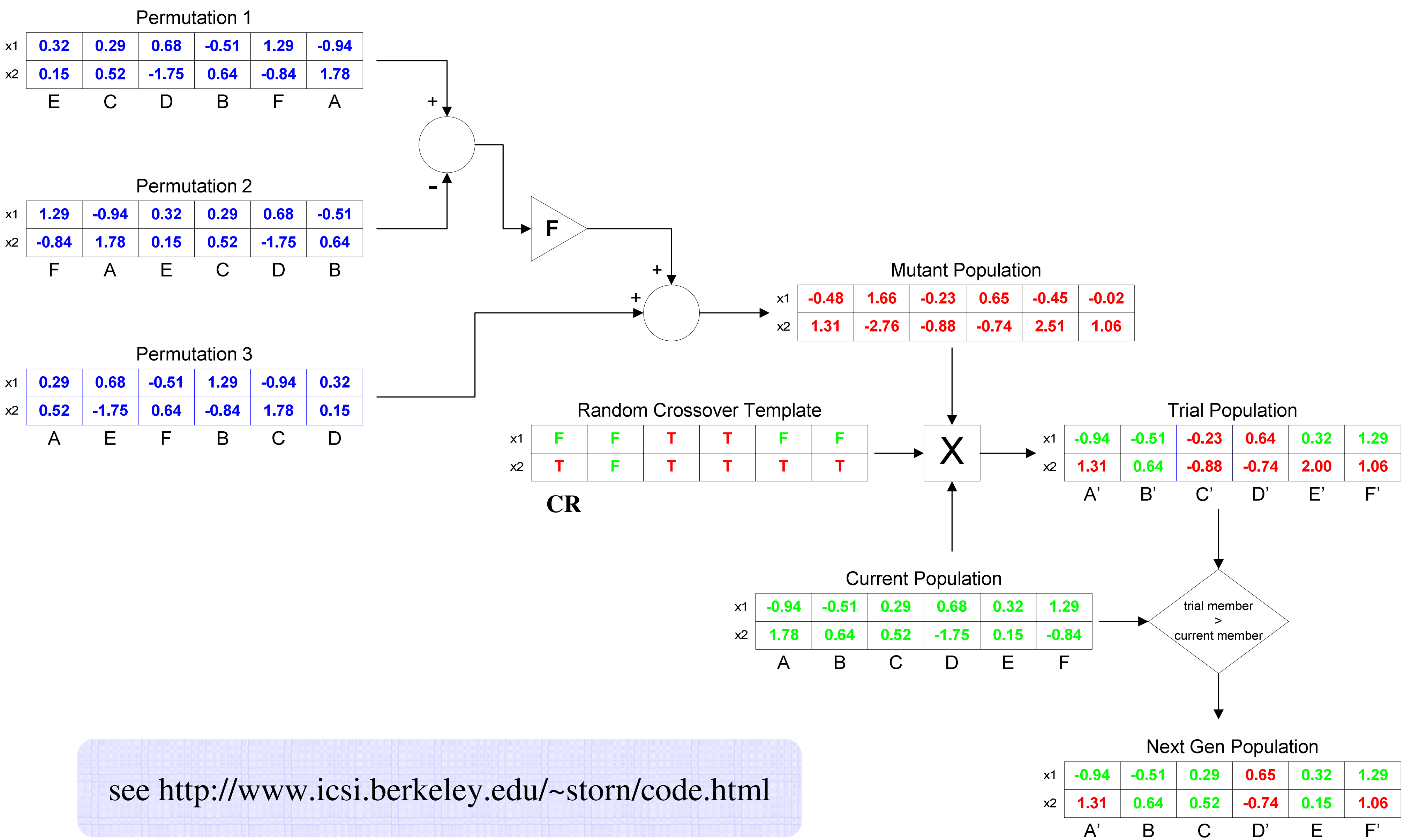
Differential Evolution

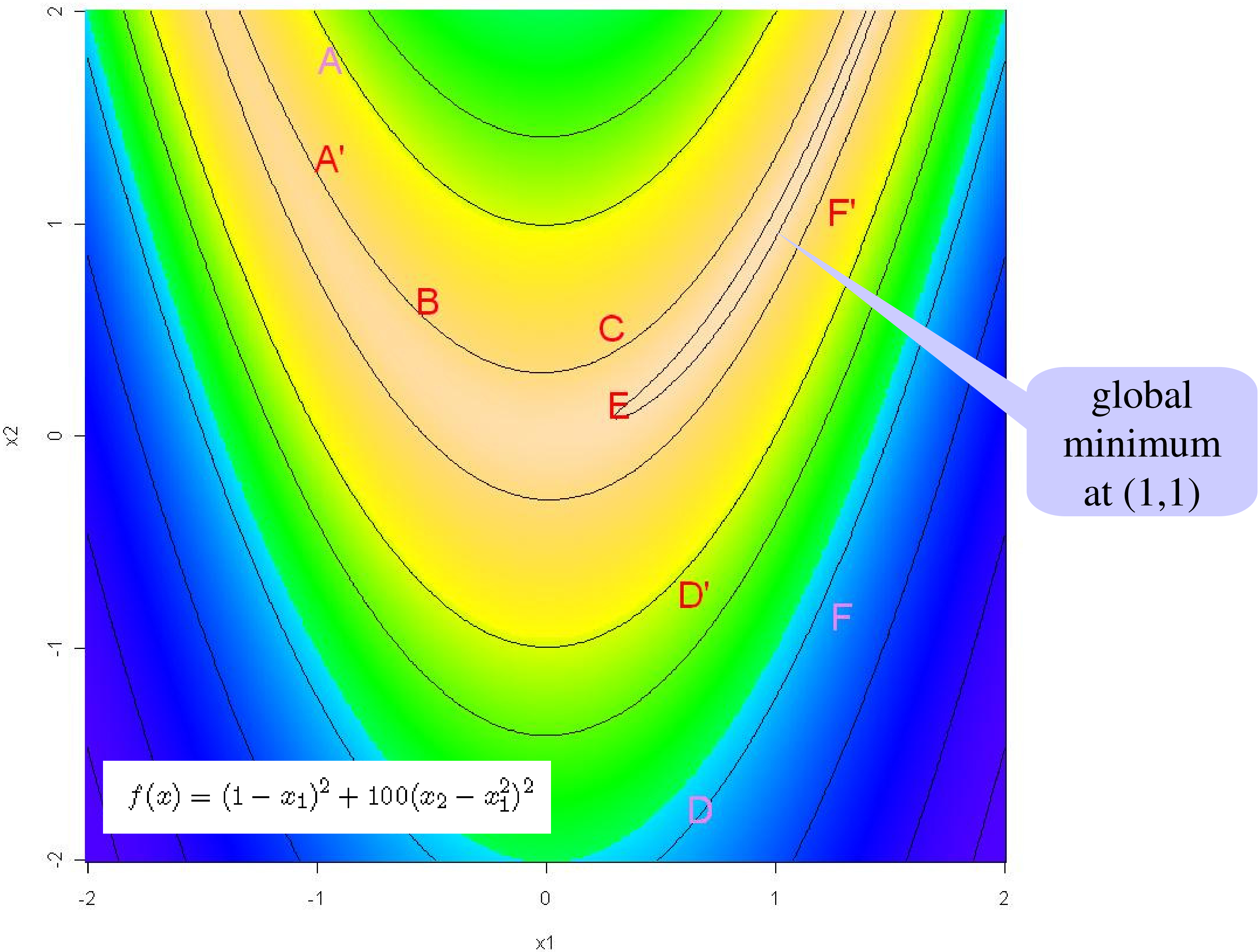
- DE is a very **simple** and yet very **powerful** population based stochastic function minimizer
- Ideal for global optimization of multidimensional multimodal functions (i.e. *really hard problems*)
- Developed in mid-1990 by Berkeley researchers Ken Price and Rainer Storm
- Implemented in R in the package DEoptim

Differential Evolution Algorithm



see http://www.icsi.berkeley.edu/~storn/code.html

DE Example


A
A'
B
C
E
F'
D'
F
D
$f(x) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2$
x1
x2
-2
-1
0
1
2
global minimum at (1,1)

Differential Evolution Function

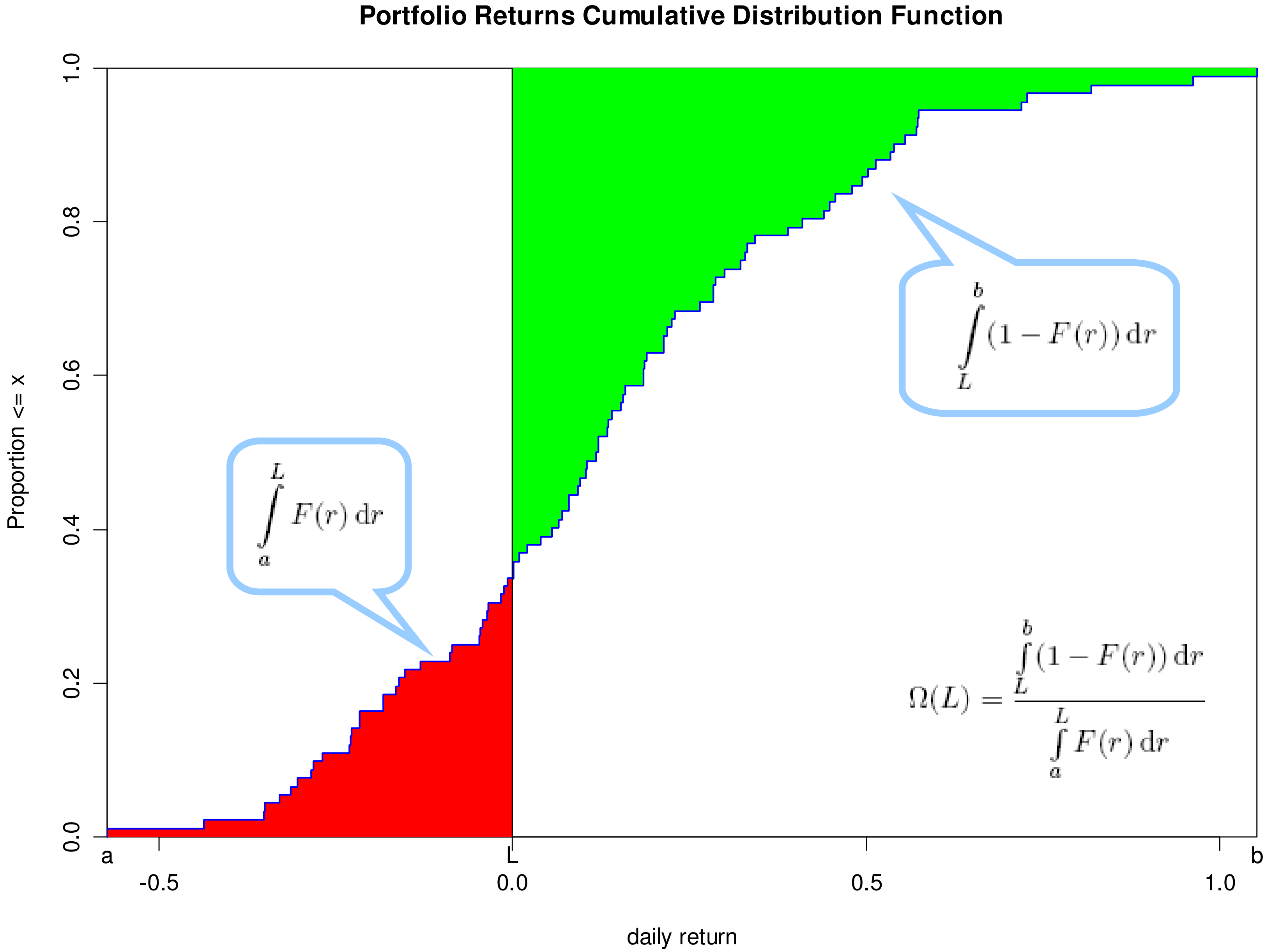
- Function
  - DEoptim {DEoptim}
- Description
  - performs evolutionary optimization via differential evolution algorithm
- Usage

```
DEoptim(FUN, lower, upper, control = list(), ...)
```

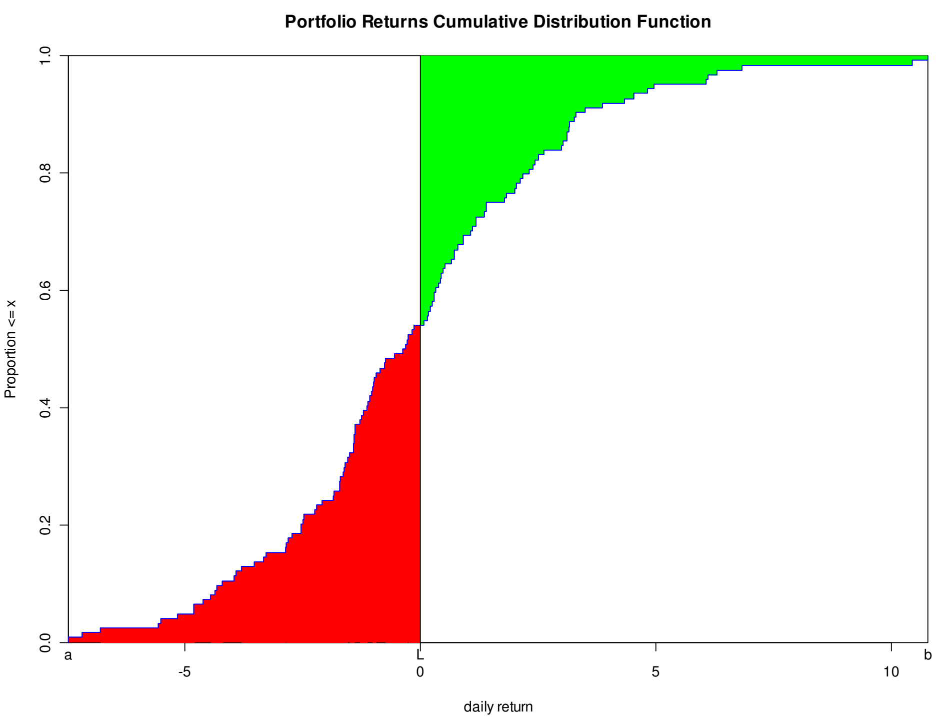
- Example

```
> lower = c(-2,-2)
> upper = c(2,2)
> res = DEoptim(banana, lower, upper)
> res$optim
$bestmem
     par1      par2
0.9987438 0.9976079
$bestval
[1] 2.986743e-06
$nfeval
[1] 5050
$iter
[1] 100
```

Omega Performance Measure



Omega Performance Measure



- Omega Performance Measure:

$$\Omega(L) = \frac{\int_L^b (1 - F(r))\,\mathrm{d}r}{\int_a^L F(r)\,\mathrm{d}r}$$

  - utilizes entire returns distribution
  - ratio of call price to put price with strike at L:

$$\Omega(L) = \frac{C(L)}{P(L)}$$

  - simple calculation:

```
omega = mean(pmax(r-L,0))/mean(pmax(L-r,0))
```

See
Keating & Shadwick 2002
Kazemi et. al., 2003

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Omega Optimization

Maximize: $\Omega(L)$

Subject to: $\sum_i |w_i| = 1$

$0 \leq w_i \leq w_i^{max}$

objective function

calls Omega() from PerformanceAnalytics

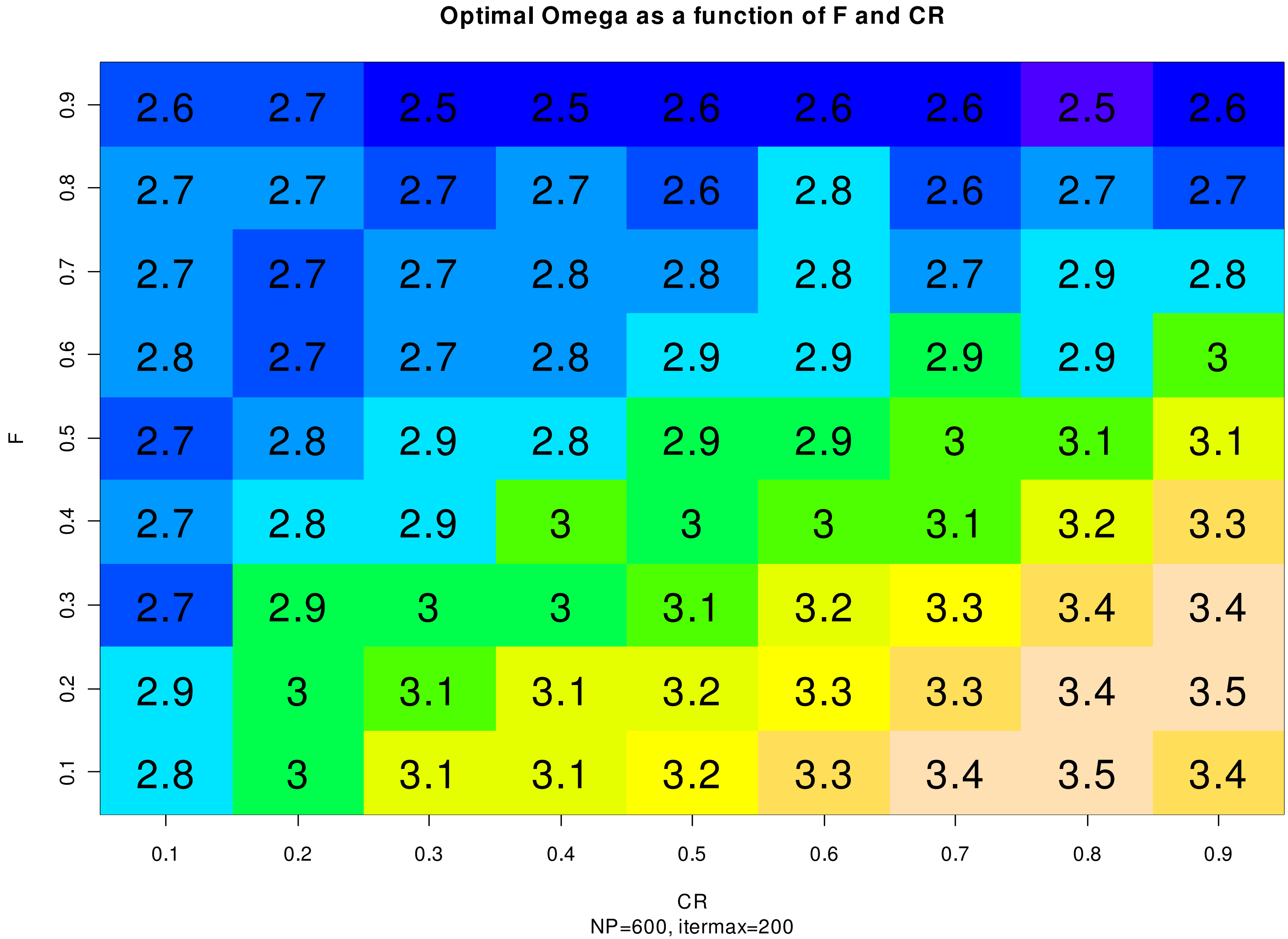
```
optOmega = function(x,ret,L)
{
  retu = ret %*% x
  obj = -Omega(retu,L=L,method="simple")
  weight.penalty = 100*(1-sum(x))^2
  return( obj +  weight.penalty )
}
```

```
> lower = rep(0,n.assets)
> upper = rep(wmax,n.assets)
```

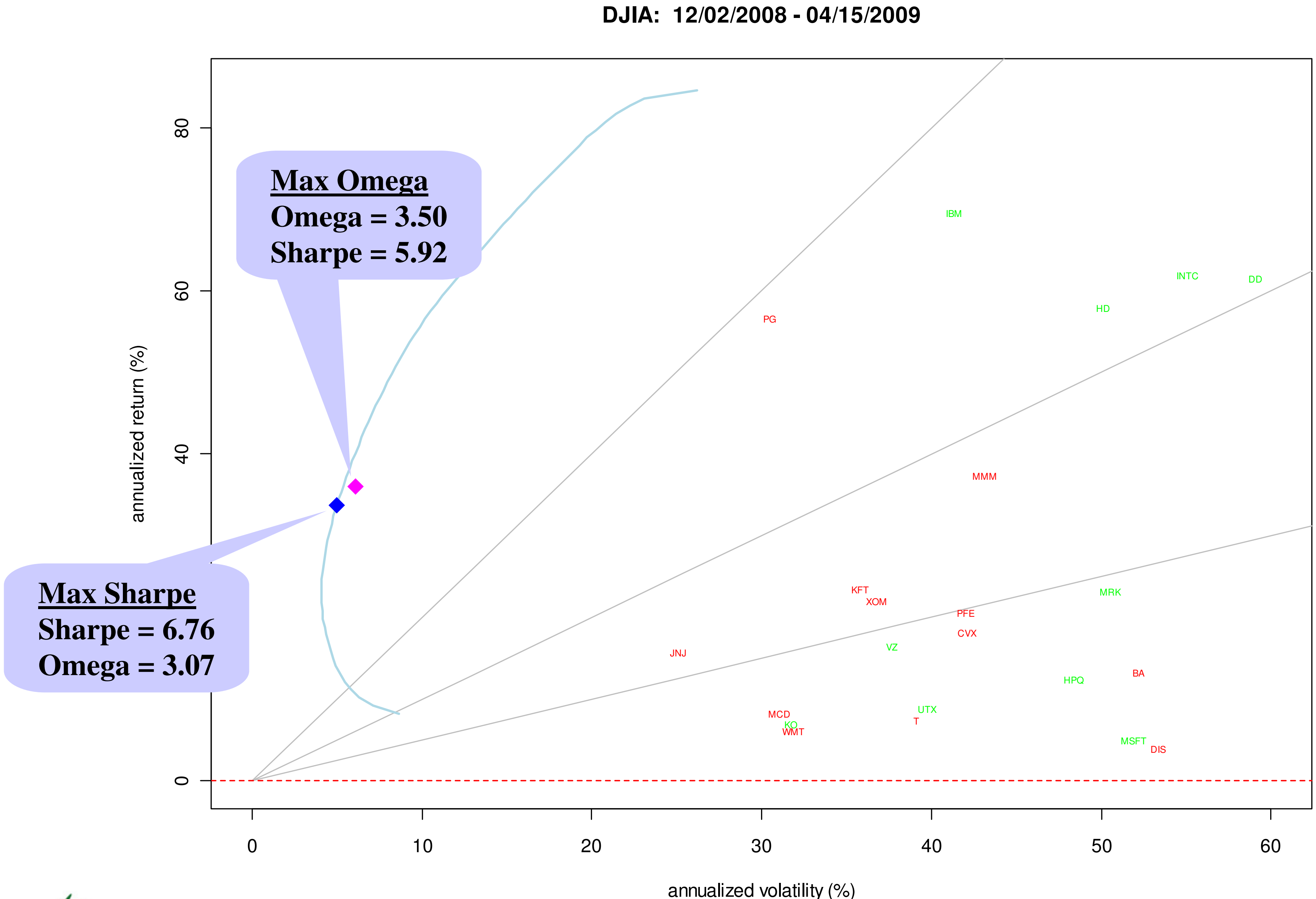
```
> res = DEoptim(optOmega,lower,upper,
  control=list(NP=2000,itermax=1000,F=0.2,CR=0.8),
  ret=coredata(r),L=L)
```

```
> w = cleanWeights(res$optim$bestmem,syms)
> w[w!=0]
 AXP   BA    C  CAT  CVX   DD  DIS   GE   GM   HD  IBM INTC  JNJ   KO  MCD  MMM
0.02 0.03 0.02 0.04 0.05 0.08 0.01 0.02 0.01 0.03 0.04 0.09 0.05 0.08 0.05 0.04
 MRK   PG    T  UTX   VZ  WMT  XOM
0.04 0.10 0.08 0.04 0.06 0.03 0.00
```

Effect of DE Parameters on Optimization

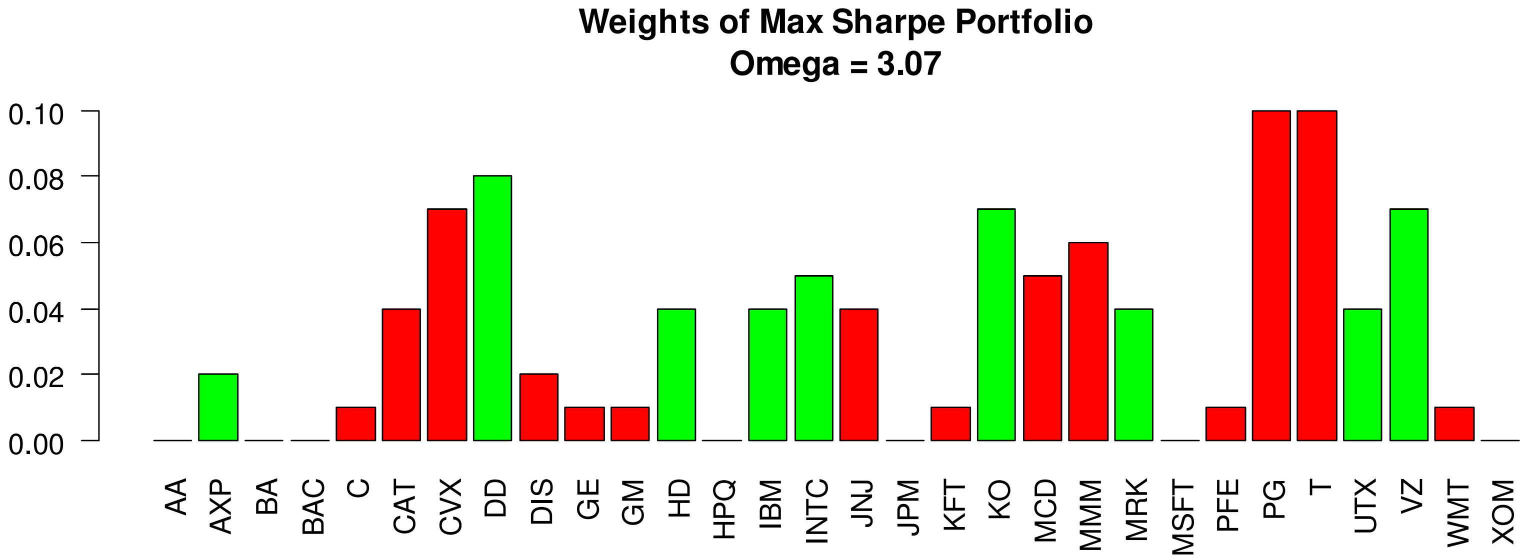


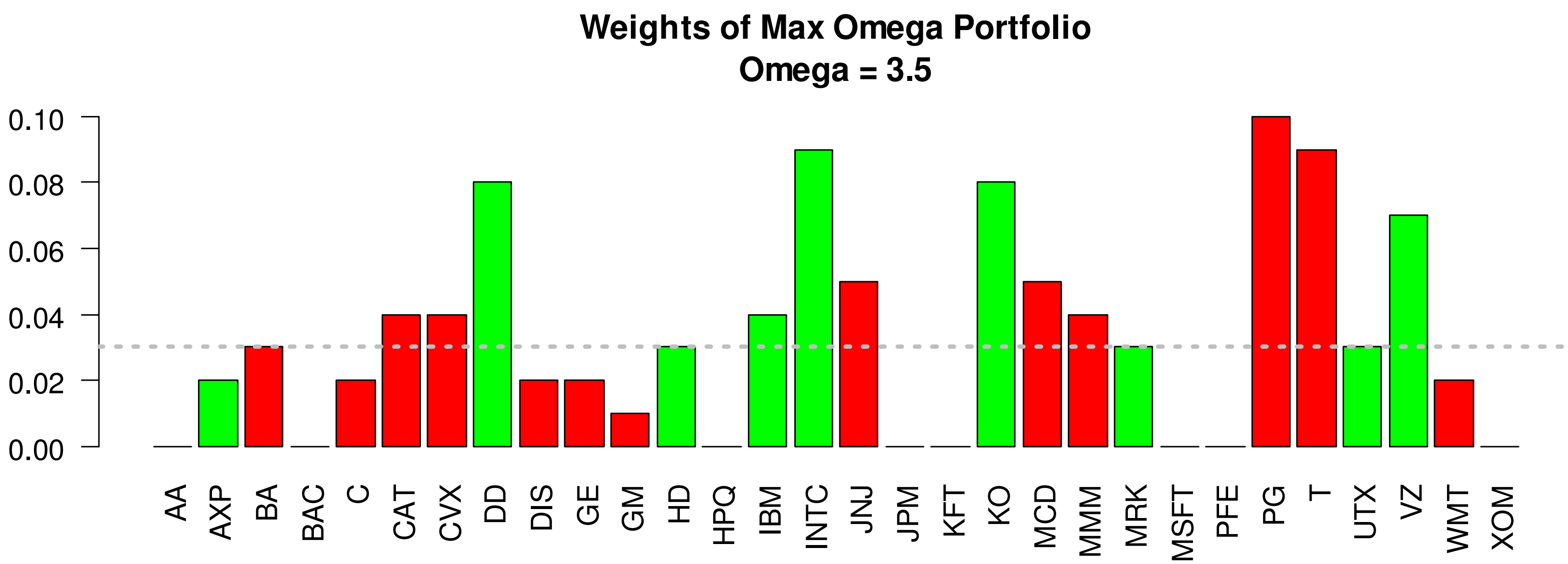
Max Omega versus Max Sharpe



Note: some assets not displayed due to cropping

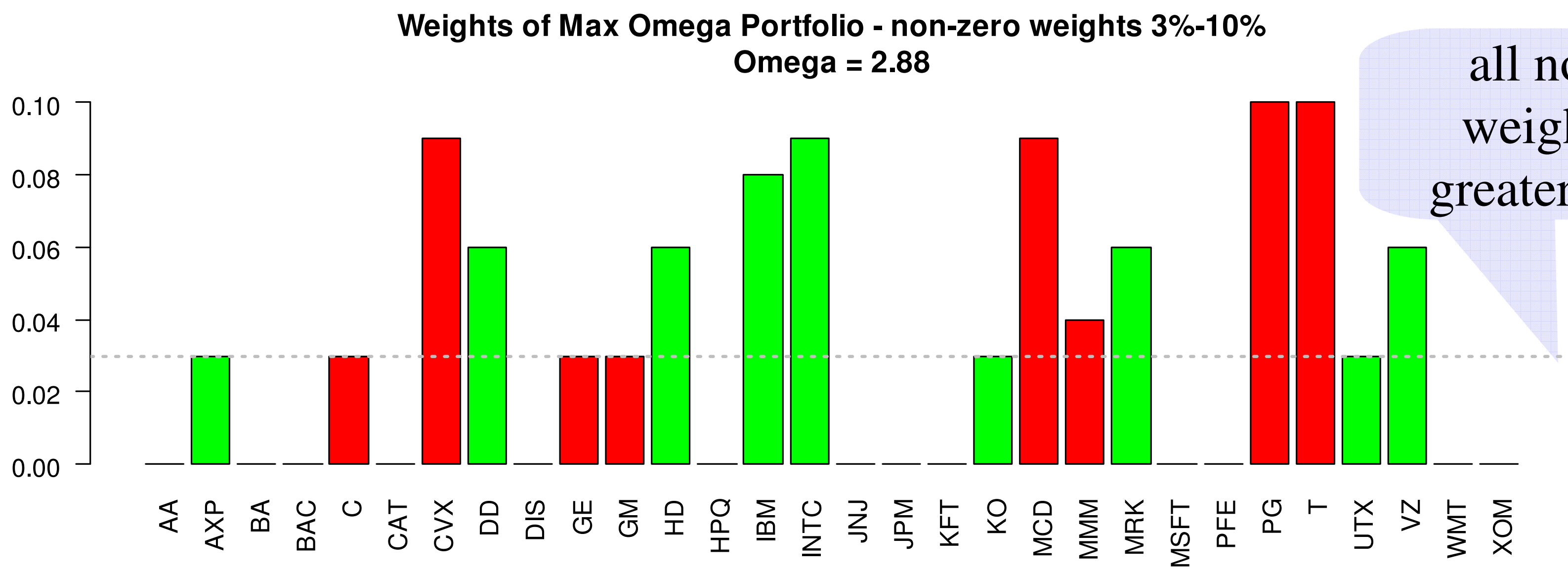
Weight Comparison





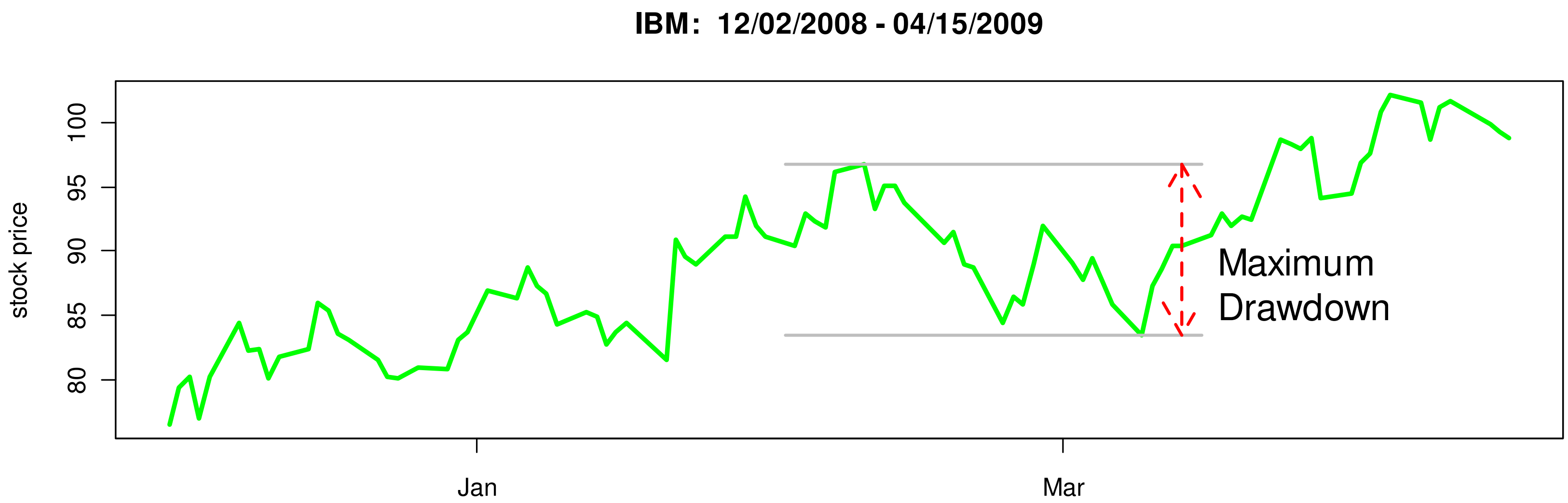
Optimization with Additional Constraints

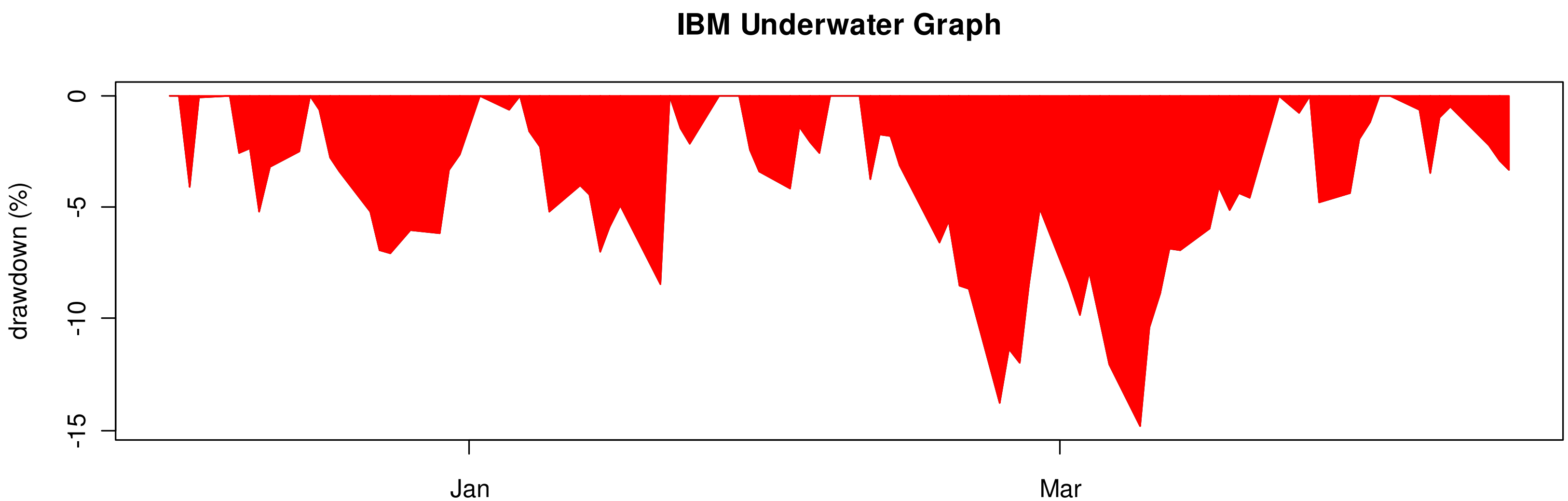
```
# max omega with non-zero weights between 3% & 10%
optOmega.gt3 = function(x,ret,L)
{
  retu = ret %*% x
  obj = -Omega(retu,L=L,method="simple")
  weight.penalty = 100*(1-sum(x))^2
  small.weight.penalty = 100*sum(x[x<0.03])
  return( obj + weight.penalty + small.weight.penalty )
}

res = DEoptim(optOmega.gt3,lower,upper,
  control=list(NP=2000,itermax=1000,F=0.2,CR=0.8),
  ret=coredata(r),L=L)
```



all non-zero weights now greater than 3%

Maximum Drawdown



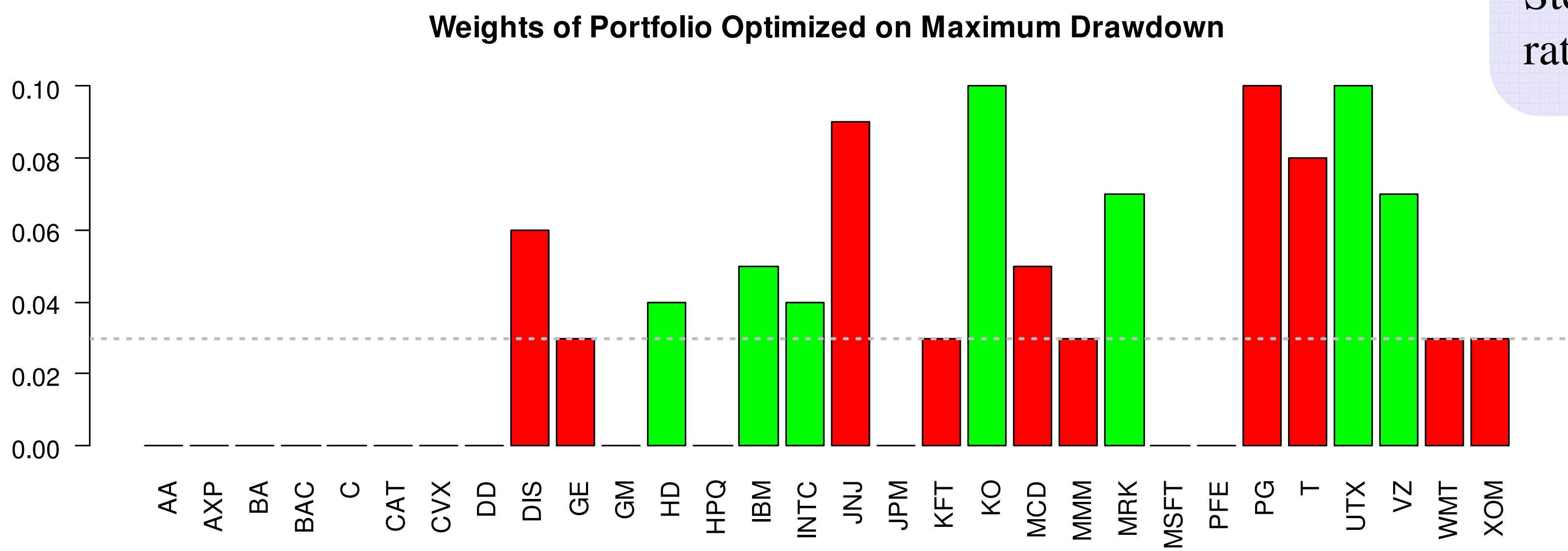


Maximum Drawdown Optimization

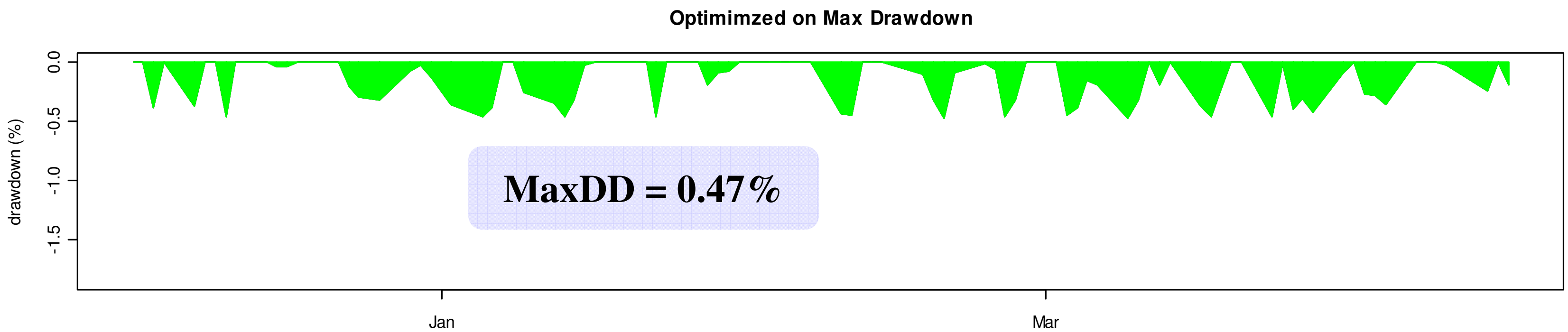
```
# max drawdown with non-zero weights between 3% & 10%
optMDD.gt3 = function(x,ret)
{
  retu = ret %*% x
  obj = mddx(retu,1)
  weight.penalty = 100*(1-sum(x))^2
  small.weight.penalty = 100*sum(x[x<0.03])
  return( obj + weight.penalty + small.weight.penalty )
}

res = DEoptim(optMDD.gt3,lower,upper,
  control=list(NP=2000,itermax=1000,F=0.2,CR=0.8),
  ret=coredata(r))
```

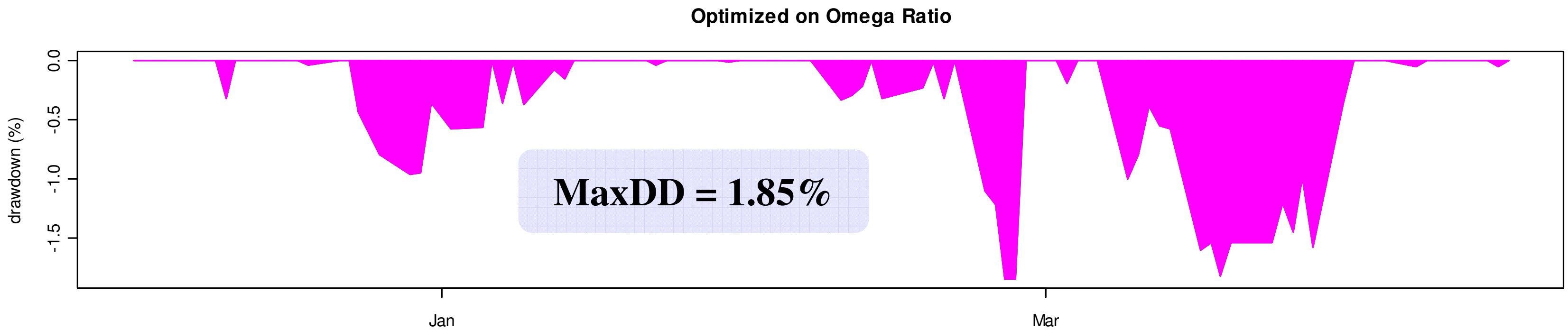
function return the mean of the top n drawdowns (in this case n=1)

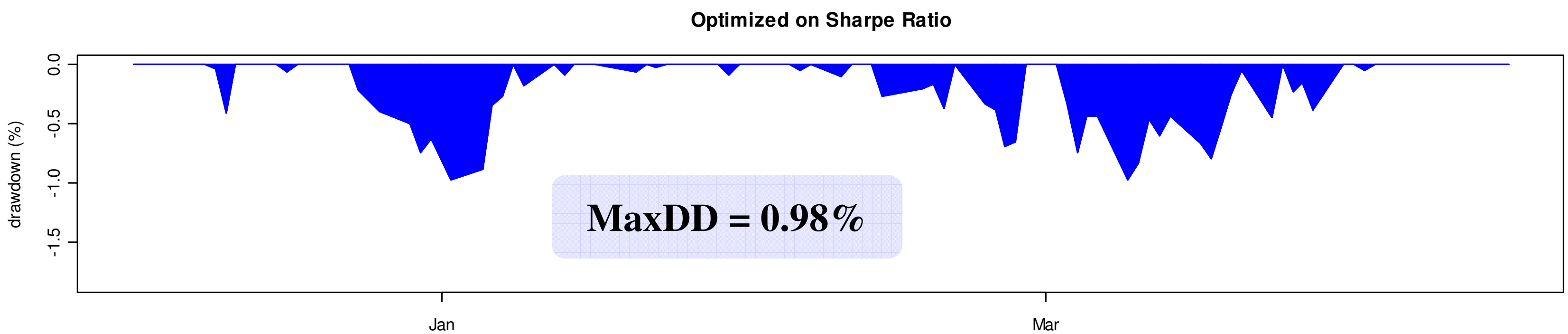
could readily implement optimization on Calmar or Sterling ratios



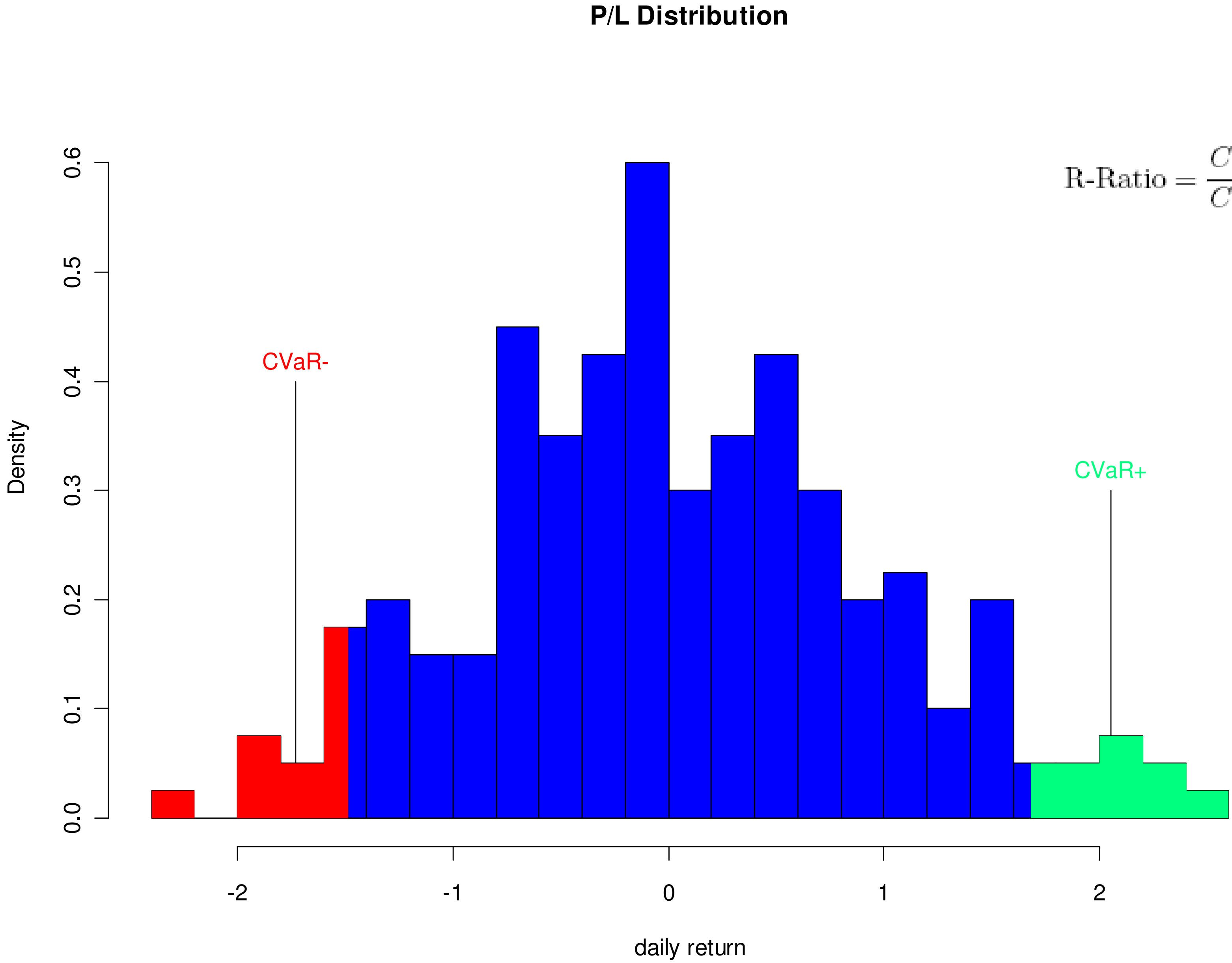
Maximum Drawdown Optimization







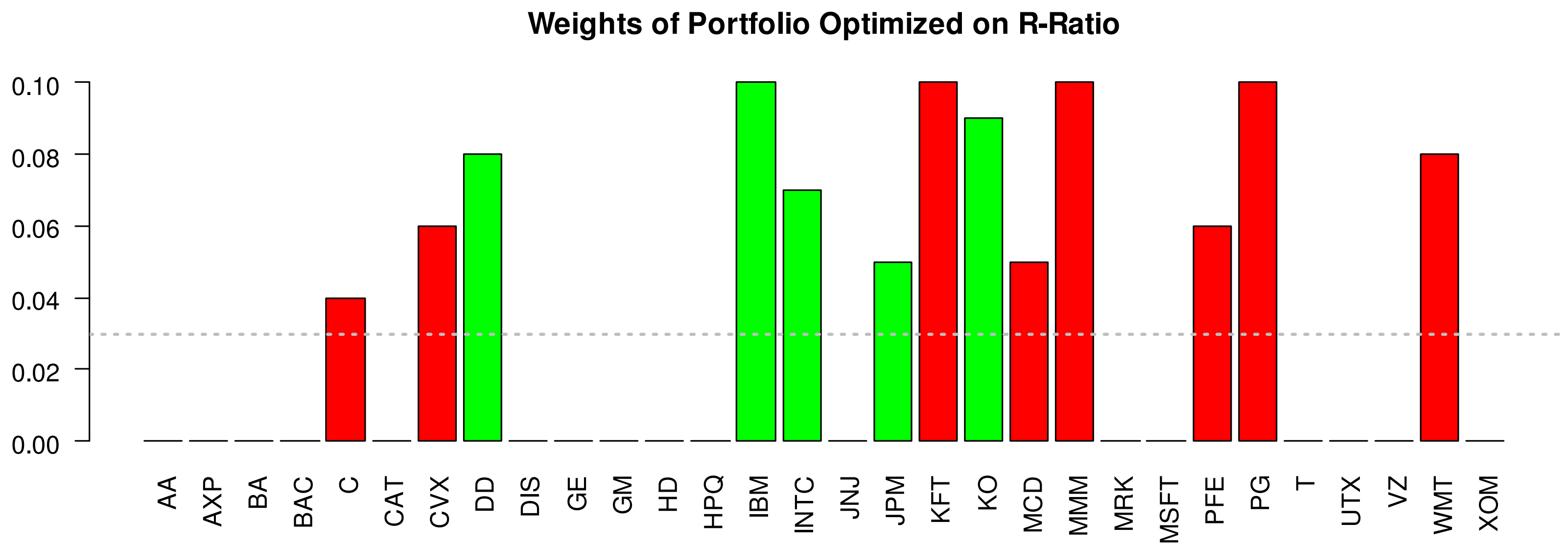
Rachev Ratio (R-Ratio)



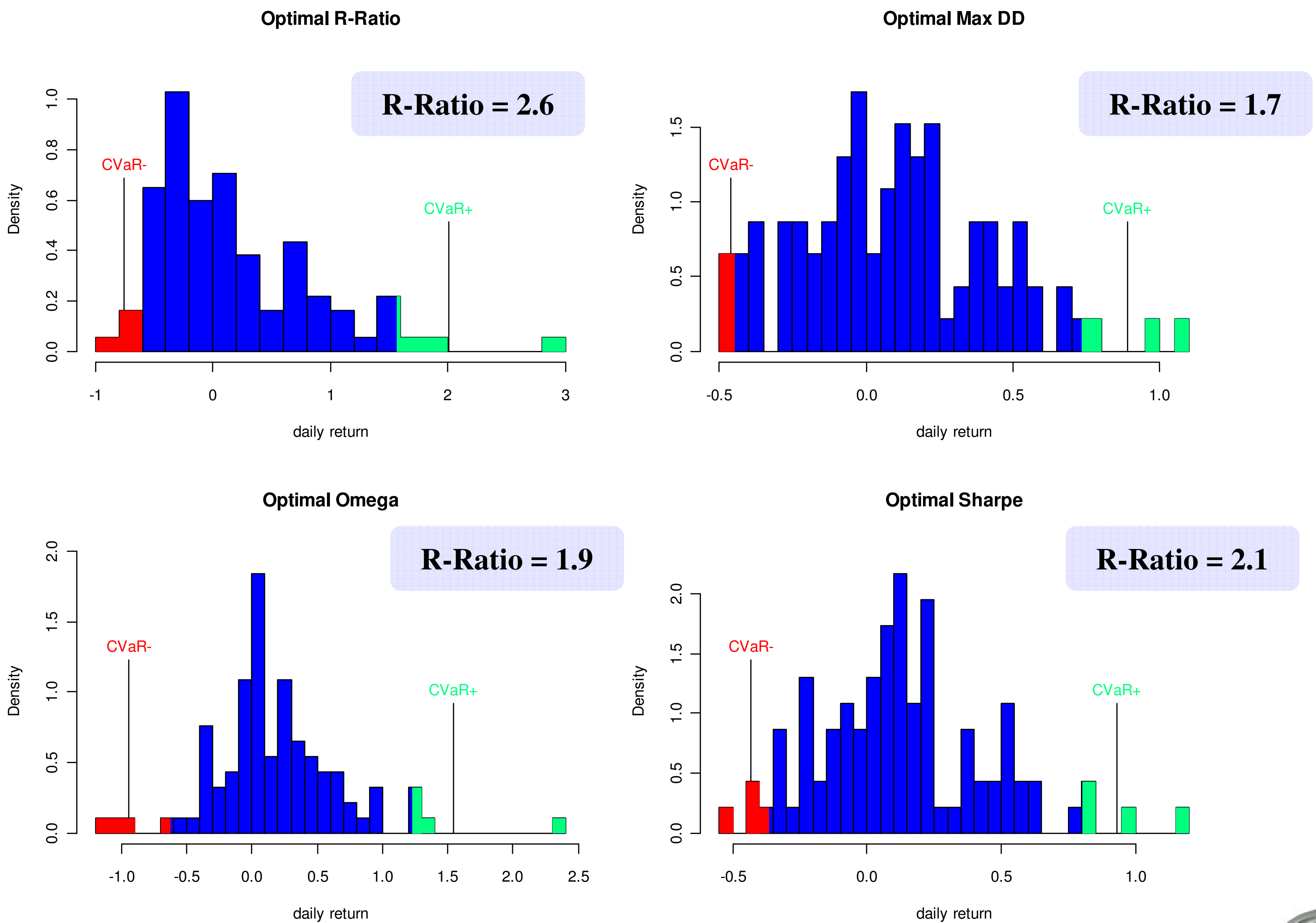
$$\text{R-Ratio} = \frac{CVaR(\alpha)^{+}}{CVaR(\alpha)^{-}}$$

R-Ratio Optimization

```
optRR.gt3 = function(x,ret)
{
  retu = ret %*% x
  obj = -CVaR(-retu)/CVaR(retu)
  weight.penalty = 100*(1-sum(x))^2
  small.weight.penalty = 100*sum(x[x<0.03])
  return( obj + weight.penalty + small.weight.penalty )
}

res = DEoptim(optRR.gt3,lower,upper,
  control=list(NP=2000,itermax=1000,F=0.2,CR=0.8),
  ret=coredata(r))
```



R-Ratio Comparison



Summary

- Mean-Variance Portfolio Optimization
  - high-level function: portfolio.optim
  - low-level function: solve.QP

- Linear Programming Optimization
  - Rglpk_solve_LP()
    - Conditional Value at Risk (CVaR):
    - MAD, Semivariance & others

- General-purpose non-linear optimization
  - DEoptim
    - Omega
    - non-linear constraints
    - maximum drawdown
    - R-Ratio
    - 130/30 portfolio optimization

Thank You

- Questions & Answers

- Contact the Author

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