

## Testing, Monitoring, and Dating Structural Changes in FX Regimes

Achim Zeileis

<http://statmath.wu.ac.at/~zeileis/>

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### Motivation

**Initial impulse:** Ajay Shah, long time *R-help* and *R-SIG-Finance* contributor, contacts Achim Zeileis, *strucchange* package maintainer.

Date: Thu, 28 Jul 2005 21:57:10 +0530  
From: Ajay Narottam Shah <ajayshah@mayin.org>  
To: Achim Zeileis <Achim.Zeileis@wu-wien.ac.at>  
Subject: Wonder if this fits (structural breaks work in a currency regime context)

...

The issues are like this. Many central banks SAY that a currency regime is X. But they routinely lie. Economists would like to know the true currency regime. And, we would like to know the date when something changed.

...

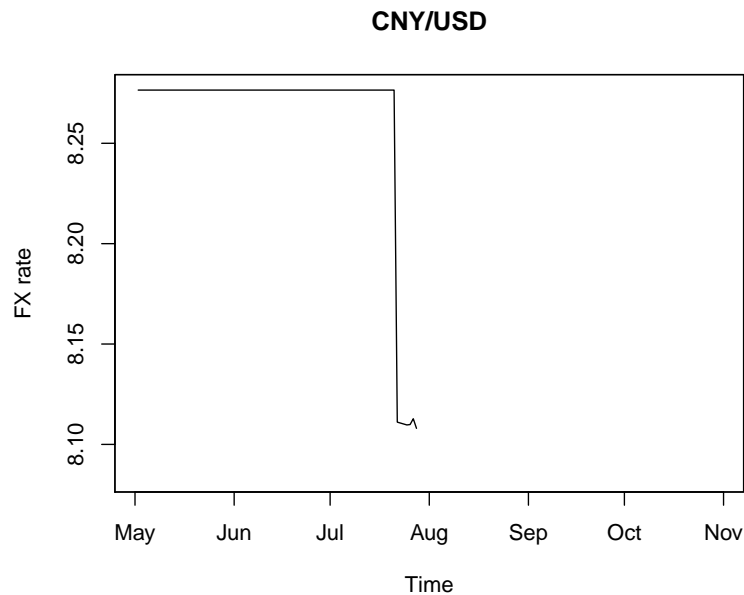
### Motivation

**Of particular interest:** China gave up on a fixed exchange rate to the US dollar (USD) on 2005-07-21. The People's Bank of China announced that the Chinese yuan (CNY) would no longer be pegged to the USD but to a basket of currencies with greater flexibility.

**Collaboration:** Ajay Shah, Ila Patnaik, and Achim Zeileis start to investigate the question *What is the new Chinese exchange rate regime?*

**First step:** Collect foreign exchange (FX) rates for various currencies for three months up to 2005-10-31.

## Motivation



## Exchange rate regimes

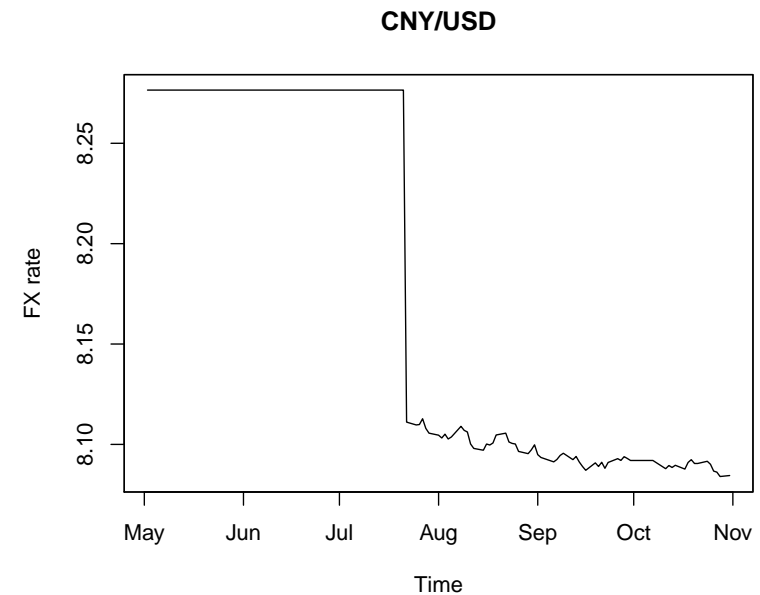
The FX regime of a country determines how it manages its currency wrt foreign currencies. Broadly, it can be

- *floating*: currency is allowed to fluctuate based on market forces,
- *pegged*: currency has limited flexibility when compared with a basket of currencies or a single currency,
- *fixed*: direct convertibility to another currency.

**Problem:** The *de facto* and *de jure* FX regime in operation in a country often differ. (*≈ politically correct version of Ajay's original e-mail*)

⇒ Data-driven classification of FX regimes

## Motivation



## Exchange rate regression

The workhorse for *de facto* FX regime classification is a linear regression model based on log-returns of cross-currency exchange rates (with respect to some floating reference currency). In the literature, this is also known as *Frankel-Wei regression*.

For modeling the log-returns of CNY a basket of regressors USD, JPY, EUR, and GBP (all log-returns wrt CHF) is employed.

Fitting the model for the first three months (up to 2005-10-31,  $n = 68$ ) shows that a plain USD peg is still in operation.

## Exchange rate regression

Ordinary least squares (OLS) estimation gives:

$$\begin{aligned} \text{CNY}_i = & \underset{(0.004)}{0.005} + \underset{(0.009)}{0.9997} \text{USD}_i + \underset{(0.011)}{0.005} \text{JPY}_i \\ & - \underset{(0.027)}{0.014} \text{EUR}_i - \underset{(0.015)}{0.008} \text{GBP}_i + \hat{\varepsilon}_i \end{aligned}$$

Only the USD coefficient is significantly different from 0 (but not from 1).

The error standard deviation is tiny with  $\hat{\sigma} = 0.028$  leading to  $R^2 = 0.998$ .

## Exchange rate regression

**In practice:** Rolling regressions are often used to answer these questions by tracking the evolution of the FX regime in operation.

**More formally:** Structural change techniques can be adapted to the FX regression to estimate and test the stability of FX regimes.

**Problem:** Unlike many other linear regression models, the stability of the error variance (fluctuation band) is of interest as well.

**Solution:** Employ an (approximately) normal regression estimated by ML where the variance is a full model parameter.

## Exchange rate regression

### Questions:

- ❶ Is this model for the period 2005-07-26 to 2005-10-31 stable or is there evidence that China kept changing its FX regime after 2005-07-26? (**testing**)
- ❷ Depending on the answer to the first question:
  - Does the CNY stay pegged to the USD in the future (starting from November 2005)? (**monitoring**)
  - When and how did the Chinese FX regime change? (**dating**)

## Model frame

**Generic idea:** Consider a regression model for  $n$  ordered observations  $y_i | x_i$  with  $k$ -dimensional parameter  $\theta$ . Ordering is typically with respect to time in time-series regressions, but could also be with respect to income, age, etc. in cross-section regressions.

To fit the model to observations  $i = 1, \dots, n$  an objective function  $\Psi(y, x, \theta)$  is used such that

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{i=1}^n \Psi(y_i, x_i, \theta).$$

This can also be defined implicitly based on the corresponding score function (or estimating function)  $\psi(y, x, \theta) = \partial \Psi(y, x, \theta) / \partial \theta$ :

$$\sum_{i=1}^n \psi(y_i, x_i, \hat{\theta}) = 0.$$

## Model frame

This class of M-estimators includes OLS and maximum likelihood (ML) estimation as well as IV, Quasi-ML, robust M-estimation etc.

Under parameter stability and some mild regularity conditions, a central limit theorem holds

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V(\theta_0)),$$

where the covariance matrix is

$$V(\theta_0) = \{A(\theta_0)\}^{-1} B(\theta_0) \{A(\theta_0)\}^{-1}$$

and  $A$  and  $B$  are the expectation of the derivative of  $\psi$  and its variance respectively.

## Model frame

**Testing:** Given that a model with parameter  $\hat{\theta}$  has been estimated for these  $n$  observations, the question is whether this is appropriate or: *Are the parameters stable or did they change through the sample period  $i = 1, \dots, n$ ?*

**Monitoring:** Given that a stable model could be established for these  $n$  observations, the question is whether it remains stable in the future or: *Are incoming observations for  $i > n$  still consistent with the established model or do the parameters change?*

**Dating:** Given that there is evidence for a structural change in  $i = 1, \dots, n$ , it might be possible that stable regression relationships can be found on subsets of the data. *How many segments are in the data? Where are the breakpoints?*

## Model frame

For the standard linear regression model

$$y_i = x_i^\top \beta + \varepsilon_i$$

with coefficients  $\beta$  and error variance  $\sigma^2$  one can either treat  $\sigma^2$  as a nuisance parameter  $\theta = \beta$  or include it as  $\theta = (\beta, \sigma^2)$ .

In the former case, the estimating functions are  $\psi = \psi_\beta$

$$\psi_\beta(y, x, \beta) = (y - x^\top \beta) x$$

and in the latter case, they have an additional component

$$\psi_{\sigma^2}(y, x, \beta, \sigma^2) = (y - x^\top \beta)^2 - \sigma^2.$$

and  $\psi = (\psi_\beta, \psi_{\sigma^2})$ . This is used for FX regressions.

## Testing

To assess the stability of the fitted model with  $\hat{\theta}$ , we want to test the null hypothesis

$$H_0 : \theta_i = \theta_0 \quad (i = 1, \dots, n)$$

against the alternative that  $\theta_i$  varies over “time”  $i$ .

Various patterns of deviation from  $H_0$  are conceivable: single/multiple break(s), random walks, etc.

To test this null hypothesis, the basic idea is to assess whether the empirical estimating functions  $\hat{\psi}_i = \psi(y_i, x_i, \hat{\theta})$  deviate systematically from their theoretical zero mean.

## Testing

To capture systematic deviations the empirical fluctuation process of scaled cumulative sums of empirical estimating functions is computed:

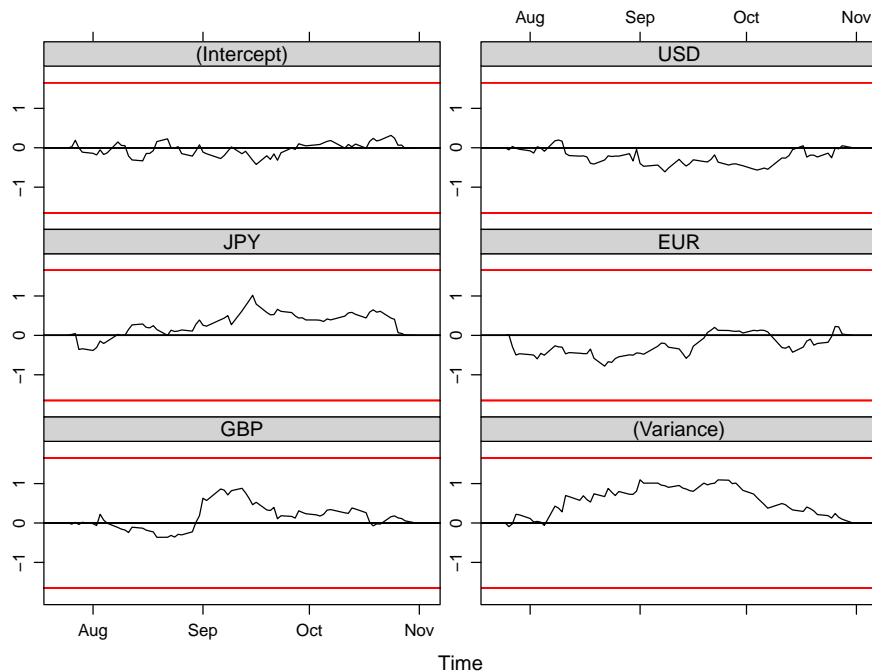
$$efp(t) = \hat{B}^{-1/2} n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \hat{\psi}_i \quad (0 \leq t \leq 1).$$

Under  $H_0$  the following functional central limit theorem (FCLT) holds:

$$efp(\cdot) \xrightarrow{d} W^0(\cdot),$$

where  $W^0$  denotes a standard  $k$ -dimensional Brownian bridge.

## Testing



## Testing

### Testing procedure:

- empirical fluctuation processes captures fluctuation in estimating functions
- theoretical limiting process is known
- choose boundaries which are crossed by the limiting process (or some functional of it) only with a known probability  $\alpha$ .
- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large  $\Rightarrow$  reject the null hypothesis.

## Testing

**More formally:** These boundaries correspond to critical values for a double maximum test statistic

$$\max_{j=1,\dots,k} \max_{i=1,\dots,n} |efp_j(i/n)|$$

which is 1.097 for the Chinese FX regression ( $p = 0.697$ ).

**Alternatively:** Employ other test statistics for aggregation.

**Special cases:** This class contains various well-known tests from the statistics and econometrics literature, e.g., Andrews' sup $LM$  test, Nyblom-Hansen test, OLS-based CUSUM/MOSUM tests.

## Testing

In empirical samples,  $efp(\cdot)$  is a  $k \times n$  array. For significance testing, aggregate it to a scalar test statistic by a functional  $\lambda(\cdot)$

$$\lambda \left( efp_j \left( \frac{i}{n} \right) \right),$$

where  $j = 1, \dots, k$  and  $i = 1, \dots, n$ .

Typically,  $\lambda(\cdot)$  can be split up into

- $\lambda_{\text{comp}}(\cdot)$  aggregating over components  $j$  (e.g., absolute maximum, Euclidian norm),
- $\lambda_{\text{time}}(\cdot)$  aggregating over time  $i$  (e.g., max, mean, range).

The limiting distribution is given by  $\lambda(W^0)$  and can easily be simulated (or some closed form results are also available).

## Testing

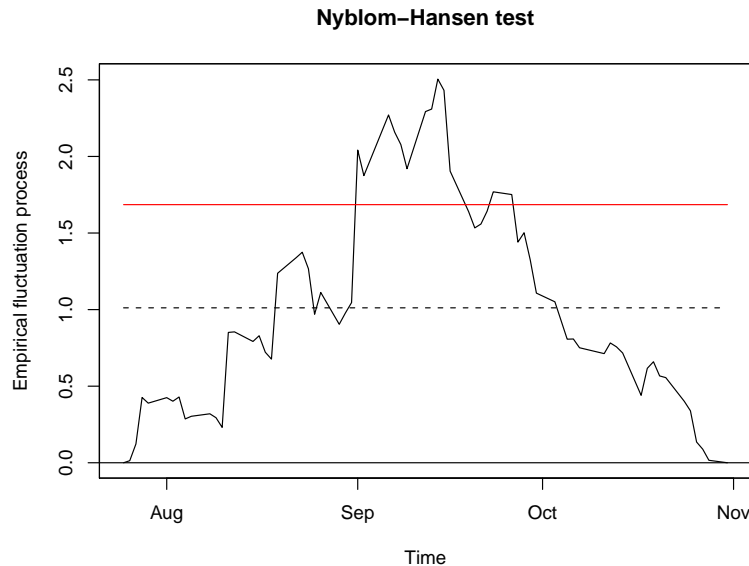
**Nyblom-Hansen test:** The test was designed for a random-walk alternative and employs a Cramér-von Mises functional.

$$\frac{1}{n} \sum_{i=1}^n \left\| efp \left( \frac{i}{n} \right) \right\|_2^2.$$

It aggregates  $efp(\cdot)$  over the components first, using the squared Euclidian norm, and then over time, using the mean.

For the Chinese FX regression this is 1.012 ( $p = 0.364$ ).

## Testing



## Testing

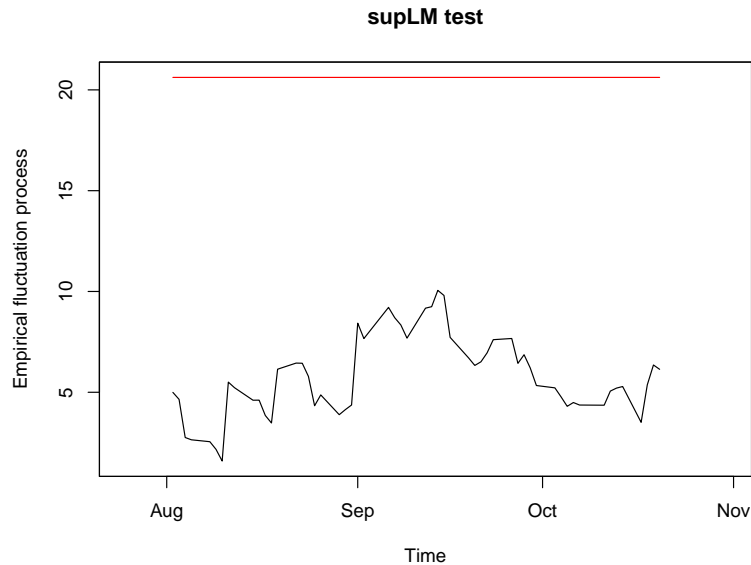
**Andrews' supLM test:** This test is designed for a single shift alternative (with unknown timing) and employs the supremum of  $LM$  statistics for this alternative.

$$\sup_{t \in \Pi} LM(t) = \sup_{t \in \Pi} \frac{\|efp(t)\|_2^2}{t(1-t)}.$$

It aggregates  $efp(\cdot)$  over the components first, using a weighted squared Euclidian norm, and then over time, using the maximum (over a compact interval  $\Pi \subset [0, 1]$ ).

For the Chinese FX regression this is 10.055 ( $p = 0.766$ ), using  $\Pi = [0.1, 0.9]$ .

## Testing



## Monitoring

**Test statistics:** Update  $efp(t)$ , and re-compute  $\lambda(efp(t))$  in the monitoring period  $1 \leq t \leq T$ .

**Critical values:** For sequential testing not only a single critical value is needed, but a full boundary function  $b(t)$  that satisfies

$$1 - \alpha = P(\lambda(W^0(t)) \leq b(t) \mid t \in [1, T])$$

Various boundary (or weighting) functions are conceivable that can direct power to early or late changes or try to spread the power evenly.

**In 2005:** Ajay Shah, Ila Patnaik, and Achim Zeileis establish a webpage and start monitoring the CNY regime. A double maximum functional with boundary  $b(t) = c \cdot t$  is employed (where  $c$  controls the significance level, using  $T = 4$  and  $\alpha = 0.05$ ).

## Monitoring

**Idea:** Fluctuation tests can be applied sequentially to monitor regression models.

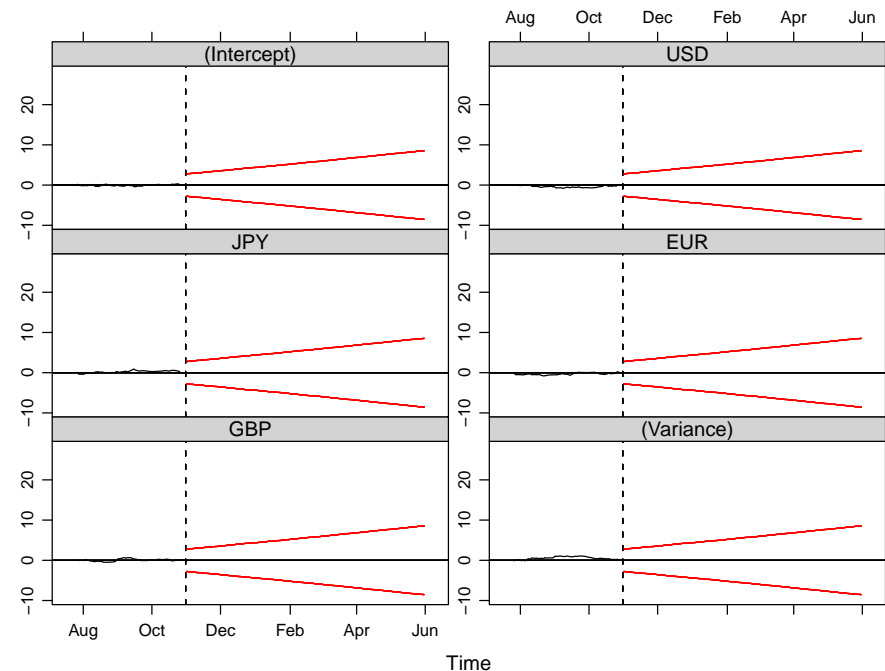
**More formally:** Sequentially test the null hypothesis

$$H_0 : \theta_i = \theta_0 \quad (i > n)$$

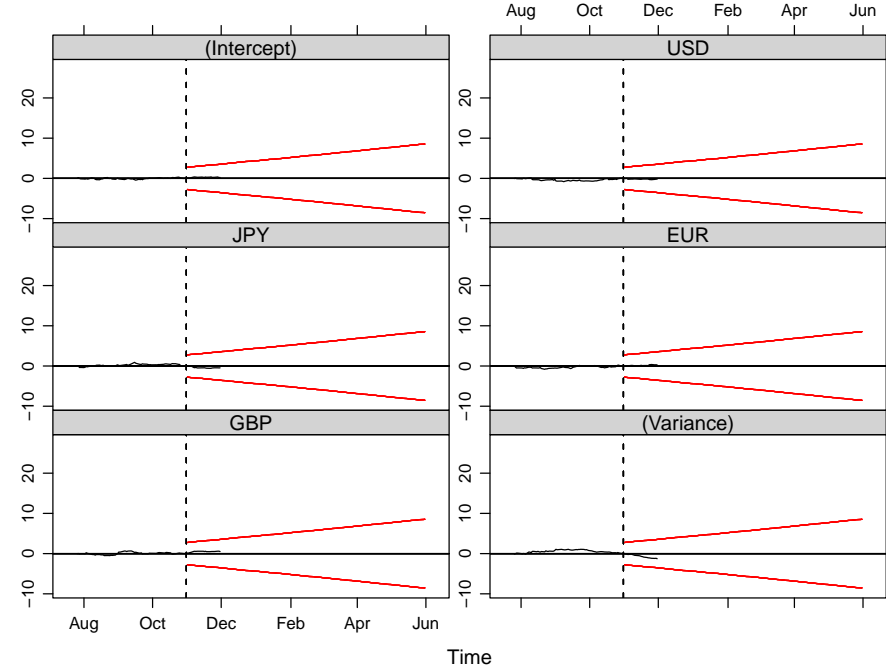
against the alternative that  $\theta_i$  changes at some time in the future  $i > n$  (corresponding to  $t > 1$ ).

**Basic assumption:** The model parameters are stable  $\theta_i = \theta_0$  in the history period  $i = 1, \dots, n$  ( $0 \leq t \leq 1$ ).

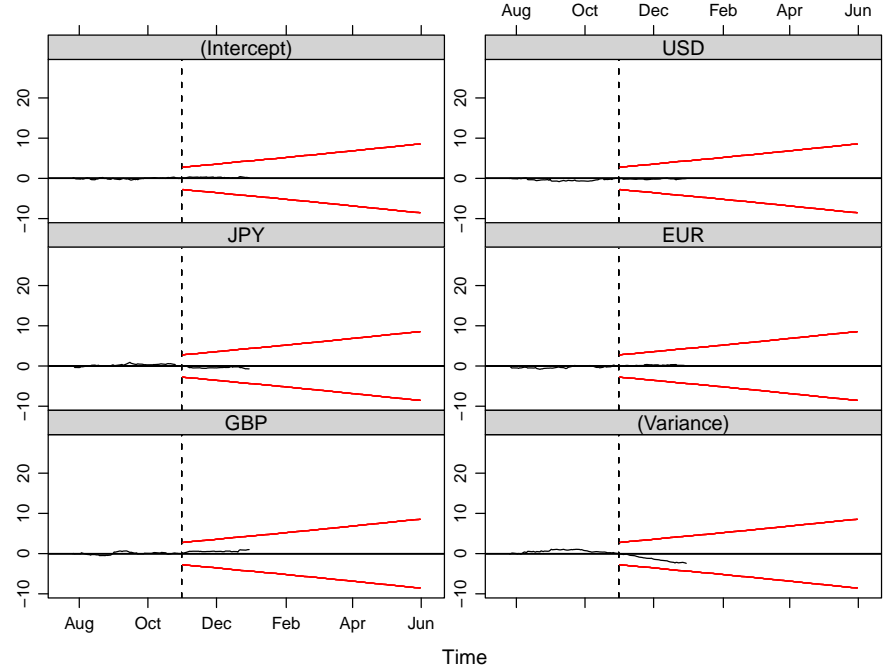
## Monitoring



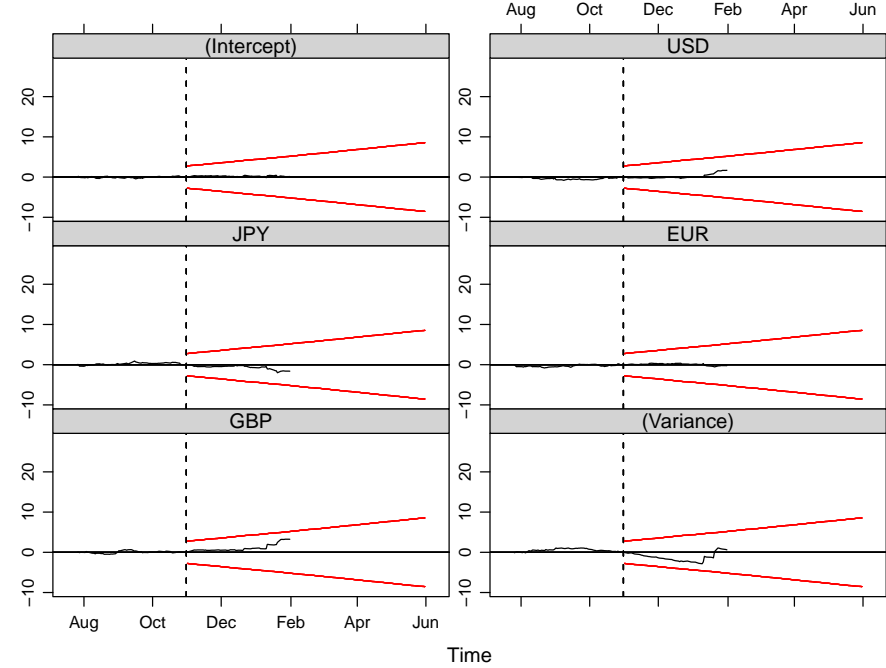
Monitoring



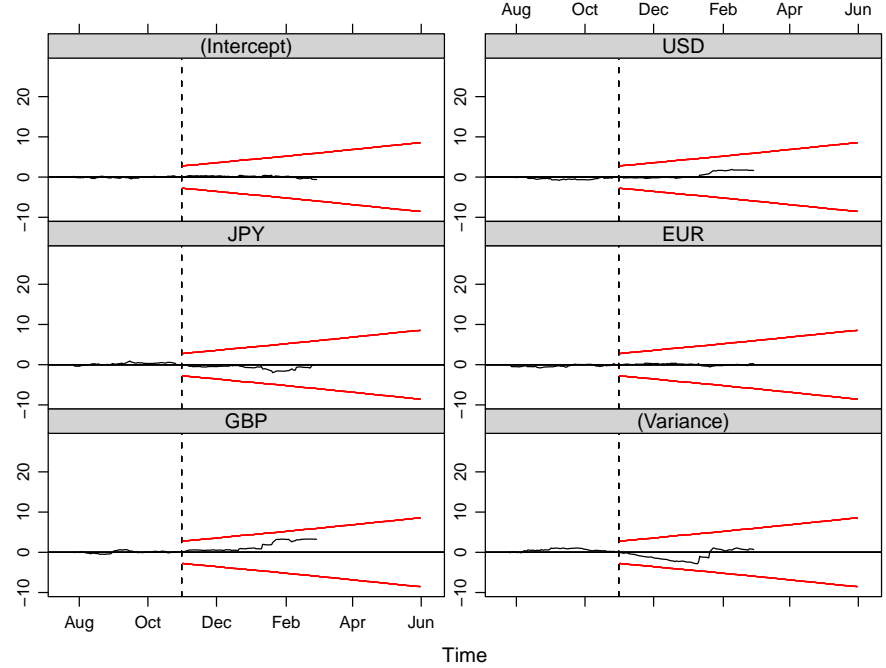
Monitoring



Monitoring

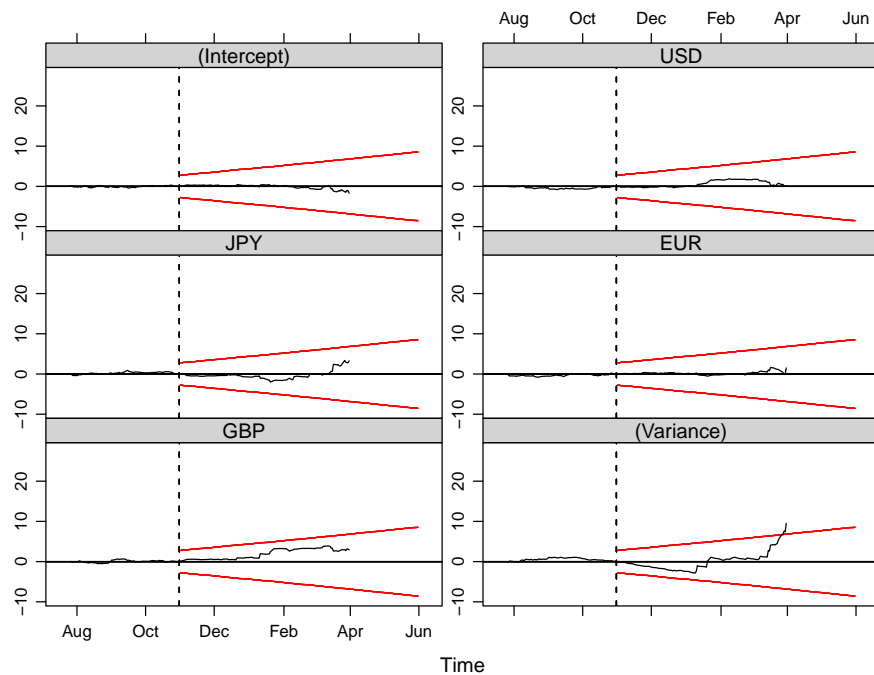


Monitoring





## Monitoring



## Monitoring

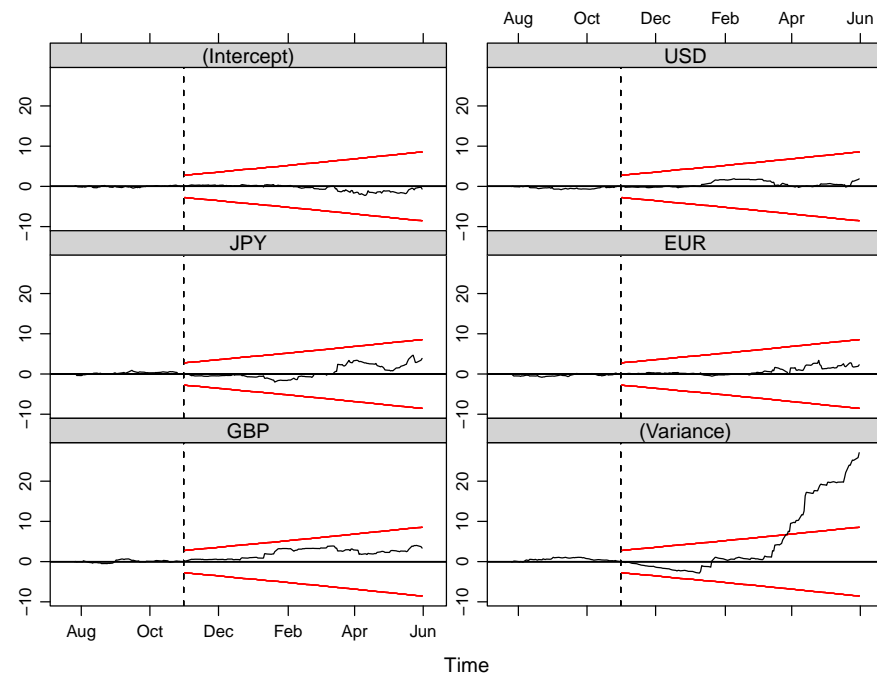
This signals a clear increase in the error variance.

The change is picked up by the monitoring procedure on 2006-03-27.

The other regression coefficients did not change significantly, signalling that they are not part of the basket peg.

Using data from an extended period up to 2009-07-31, we fit a segmented model to determine where and how the model parameters changed.

## Monitoring



## Dating

**Segmented regression model:** A stable model with parameter vector  $\theta^{(j)}$  holds for the observations in  $i = i_{j-1} + 1, \dots, i_j$ . The segment index is  $j = 1, \dots, m + 1$ .

The set of  $m$  breakpoints  $\mathcal{I}_{m,n} = \{i_1, \dots, i_m\}$  is called  $m$ -partition. Convention:  $i_0 = 0$  and  $i_{m+1} = n$ .

The value of the segmented objective function  $\Psi$  is

$$PSI(i_1, \dots, i_m) = \sum_{j=1}^{m+1} psi(i_{j-1} + 1, i_j),$$

$$psi(i_{j-1} + 1, i_j) = \sum_{i=i_{j-1}+1}^{i_j} \Psi(y_i, x_i, \hat{\theta}^{(j)}).$$

## Dating

Thus,  $psi(i_{j-1} + 1, i_j)$  is the minimal value of the objective function for the model fitted on the  $j$ th segment.

Dating tries to find

$$(\hat{i}_1, \dots, \hat{i}_m) = \underset{(i_1, \dots, i_m)}{\operatorname{argmin}} PSI(i_1, \dots, i_m)$$

over all partitions  $(i_1, \dots, i_m)$  with  $i_j - i_{j-1} + 1 \geq \lfloor nh \rfloor \geq k$ .

Bellman principle of optimality:

$$PSI(\mathcal{I}_{m,n}) = \min_{mn_h \leq i \leq n-n_h} [PSI(\mathcal{I}_{m-1,i}) + psi(i+1, n)]$$

## Dating

Thus, for a given number of breaks  $m$ , the optimal breaks  $\hat{i}_1, \dots, \hat{i}_m$  be found.

To determine the number of breaks, some model selection has to be done, e.g., via information criteria or sequential tests. Here, we use the LWZ criterion (modified BIC):

$$\begin{aligned} IC(m) &= 2 \cdot NLL(\mathcal{I}_{m,n}) + \text{pen} \cdot ((m+1)k + m), \\ \text{pen}_{\text{BIC}} &= \log(n), \\ \text{pen}_{\text{LWZ}} &= 0.299 \cdot \log(n)^{2.1}. \end{aligned}$$

## Dating

It is well-known that this problem can be solved by a dynamic programming algorithm of order  $O(n^2)$  that essentially relies on a triangular matrix of  $psi(i, j)$  for all  $1 \leq i < j \leq n$ .

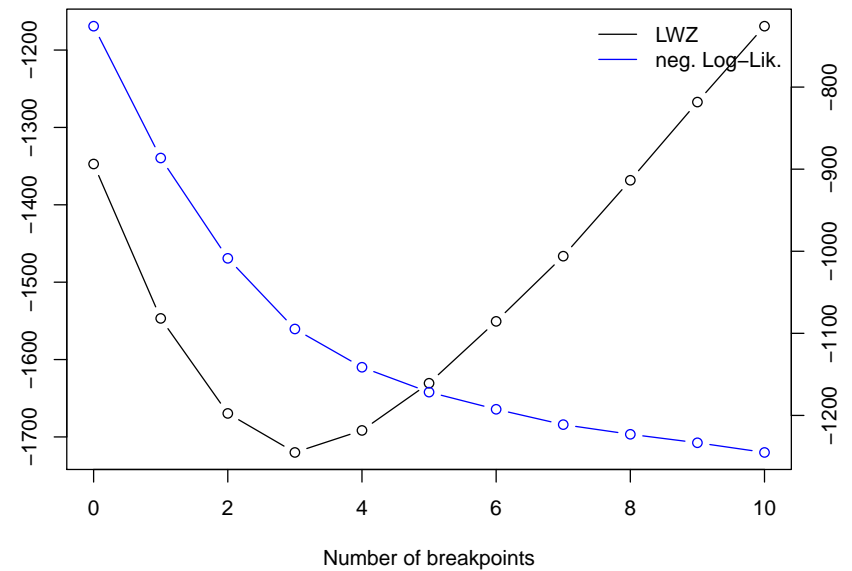
In linear regressions this approach has been popularized by Bai & Perron and it is common practice to use the residual sum of squares as objective function:

$$\Psi_{\text{RSS}}(y_i, x_i, \beta) = (y_i - x_i^\top \beta)^2.$$

To capture changes in the variances as well the (negative) log-likelihood from a normal model can be employed:

$$\Psi_{\text{NLL}}(y_i, x_i, \beta, \sigma) = -\log \left( \sigma^{-1} \phi \left( \frac{y_i - x_i^\top \beta}{\sigma} \right) \right).$$

## Dating



## Dating

The estimated breakpoints and parameters are:

start/end	$\beta_0$	$\beta_{USD}$	$\beta_{JPY}$	$\beta_{EUR}$	$\beta_{GBP}$	$\sigma$	$R^2$
2005-07-26	<b>-0.005</b>	<b>0.999</b>	0.005	-0.015	0.007	0.028	0.998
2006-03-14	(0.002)	(0.005)	(0.005)	(0.017)	(0.008)		
2006-03-15	<b>-0.025</b>	<b>0.969</b>	-0.009	0.026	-0.013	0.106	0.965
2008-08-22	(0.004)	(0.012)	(0.010)	(0.023)	(0.012)		
2008-08-25	-0.015	<b>1.031</b>	-0.026	0.049	0.007	0.263	0.956
2008-12-31	(0.030)	(0.044)	(0.030)	(0.059)	(0.035)		
2009-01-02	0.001	<b>0.981</b>	0.008	-0.008	0.009	0.044	0.998
2009-07-31	(0.004)	(0.005)	(0.004)	(0.009)	(0.004)		

corresponding to

- ① tight USD peg with slight appreciation,
- ② slightly relaxed USD peg with some more appreciation,
- ③ slightly relaxed USD peg without appreciation,
- ④ tight USD peg without appreciation.

## Dating

**Epilogue:** What happened since summer 2009?

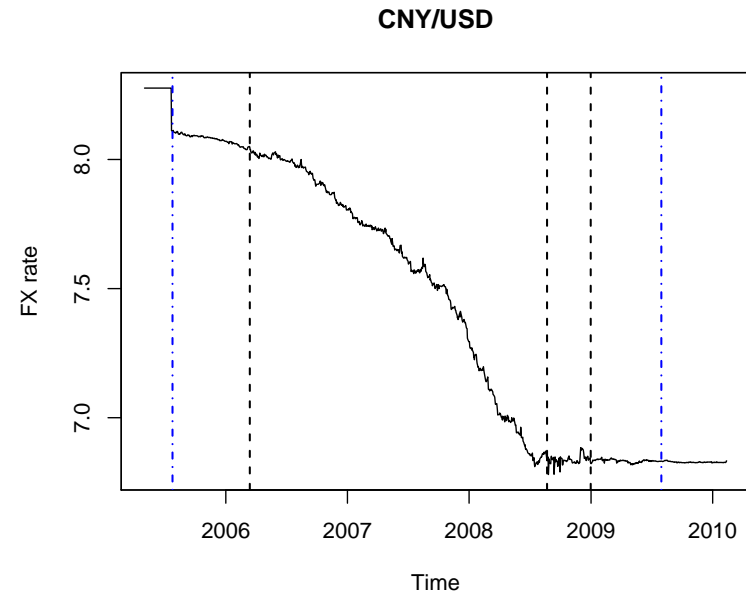
Estimation based on 2009-08-04 through 2010-01-29 ( $n = 122$ ) gives:

$$\begin{aligned}
 \text{CNY}_i = & \underset{(0.002)}{0.001} + \underset{(0.003)}{0.9953} \text{USD}_i + \underset{(0.003)}{0.002} \text{JPY}_i \\
 & + \underset{(0.011)}{0.007} \text{EUR}_i + \underset{(0.003)}{0.004} \text{GBP}_i + \hat{\varepsilon}_i
 \end{aligned}$$

Only the USD coefficient is significantly different from 0 (but not from 1).

The error standard deviation became even smaller with  $\hat{\sigma} = 0.018$  leading to  $R^2 = 0.999$ .

## Dating



## Software

All methods are implemented in the R system for statistical computing and graphics and are freely available in the contributed packages *strucchange* and *fxregime* from the Comprehensive R Archive Network:

<http://www.R-project.org/>  
<http://CRAN.R-project.org/>

## Software: *strucchange*

Classical structural change tools for OLS regression:

- Testing: `efp()`, `Fstats()`, `sctest()`.
- Monitoring: `mefp()`, `monitor()`.
- Dating: `breakpoints()`.
- Vignette: "strucchange-intro".

Object-oriented structural change tools:

- Testing: `gefp()`, `efpFunctional()` (including special cases: `maxBB`, `meanL2BB`, `supLM`, ...).
- Monitoring: Object-oriented implementation still to do.
- Dating: Some currently unexported support in `gbreakpoints()` in *fxregime*.
- Vignette: None, but CSDA paper.

## Application: Indian FX regimes

India also has an expanding economy with a currency receiving increased interest over the last years. We track the evolution of the INR FX regime since trading in the INR began.

```
R> head(FXRatesCHF[, c(1:6, 13)], 3)
      USD  JPY  DUR EUR  DEM  GBP INR
1971-01-04 0.232 82.8 0.429 NA 0.844 0.0967 NA
1971-01-05 0.232 83.0 0.429 NA 0.845 0.0968 NA
1971-01-06 0.232 83.0 0.429 NA 0.845 0.0968 NA

R> inr <- fxreturns("INR", data = FXRatesCHF,
+   other = c("USD", "JPY", "DUR", "GBP"), frequency = "weekly",
+   start = as.Date("1993-04-01"), end = as.Date("2008-01-04"))
R> head(inr, 3)
      INR  USD  JPY  DUR  GBP
1993-04-09 0.9773 0.9773 0.0977 0.567 -0.02236
1993-04-16 -0.0339 -0.0339 -0.5387 0.625 0.14295
1993-04-23 3.2339 3.2339 1.4331 1.264 0.00876
```

## Software: *fxregime*

Structural change tools for exchange rate regression based on normal (quasi-)ML:

- Data: `FXRatesCHF` ("zoo" series with US Federal Reserve exchange rates in CHF for various currencies).
- Preprocessing: `fxreturns()`.
- Model fitting: `fxlm()`.
- Testing: `gefp()` from *strucchange*.
- Monitoring: `fxmonitor()`.
- Dating: `fxregimes()` based on currently unexported `gbreakpoints()`; `refit()` method for fitting segmented regression.
- Vignettes: "CNY", "INR".

## Application: Indian FX regimes

Using weekly returns from 1993-04-09 through to 2008-01-04 (yielding  $n = 770$  observations), we fit a single FX regression using the same basket as above.

```
R> inr_lm <- fxlm(INR ~ USD + JPY + DUR + GBP, data = inr)
R> coef(inr_lm)
(Intercept)      USD      JPY      DUR      GBP
    0.0280    0.9185    0.0405    0.1046    0.0484
(Variance)
    0.3375
```

## Application: Indian FX regimes

As we would expect multiple changes, we assess its stability with the Nyblom-Hansen test. Alternatively, a MOSUM test could be used. The double maximum test has less power.

```
R> inr_efp <- gefp(inr_lm, fit = NULL)
R> sctest(inr_efp, functional = meanL2BB)
```

### M-fluctuation test

```
data:  inr_efp
f(efp) = 3.11, p-value = 0.005
```

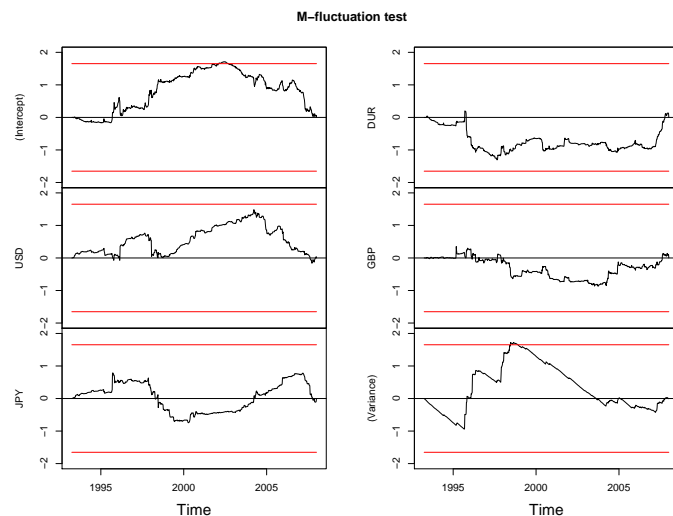
```
R> sctest(inr_efp, functional = maxBB)
```

### M-fluctuation test

```
data:  inr_efp
f(efp) = 1.72, p-value = 0.03099
```

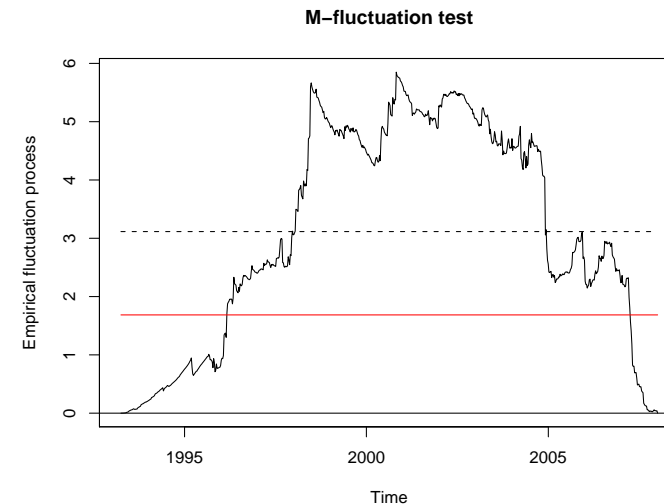
## Application: Indian FX regimes

```
R> plot(inr_efp, functional = maxBB, aggregate = FALSE,
+       ylim = c(-2, 2))
```



## Application: Indian FX regimes

```
R> plot(inr_efp, functional = meanL2BB)
```



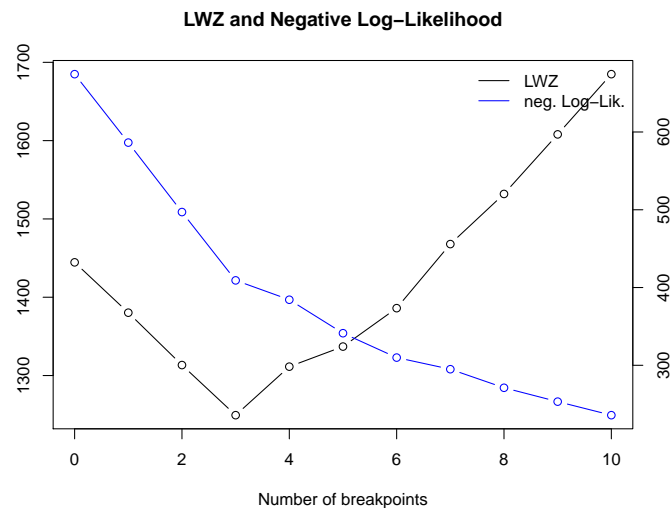
## Application: Indian FX regimes

Dating is computationally more demanding. The dynamic programming algorithm can be parallelized, though. This is easily available (thanks to Anmol Sethy) by means of optional *foreach* support in *fxregime*.

```
R> library("foreach")
R> library("doMC")
R> registerDoMC(2)
R> inr_reg <- fxregimes(INR ~ USD + JPY + DUR + GBP, data = inr,
+   h = 20, breaks = 10, hpc = "foreach")
```

## Application: Indian FX regimes

```
R> plot(inr_reg)
```



## Application: Indian FX regimes

Somewhat more compactly:

start/end	$\beta_0$	$\beta_{USD}$	$\beta_{JPY}$	$\beta_{DUR}$	$\beta_{GBP}$	$\sigma$	$R^2$
1993-04-09 1995-03-03	-0.006 (0.017)	<b>0.972</b> (0.018)	0.023 (0.014)	0.011 (0.032)	0.020 (0.024)	0.157	0.989
1995-03-10 1998-08-21	<b>0.161</b> (0.071)	<b>0.943</b> (0.074)	0.067 (0.048)	-0.026 (0.155)	0.042 (0.080)	0.924	0.729
1998-08-28 2004-03-19	0.019 (0.016)	<b>0.993</b> (0.016)	0.010 (0.010)	<b>0.098</b> (0.034)	-0.003 (0.021)	0.275	0.969
2004-03-26 2008-01-04	-0.058 (0.042)	<b>0.746</b> (0.045)	<b>0.126</b> (0.042)	<b>0.435</b> (0.116)	<b>0.121</b> (0.056)	0.579	0.800

corresponding to

- ① tight USD peg,
- ② flexible USD peg,
- ③ tight USD peg,
- ④ flexible basket peg.

## Application: Indian FX regimes

Various methods for extracting information can be applied directly. Otherwise, refitting of FX regressions gives access to all quantities that might be of interest.

```
R> coef(inr_reg)[, 1:5]
```

```
(Intercept)  USD    JPY    DUR    GBP
1993-04-09--1995-03-03  -0.00574  0.972  0.02347  0.0113  0.02037
1995-03-10--1998-08-21   0.16113  0.943  0.06692 -0.0261  0.04236
1998-08-28--2004-03-19   0.01861  0.993  0.00976  0.0983 -0.00322
2004-03-26--2008-01-04  -0.05761  0.746  0.12561  0.4354  0.12137
```

```
R> inr_rf <- refit(inr_reg)
```

```
R> sapply(inr_rf, function(x) summary(x)$r.squared)
```

```
1993-04-09--1995-03-03 1995-03-10--1998-08-21 1998-08-28--2004-03-19
0.989                    0.729                    0.969
2004-03-26--2008-01-04
0.800
```

## Next steps

**Current activities:** Application to wider range of currencies.

**Of particular interest:** Classification of exchange rate regimes and monitoring.

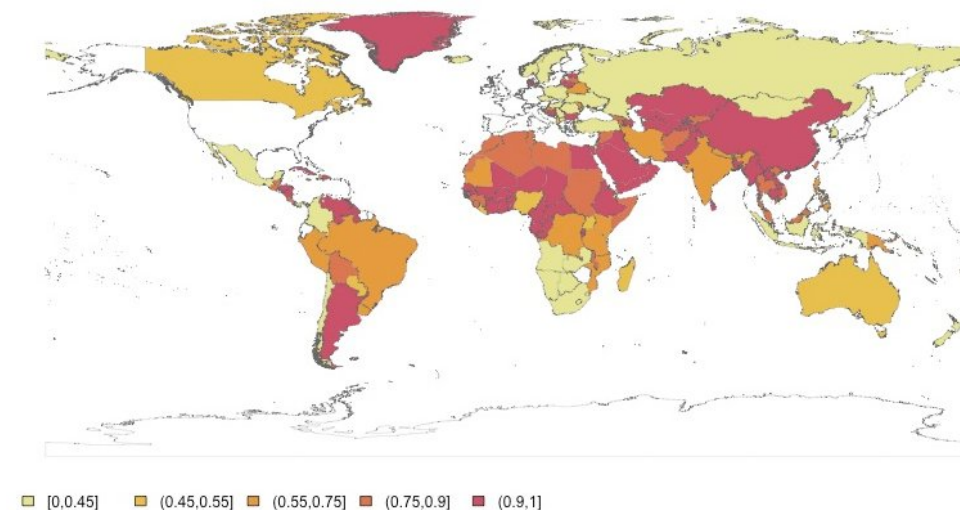
**Open problems:**

- Fully automatic selection of breakpoints.
- Sequential usage of BIC/LWZ, i.e., with growing sample size  $n$ .
- Differences between subsequent regimes that are statistically significant but not practically relevant (or vice versa).

**First steps:** Anmol Sethy started to build infrastructure for larger FX rates database from mixed sources.

**First results:** World map of  $R^2$  from FX regressions (basket: USD, EUR, JPY, GBP), November 2009, based on segmented weekly data.

## Next steps



## Summary

- Exchange rate regime analysis can be complemented by structural change tools.
- Both coefficients (currency weights) and error variance (fluctuation band) can be assessed using an (approximately) normal regression model.
- Estimation, testing, monitoring, and dating are all based on the same model, i.e., the same objective function.
- Traditional significance tests can be complemented by graphical methods conveying timing and component affected by a structural change.
- Software is freely available, both for the general method and the application to FX regimes.

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