Can you do better than cap-weighted equity benchmarks?

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INVESTING ALWAYS INVOLVES RISK.
Outline

1. Introduction to efficient indexes
2. Overview of modeling
3. Analysis of results
4. Wrap-Up
Outline

1. Introduction to efficient indexes
2. Overview of modeling
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4. Wrap-Up
The tangency portfolio

Efficient Frontier

sdP

muP

F

T

MV

0 5 10 15
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5
The tangency portfolio

Efficient Frontier

\[ T \approx \text{S&P500} \, ? \]
Is Your Index Fund Broken?

Worth Their Weighting?

Over 11 years ended Jan. 1, 2010, indexes that aren’t tied to market values outperformed.

<table>
<thead>
<tr>
<th></th>
<th>Efficient</th>
<th>Minimum Volatility</th>
<th>Fundamental</th>
<th>S&amp;P 500 Equal-Weighted</th>
<th>S&amp;P 500</th>
<th>Russell 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap-Weighted</td>
<td></td>
<td>2</td>
<td>4</td>
<td>6%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-cap-Weighted</td>
<td>6%</td>
<td>4</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: The Journal of Indexes

Jack Hough, SmartMoney, "Is Your Index Fund Broken?", January 31, 2011
The efficient market inefficiency of capitalization-weighted stock portfolios

“Matching the market is an inefficient investment strategy.”

Robert A. Haugen and Nardin L. Baker

Motivation for research

Efficient Indexation

- maximize Sharpe ratio

\[ w^* = \arg \max_w \frac{w' \mu}{\sqrt{w' \Sigma w}} \]

- covariance matrix
  - derived from principal component analysis (PCA)

- expected returns
  - form deciles by downside risk
  - expected return equals mean of each decile

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Research project

Goal

- Compare performance of alternative index constructions using S&P 500 constituents

Methodology

- use a rolling 2-year window of current constituent returns and re-balance at the start of each month
- generate 48-months of out-of-sample index returns (Jan-2007 to Dec-2010)
- S&P 500 returns were calculated using constituent weights (apples-to-apples comparisons without factoring in transaction costs)

Constraint

- positive weights (max of 25%)

Focus of research

- minimum risk (minimum variance and minimum CVaR) portfolios
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Global minimum variance portfolio

Efficient Frontier

muP

sdP

Global Minimum Variance Portfolio

F

T

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M-V optimization and Quadratic Programming

**general QP problem**

\[
\min_b \quad \frac{1}{2} b^T Db - b^T d \\
\text{s.t.} \quad A^T b \geq b_0 \\
\quad b \geq 0
\]

**mean-variance portfolio optimization**

\[
\min_b \quad \omega^T \Sigma \omega \\
\text{s.t.} \quad \omega^T \mu = \mu_p \\
\quad \omega^T 1 = 1 \\
\quad \omega_{min} \geq \omega_i \geq \omega_{max}
\]

**R Code: the solve.QP function**

```r
> library(quadprog)
> args(solve.QP)

function (Dmat, dvec, Amat, bvec, meq = 0, factorized = FALSE) NULL
```

objective function assignments: \(2\Sigma \rightarrow D \quad \omega \rightarrow b \quad 0 \rightarrow d\)
The general form of a factor model for asset returns is:

\[ R_{j,t} = \beta_{0,j} + \beta_{1,j} F_{1,t} + \cdots + \beta_{p,j} F_{p,t} + \epsilon_{j,t} \]

where

- \( R_{j,t} \) is either return or excess return on the jth asset at time t
- \( F_{1,t}, \ldots, F_{p,t} \) are factors (aka risk factors) at time t
- \( \epsilon_{1,t}, \ldots, \epsilon_{n,t} \) are uncorrelated, mean-zero, unique risks

The factor model in matrix form is:

\[ R_t = \beta_0 + \beta^T F_t + \epsilon_t \]
Given the following covariance matrices:

\[ \Sigma_\epsilon = \begin{pmatrix}
\sigma_{\epsilon,1}^2 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sigma_{\epsilon,n}^2
\end{pmatrix} \]

\[ \Sigma_F = p \times p \text{ covariance matrix of } (F_t) \]

The returns covariance matrix is:

\[ \Sigma_R = \beta^T \Sigma_F \beta + \Sigma_\epsilon \]
Covariance matrix estimation

- Estimating the covariance matrix based on a factor model is a bias-versus-variance trade-off
  - sample covariance matrix is unbiased but may have significant estimation error
  - estimating the covariance matrix via a factor model may be biased but also may significantly reduce estimation error by significantly reducing the number of estimates

- Sample covariance matrix for n-assets
  - \( n(n + 1)/2 \) estimations
  - for 500 assets, 125,250 estimates are required

- Covariance matrix with n-assets and a factor model with p-factors
  - \( np + n + p^2 \) estimations
  - for 500 assets and 10 factors, 5,600 estimates are required
Industry factor model

Model background
- Sheikh, "Barra’s Risk Models", 1995

Response
- daily equity returns

Explanatory variables
- company industry classification

Model details
Cross-sectional factor models

Differences between time-series factor models and cross-sectional factor models:

<table>
<thead>
<tr>
<th>Model type</th>
<th>Assets</th>
<th>Time Periods</th>
<th>Factors</th>
<th>Betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>time-series</td>
<td>one asset at a time</td>
<td>all time periods</td>
<td>known</td>
<td>estimated</td>
</tr>
<tr>
<td>cross-section</td>
<td>all assets</td>
<td>one period at a time</td>
<td>estimated</td>
<td>known</td>
</tr>
</tbody>
</table>

Cross-sectional factor model for the jth asset at some fixed t:

\[ R_j = \beta_0 + \beta_1 F_{1,j} + \cdots + \beta_p F_{p,j} + \epsilon_j \]
Industry factor model

General industry factor model has the following form:

\[ R_j = \beta_1 F_{1,j} + \beta_2 F_{2,j} + \cdots + \beta_p F_{p,j} + \epsilon_j \]

\[ \beta_i = \begin{cases} 
1, & \text{if asset } j \text{ in industry } i \\
0, & \text{if asset } j \text{ not in industry } i 
\end{cases} \]

- Factor realizations represent a weighted average return in time period \( t \) of all of the asset returns for companies operating in industry \( j \)

- S&P Sector GICS codes for 10 sectors (10 sectors):
  - energy
  - materials
  - industrial
  - discretionary
  - staples
  - health
  - financial
  - info tech
  - telecom
  - utilities
Recall the general form of a factor model:

\[ R_t = \beta_0 + \beta^T F_t + \epsilon_t \]

- In statistical factor models:
  - factor realizations are not directly observable
  - no external knowledge of betas (as in cross-sectional models)
  - factor realizations and betas must be extracted from the returns data using statistical methods

- Principal component analysis - eigen decomposition of covariance matrix
PCA statistical factor model

Model background


Response

- daily equity returns

Explanatory variables

- principal components

Model details

Conditional Value-at-Risk

![Conditional Value-at-Risk Diagram](image)

- \( \alpha \)
- \( 1 - \alpha \)
- Value-at-Risk
- CVaR
- P&L Distribution

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It can be shown that minimizing the CVaR of a portfolio is a linear programming problem that can be carried out with a general-purpose LP solver†

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†Yollin, "R Tools for Portfolio Optimization", R/Finance 2009
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Cumulative return comparisons

Cumulative Returns

SP500
equal weights
min var sample cov
cumulative return

Yollin/Kumar (Copyright © 2011)  Beating the benchmark  R/Finance 2011 24 / 32
Cumulative return comparisons

Drawdown from Peak Equity Attained

-0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0.0

SP500
equal weights
min var sample cov
drawdown

Jan 07 Jul 07 Jan 08 Jul 08 Jan 09 Jul 09 Jan 10 Jul 10 Dec 10

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Cumulative return comparisons

Cumulative Returns

SP500
min var industry
min var PCA
min CVaR

cumulative return

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Cumulative return comparisons

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SP500
min var industry
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Drawdown from Peak Equity Attained

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## Summary

<table>
<thead>
<tr>
<th></th>
<th>SP500</th>
<th>minVaRSample</th>
<th>minVarIndustry</th>
<th>minVarPCA</th>
<th>minCVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Return</td>
<td>-0.106</td>
<td>-0.086</td>
<td>0.055</td>
<td>0.032</td>
<td>0.025</td>
</tr>
<tr>
<td>Annualized Return</td>
<td>-0.028</td>
<td>-0.022</td>
<td>0.013</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td>Annualized StdDev</td>
<td>0.241</td>
<td>0.138</td>
<td>0.161</td>
<td>0.174</td>
<td>0.139</td>
</tr>
<tr>
<td>Conditional VaR</td>
<td>-0.159</td>
<td>-0.105</td>
<td>-0.118</td>
<td>-0.126</td>
<td>-0.100</td>
</tr>
<tr>
<td>Max DrawDown</td>
<td>0.549</td>
<td>0.337</td>
<td>0.370</td>
<td>0.406</td>
<td>0.329</td>
</tr>
</tbody>
</table>

- All minimum variance portfolios and the minimum CVaR portfolio outperformed the S&P 500 Index during the testing period
  - Higher annualized return
  - Lower annualized volatility
  - Smaller conditional value-at-risk
  - Smaller maximum drawdown

- Returns are difficult (impossible) to forecast and these techniques don’t require them

Can you do better than cap-weighted equity benchmarks? Maybe!
Special thanks

SunGard Financial Systems

- Historical S&P 500 constituent weights
- Historical stock prices
Special thanks

Revolution Analytics

- Revolution R Enterprise and RevoScaleR
Questions and comments

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