

# Modeling Low default Credit Portfolio in R.

By

Angshuman Dey

Brian G. Peterson

Gaurav Yadav

**Low default portfolios (LDP) can be defined as those portfolios where there is extremely low or no occurrence of default events.**

*These low-default portfolios are characterized by the lack of sufficient data and the resulting difficulty in back-testing the Probability of Default.*

**Examples of low-default portfolios are portfolios with exposures to:**

- Banks, insurance companies, sovereigns, highly-rated corporate obligors and most forms of specialized lending like project finance.

- *However, as a fallout of credit crisis, there are instances of occurrence of default in highly rated categories also.*

**Low default portfolios can not be excluded from the purview of computation of capital requirement because of scarcity of the data**

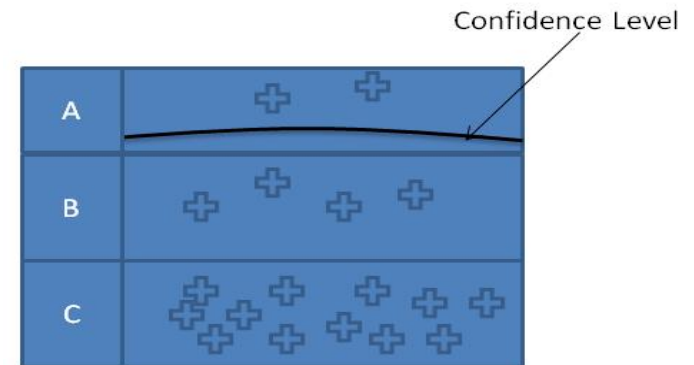
## Application of traditional methods doesn't necessarily work out because of following reasons:

- Statistical models are largely dependent on the historical data and due to scarcity of such data result would be unreliable and sometimes wrong.
- Due to inherent nature of the sector qualitative factors plays a dominant role in determining the risk profile of the counterparty.

**The methodology suggested by Pluto and Tasche is based on most prudent estimation principle , detailed out in the subsequent slides.**

### Most Prudent Principle

The most prudent principle attempts to estimate with a degree of conservatism the maximum probability of default for a rating grade such that no more defaults occur than the current grade and lower grades.



Obligor Grade A > Grade B > Grade C

+ Default event

## Approach and Assumption

- Estimate PDs for portfolios with no defaults, or a very low number of defaults in the overall portfolio.
- Uses all available quantitative information of the rating system and its grades.
- Estimate the PDs by upper confidence bounds
- PD ordering as indicated by the rating grades.
- Applied under an assumption of independent default events but can be adapted to the case of correlated defaults

## Methodology Overview

- Define number of borrowers in each grade and defaults in each grade
- Define number of years for which through the cycle PD has to be estimated based on inter-temporal correlation matrix between borrower grades.
- Define asset correlation and confidence level at which the PDs have to be estimated.
- Define borrower's asset value change process incorporating systematic factor and idiosyncratic components.
- Estimate probability that the borrower defaults under the assumption of systematic and the idiosyncratic factors are independent; implying that the change in asset value is normally distributed. Further, the correlation matrix describes the joint distribution of inter-temporal default events and the underlying dependence structure is normally distributed.
- Estimation of probability of default in each borrower category based on '*most prudent principle*'.

# Case Study

## Data

Following is a brief snapshot of the primary data used:

- Number of obligor grades = 3
- Number of obligors in each of the grades and the default events:

Obligor Grades	A	B	C
Number of obligors	100	400	300
Defaults observed during one year	0	2	1

## Correlation Matrix

Years	1	2	3	4	5
1	100.00%	30.00%	9.00%	2.70%	0.81%
2	30.00%	100.00%	30.00%	9.00%	2.70%
3	9.00%	30.00%	100.00%	30.00%	9.00%
4	2.70%	9.00%	30.00%	100.00%	30.00%
5	0.81%	2.70%	9.00%	30.00%	100.00%

## Example

### Replicate the original Pluto and Tasche paper:

- Utilize the data from the paper
- Little bond data is publicly available, and portfolio data is proprietary
- Replicate case “Correlated Default Events Extending to Multi-Period Case”
- Utilize the correlation matrix provided
- Stress the matrix per Basel II

# Results & Chart

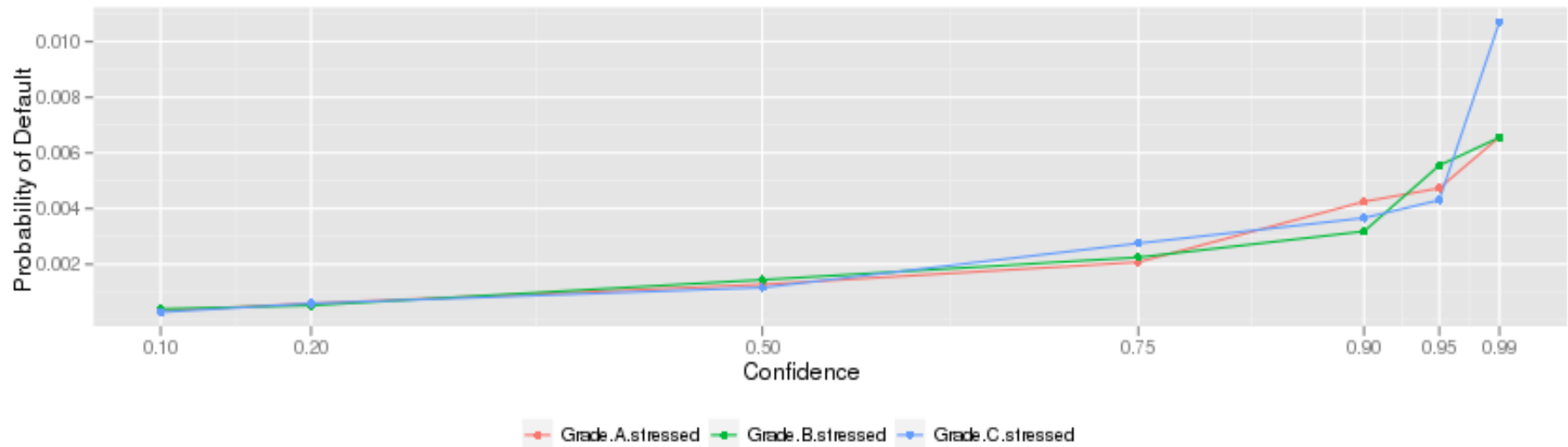
Following are the key assumptions relating to the user input data requirements:

- **Default correlation matrix**
- **Minimum value of asset correlation for stressed scenario as per Basel II corporate risk weight function (12%)**
- **The time horizon is taken as 5 years as the PD estimates are to be based on a time series of minimum 5 years as per Basel II.**

We derived the probability of default at multiple confidence levels for each of the grade. The results are presented graphically here, showing monotonically increasing default probabilities in all the categories as we seek to increase the confidence level.

99.9% confidence PD exceeds 40% for all grades, reaching as high as 81% (not shown because of scale)

## Stressed





Low default portfolios (LDP) can be defined as those portfolios where there is extremely low or no occurrence of default events. These low-default portfolios are characterized by the lack of sufficient data and the resulting difficulty in back-testing the Probability of Default.

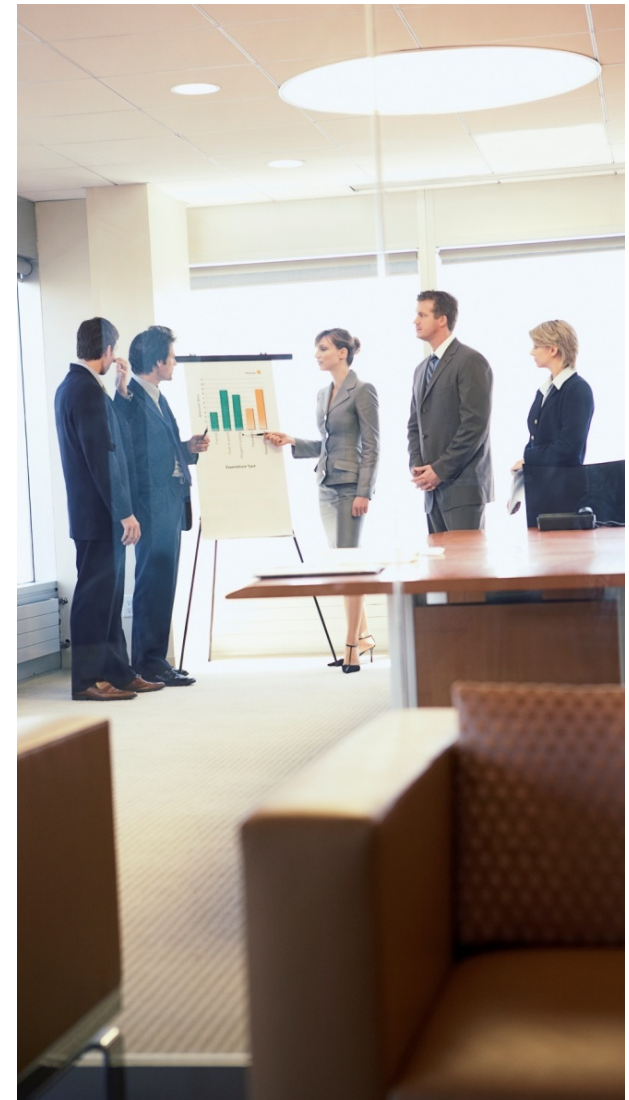
Examples of low-default portfolios are portfolios with exposures to banks, insurance companies, sovereigns, highly-rated corporate obligors and most forms of specialized lending like project finance. However, as a fallout of credit crisis, there are instances of occurrence of default in highly rated categories also.

Low default portfolios can not be excluded from the perview of computation of capital requirement because of scarcity of the data

Application of traditional methods doesn't necessarily works out because of following reasons:

- Statistical models are largely dependent on the historical data and due to scarcity of such data result would be unreliable and sometimes wrong.
- Due to inherent nature of the sector qualitative factors plays a dominant role in determining the risk profile of the counterparty.

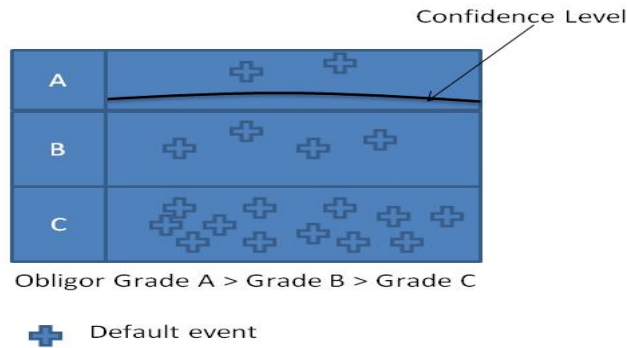
The methodology suggested by Pluto and Tasche is based on *most prudent estimation* principle , detailed out in the subsequent slides.



# Methodology Overview

## Most Prudent Principle

The most prudent principle attempts to estimate with a degree of conservatism the maximum probability of default for a rating grade such that no more defaults occur than the current grade and lower grades.



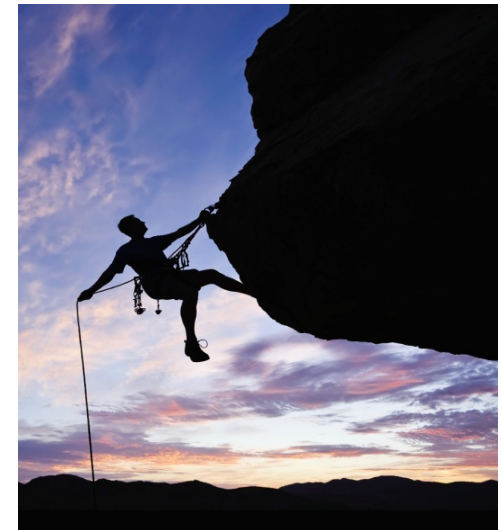
## Approach and Assumption

The methodology developed by Pluto and Tasche is intended to estimate PDs for portfolios without any defaults, or a very low number of defaults in the overall portfolio. It uses all available quantitative information of the rating system and its grades.

This means to estimate the PDs by upper confidence bounds while guaranteeing at the same time a PD ordering that respects the differences in credit quality indicated by the rating grades. The methodology is most easily applied under an assumption of independent default events but can be adapted to the case of correlated defaults.

## Methodology

- Define number of borrower in each grade and defaults in each grade
- Define number of years for which through the cycle PD has to be estimated based on inter-temporal correlation matrix between borrower grades.
- Define asset correlation and confidence level at which the PDs have to be estimated.
- Define borrower's asset value change process incorporating systematic factor and idiosyncratic components.
- Estimate probability that the borrower defaults under the assumption of systematic and the idiosyncratic factors are independent; implying that the change in asset value is normally distributed. Further, the correlation matrix describes the joint distribution of inter-temporal default events and the underlying dependence structure is normally distributed.
- Estimation of probability of default in each borrower category based on 'most prudent principle'.



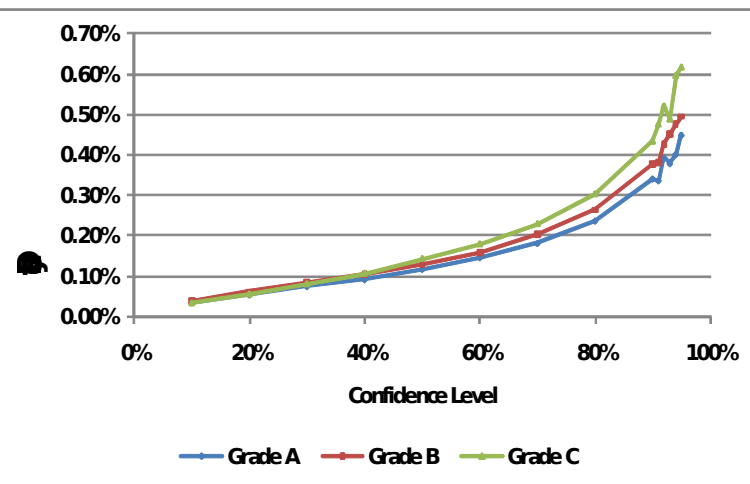
# Case Study

## Data

Following is a brief snapshot of the primary data used:

- Number of obligor grades = 3
- Number of obligors in each of the grade and the default events:

Obligor Grades	A	B	C
Number of obligors	100	400	300
Defaults observed during one year	0	2	1



## Example

We did perform the test with the test data provided in the original paper , in view of the scarcity of the data and it's confidentiality. The computation was performed for 'Correlated default events extending to multi-period case'.

*We did use the correlation matrix given to generate PD values and also checked the consistency of the solution by stressing the correlation matrix.*

Following are the key assumptions relating to the user input data requirements:

- Default correlation matrix
- Minimum value of asset correlation as per Basel II corporate risk weight function (12%)
- The time horizon is taken as 5 years as the PD estimates are to be based on a time series of minimum 5 years as per Basel II.

Based on the above mentioned assumptions we followed the sequence of steps mentioned in the 'Methodology' . We derived the probability of default at multiple confidence level for each of the grade.

The results are presented graphically here ,showing monotonically increasing default probabilities in all the categories as we seek to increase the confidence level.