The peer performance of hedge funds

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Work in progress – Comments welcome

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How to do performance analysis?

- Compute Sharpe ratio, Jensen's alpha, Treynor ratio, information ratio, modified Sharpe ratio, ... and interpret this $\hat{\alpha}_i$, in comparison with those of "peer" funds $\hat{\alpha}_j$ $(j = 1, \ldots, n)$.

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- Compute Sharpe ratio, Jensen's alpha, Treynor ratio, information ratio, modified Sharpe ratio, ... and interpret this $\hat{\alpha}_i$, in comparison with those of "peer" funds $\hat{\alpha}_j$ $(j = 1, \ldots, n)$.
- (Statistically significant) differences can be because of luck. E.g. if $H_0 : \alpha_i = \alpha_j$ for all n, and we test at a 5% significance level, we will still reject H_0 for $5\%n$ funds.

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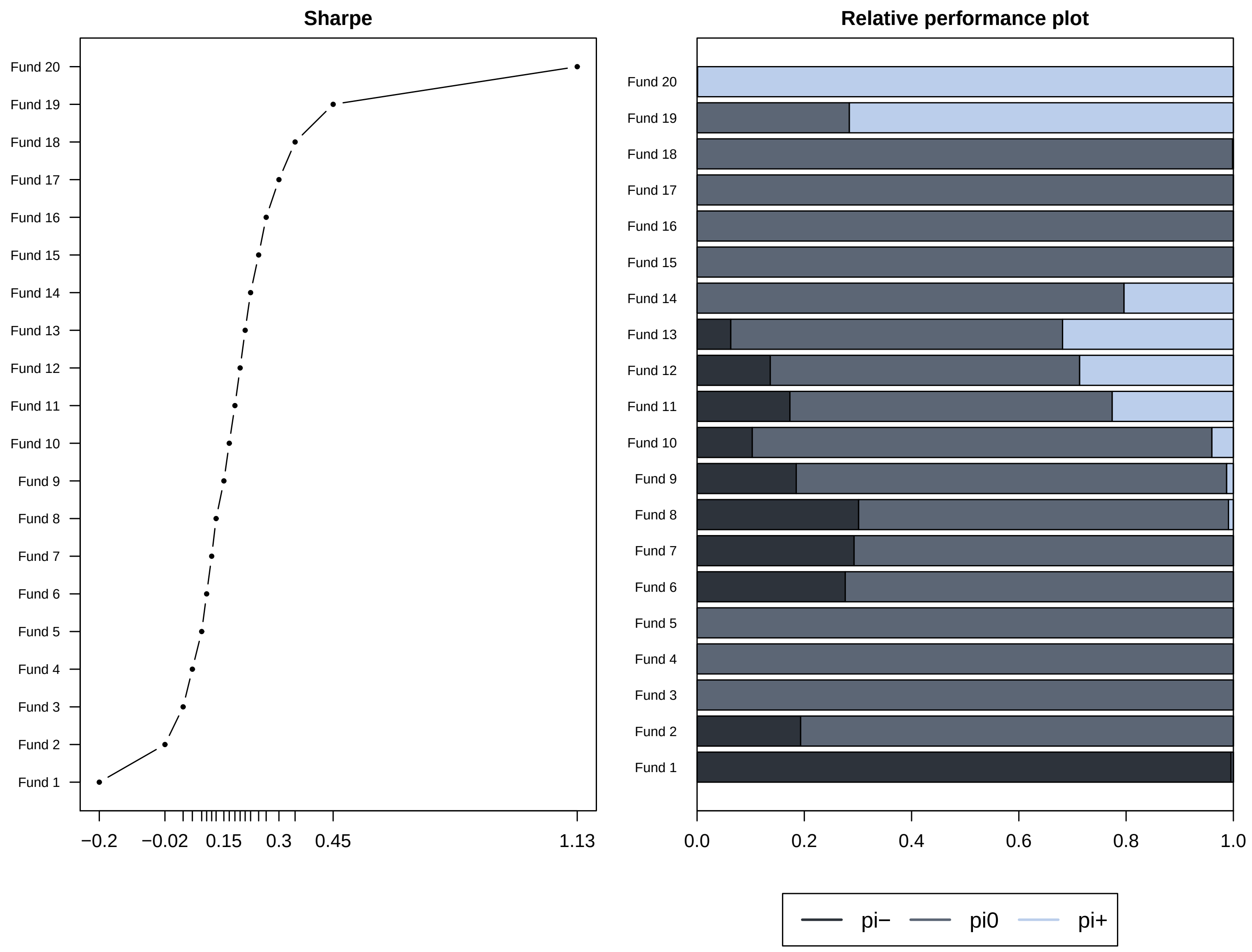
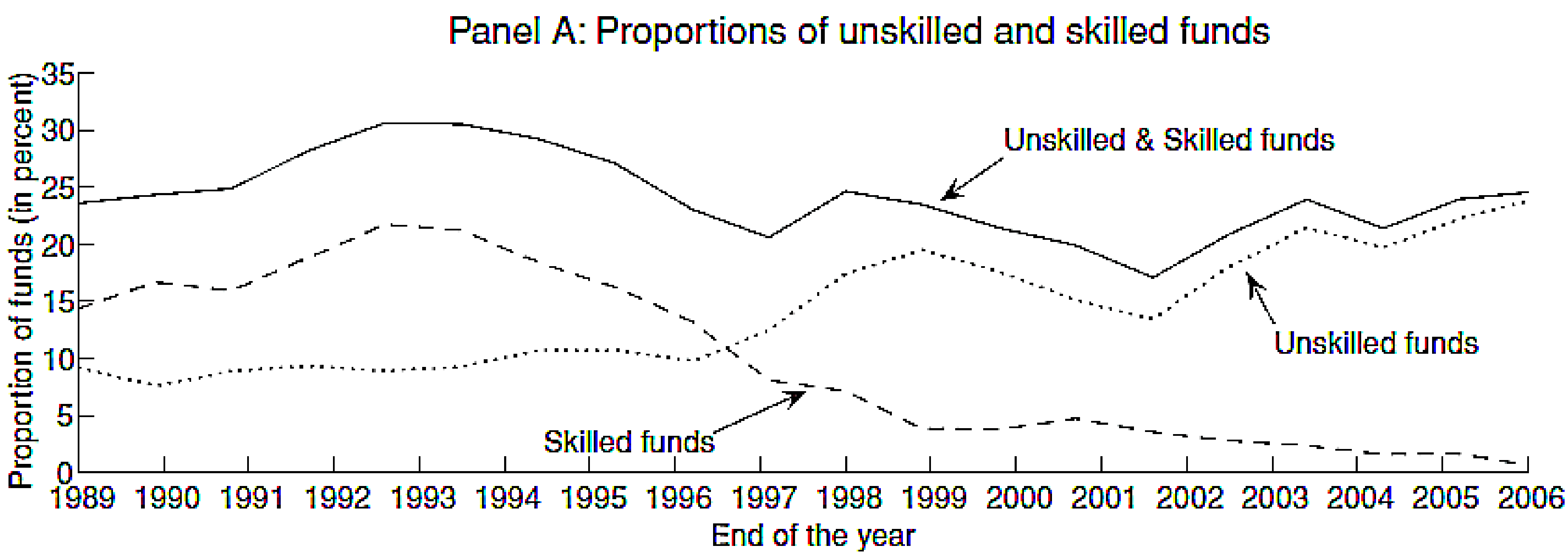
- (Statistically significant) differences can be because of luck. E.g. if $H_0 : \alpha_i = \alpha_j$ for all n, and we test at a 5% significance level, we will still reject H_0 for $5\%n$ funds.
- Avoid this by estimating the population parameters:

$$\underbrace{\pi_i^0 = \frac{\sharp(\alpha_i = \alpha_j)}{n}}_{\text{Equal performance ratio}} \quad \underbrace{\pi_i^+ = \frac{\sharp(\alpha_i > \alpha_j)}{n}}_{\text{Outperformance ratio}} \quad \underbrace{\pi_i^- = \frac{\sharp(\alpha_i < \alpha_j)}{n}}_{\text{Underperformance ratio}}$$


Sharpe
Fund 20
Fund 19
Fund 18
Fund 17
Fund 16
Fund 15
Fund 14
Fund 13
Fund 12
Fund 11
Fund 10
Fund 9
Fund 8
Fund 7
Fund 6
Fund 5
Fund 4
Fund 3
Fund 2
Fund 1
−0.2
−0.02
0.15
0.3
0.45
1.13
Relative performance plot
0.0
0.2
0.4
0.6
0.8
1.0
pi−
pi0
pi+

Previous literature

- Barras, Scaillet, Wermers (JF 2010) estimate the percentage of mutual funds that have a positive alpha (using data from 1974).



Panel A: Proportions of unskilled and skilled funds
Proportion of funds (in percent)
35
30
25
20
15
10
5
0
Unskilled & Skilled funds
Unskilled funds
Skilled funds
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
End of the year

Our question

- Main differences with BSW:
  - ✓ BSW compares fund returns with return of a single benchmark. We do pairwise comparisons.
  - ✓ Their tool answers the question: "How many funds outperform the benchmark?"

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- Main differences with BSW:
  - ✓ BSW compares fund returns with return of a single benchmark. We do pairwise comparisons.
  - ✓ Their tool answers the question: "How many funds outperform the benchmark?"
  - ✓ Our tool answers:
    - Micro-level question: How well does a fund perform compared to others?
    - Macro-level question: How well does a fund strategy perform compared to others?

How to estimate π_i^0, π_i^+ *and* π_i^-?

- Compute p-values two-sided test $H_0 : \alpha_i = \alpha_j$ using studentized test statistic of Ledoit and Wolf (JEMF 2008)

How to estimate π_i^0, π_i^+ and π_i^-?

- Compute p-values two-sided test $H_0 : \alpha_i = \alpha_j$ using studentized test statistic of Ledoit and Wolf (JEMF 2008)
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- Use p-values to estimate π_i^0 following the procedure of Storey (JRS 2002)
- Attribution of $1 - \pi_i^0$ to π_i^+ and π_i^-
- Computationally intensive on a universe of n funds: $(n^2 - n)/2$ comparisons. R package CompStrat available from `www.econ.kuleuven.be/kris.boudt/public` uses SNOW to split the task among cores.

Outline

- Outline
  1. Introduce the estimator;
  2. Simulation study and practical examples using the CompStrat package.
- This is still work in progress!

Testing the equality of Sharpe ratios

Performance analysis in R


```
> library(PerformanceAnalytics)
> data(managers)
> round(SharpeRatio(managers[,1:6],FUN="StdDev"),3)
                HAM1  HAM2  HAM3  HAM4  HAM5  HAM6
StdDev Sharpe: 0.434 0.385 0.341 0.207 0.089 0.464
> library(CompStrat)
> sharpeTesting(x=managers[,1],y=managers[,2])
```

Testing the equality of Sharpe ratios


```
> sharpeTesting(x=managers[,1],y=managers[,2])
$n
[1] 125
$sharpe
     HAM1      HAM2
0.4339932 0.3959264
$dsharpe
[1] 0.06161532
$tstat
[1] 0.5323056
$pval
[1] 0.5945143
```

Testing the equality of Sharpe ratios


```
> sharpeTesting(x=managers[,1],y=managers[,5])
$n
[1] 77
$sharpe
     HAM1      HAM5
0.4339932 0.1173701
$dsharpe
[1] 0.2972916
$tstat
[1] 2.263586
$pval
[1] 0.02359959
```

Testing the equality of Sharpe ratios (Ledoit and Wolf, JEMF 2008)

- Studentized test-statistic:

$$\frac{\hat{\alpha}_i - \hat{\alpha}_j}{\widehat{\text{SE}}(\hat{\alpha}_i - \hat{\alpha}_j)} \overset{a}{\sim} N(0,1)$$

with $\widehat{\text{SE}}(\hat{\alpha}_i - \hat{\alpha}_j)$ a function of the covariance matrix of the mean and StdDev estimates.

- In small samples: Bootstrap.

```
control = list(nBoot = 250, type = 2)
out <- sharpeTesting(x, y, control)
```

Estimation of π_i^0

Storey's Procedure for $\hat{\pi}^0$

- Compute two-sided p-values under the null of equality of the Sharpe ratios
  - ✓ If H_0 is true: p-values are uniformly distributed between 0 and 1.
  - ✓ If H_0 is false: p-values are close to zero

Monte Carlo

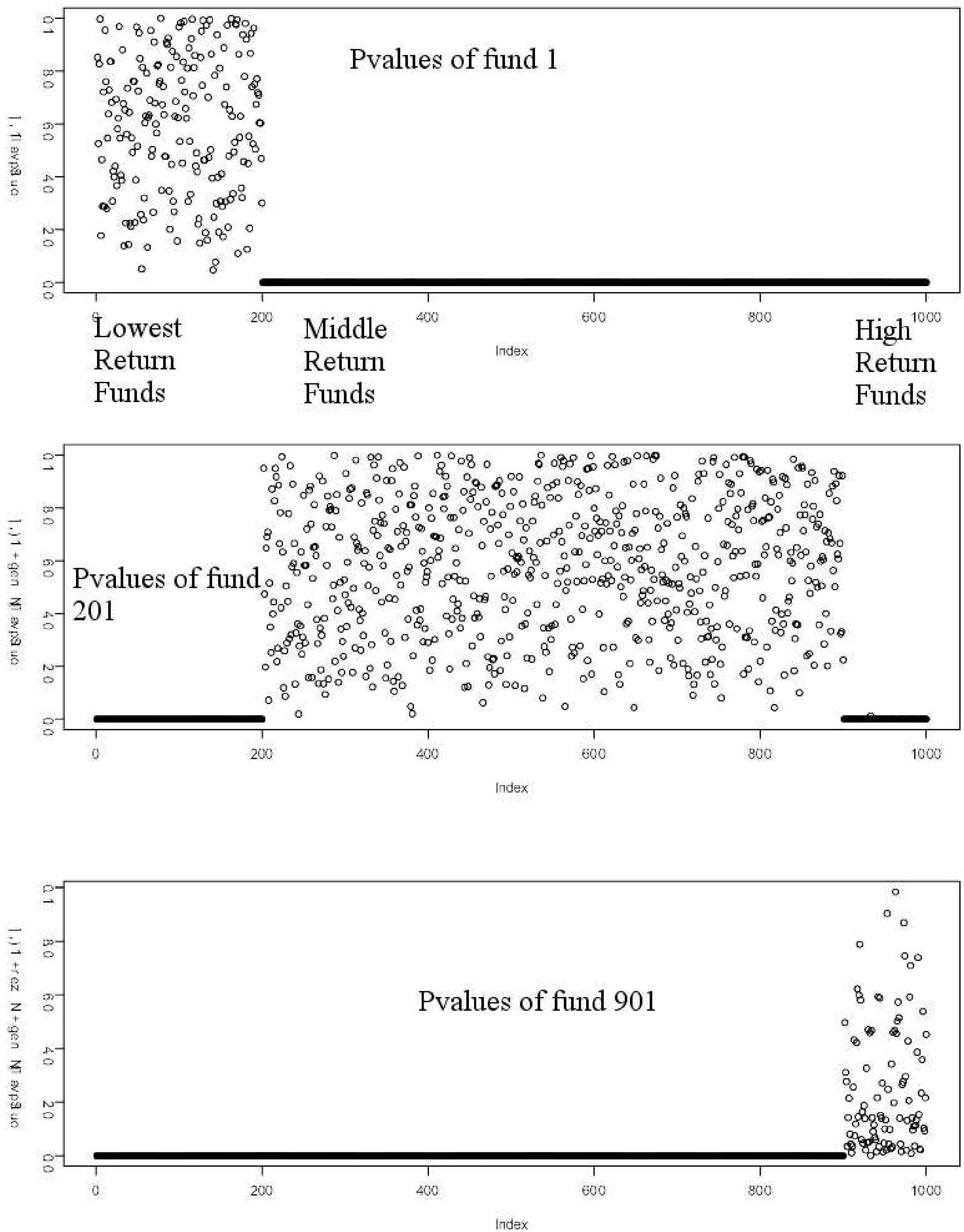
- Suppose universe of 1000 funds:

  1. Group 1: 200 funds with mean monthly return of 0%, $\sigma = 1\%$
  2. Group 2: 700 funds with mean monthly return of 0.5%, $\sigma = 1\%$
  3. Group 3: 100 funds with mean monthly return of 1%, $\sigma = 1\%$

Simulated pvalues


```
# Generate data

# Main function for testing equality of Sharpe ratios on a univers
# Output is $n \times n$ matrix of pvalues (among others)
out = sharpeScreening(rets)
```


Pvalues of fund 1
Lowest Return Funds
Middle Return Funds
High Return Funds
Index
Pvalues of fund 201
Index
Pvalues of fund 901
Index

Storey's Procedure for $\hat{\pi}^0$

- For λ sufficiently large (e.g. 0.7): all p-values exceeding λ correspond to funds for which H0 is true. If n^0 such funds, we expect $(1-\lambda)n_i^0$ p-values exceeding λ.
- Hence the estimator:

$$\hat{n}_i^0 = \frac{1}{1-\lambda}\sum_{j=1}^{n}(\text{p-values}_{\alpha_i=\alpha_j} > \lambda)$$

$$\hat{\pi}_i^0 = \hat{n}_i^0/n$$

Monte Carlo study: Setup

- Same universe as before:
  1. 20% funds with mean monthly return of 0%;
  2. 70% funds with mean monthly return of 0.5%;
  3. 10% funds with mean monthly return of 1%.
- We consider different values for σ, T and n.

Monte Carlo study: Results

- No bias:
  - ✓ If the 3 distributions are very distinct;
  - ✓ and/or if T is sufficiently large.
- Otherwise distributions of test statistics overlap, leading to upward bias in $\hat{\pi}_0$ in finite samples.
- Small impact of increasing n.

Attribution of $1 - \hat{\pi}_i^0$ to π_i^+ and π_i^-

Procedure for $\hat{\pi}_i^+$, $\hat{\pi}_i^-$

- Next step is to attribute $n - \hat{n}^0$ to n_i^+ and n_i^-

$$n_i^+ = \sharp(\alpha_i > \alpha_j) \quad = \quad \sharp(\hat{\alpha}_i > \hat{\alpha}_j) - \sharp(\hat{\alpha}_i > \hat{\alpha}_j | \alpha_i = \alpha_j)$$
$$-\sharp(\hat{\alpha}_i > \hat{\alpha}_j | \alpha_i < \alpha_j) + \sharp(\hat{\alpha}_i < \hat{\alpha}_j | \alpha_i > \alpha_j)$$

Procedure for $\hat{\pi}_i^+$, $\hat{\pi}_i^-$

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$$
\begin{aligned}
n_i^+ = \sharp(\alpha_i > \alpha_j) &= \sharp(\hat{\alpha}_i > \hat{\alpha}_j) - \sharp(\hat{\alpha}_i > \hat{\alpha}_j | \alpha_i = \alpha_j) \\
&\quad - \sharp(\hat{\alpha}_i > \hat{\alpha}_j | \alpha_i < \alpha_j) + \sharp(\hat{\alpha}_i < \hat{\alpha}_j | \alpha_i > \alpha_j) \\
&\approx \sharp(\hat{\alpha}_i > \hat{\alpha}_j) - \hat{n}_i^0/2
\end{aligned}
$$

Procedure for $\hat{\pi}_i^+$, $\hat{\pi}_i^-$

- Next step is to attribute $n - \hat{n}^0$ to n_i^+ and n_i^-

$$\begin{aligned} n_i^+ = \sharp(\alpha_i > \alpha_j) &= \sharp(\hat{\alpha}_i > \hat{\alpha}_j) - \sharp(\hat{\alpha}_i > \hat{\alpha}_j | \alpha_i = \alpha_j) \\ &\quad - \sharp(\hat{\alpha}_i > \hat{\alpha}_j | \alpha_i < \alpha_j) + \sharp(\hat{\alpha}_i < \hat{\alpha}_j | \alpha_i > \alpha_j) \\ &\approx \sharp(\hat{\alpha}_i > \hat{\alpha}_j) - \hat{n}_i^0 / 2 \end{aligned}$$

- We therefore propose the following estimators:

$$\hat{\pi}_i^+ = \frac{\sharp(\hat{\alpha}_i > \hat{\alpha}_j) - \hat{n}_i^0/2}{n} \quad \hat{\pi}_i^- = \frac{\sharp(\hat{\alpha}_i < \hat{\alpha}_j) - \hat{n}_i^0/2}{n}$$

Procedure for $\hat{\pi}_i^+$, $\hat{\pi}_i^-$

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\end{aligned}
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$$
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$$

- If $\hat{\pi}_i^+ < 0, \hat{\pi}_i^+ = 0, \hat{\pi}_i^- = 1 - \hat{\pi}_i^0$.
- If $\hat{\pi}_i^- < 0, \hat{\pi}_i^- = 0, \hat{\pi}_+^- = 1 - \hat{\pi}_i^0$.
- Note additivity: $\hat{\pi}_i^0 + \hat{\pi}_i^+ + \hat{\pi}_i^- = 1$.

Monte Carlo study on precision $\hat{\pi}_i^+$, $\hat{\pi}_i^-$

- No bias:
  - ✓ If the 3 distributions are very distinct;
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- Otherwise distributions of test statistics overlap, leading to upward bias in $\hat{\pi}_0$ and downward bias in $\hat{\pi}_i^+$ and $\hat{\pi}_i^-$ in finite samples.
- Small impact of increasing n.

Application

Data

- Hedge Fund Research database, August 2005-August 2011 (six years).
- We focus our analysis on four strategies: Equity Hedge, Event-Driven, Relative Value and Macro.
- After filters: 987 US funds.

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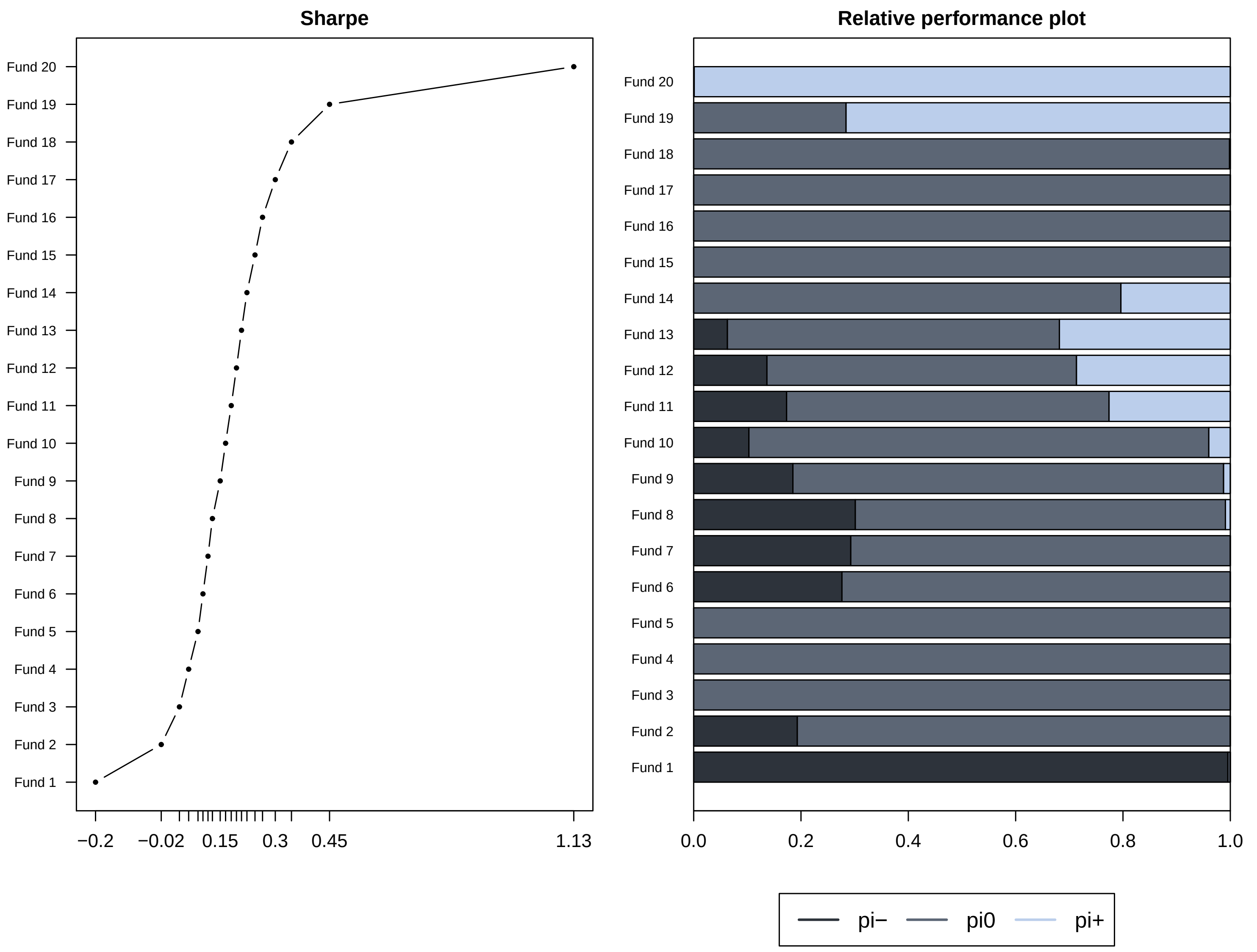
- After filters: 987 US funds.
- Applications:
  1. Descriptive: Under, equal and outperformance of a fund (style) compared to other funds (styles);
  2. Screening.


Sharpe
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0.45
1.13
Relative performance plot
0.0
0.2
0.4
0.6
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1.0
pi−
pi0
pi+

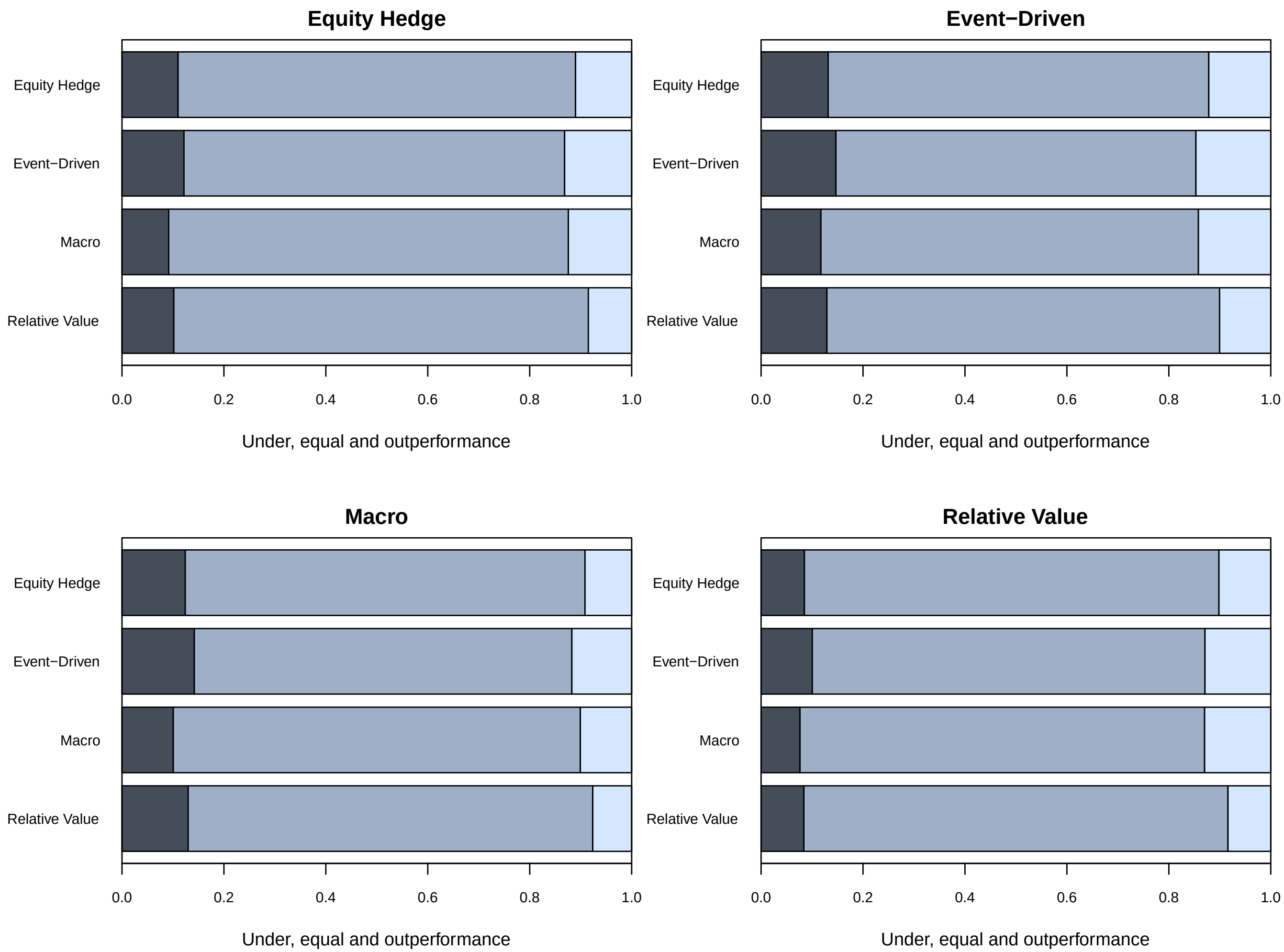
Relative performance charts

- At the hedge fund level: Under, equal and outperformance of the funds belonging to style k compared to funds belonging to another style

  ✓ $\hat{\pi}^0$: percentage of pairs of funds in the two styles for which the risk adjusted performance is the same

  ✓ $\hat{\pi}^+$: percentage of pairs of funds in the two styles for which the risk adjusted performance of the fund in style k is higher

  ✓ $\hat{\pi}^-$: percentage of pairs of funds in the two styles for which the risk adjusted performance of the fund in style k is lower



Equity Hedge
Event−Driven
Macro
Relative Value
Equity Hedge
Event−Driven
Macro
Relative Value
0.0
0.2
0.4
0.6
0.8
1.0
Under, equal and outperformance

As a screening tool

- Select funds based on their individual Sharpe ratio and the Sharpe ratio adjusted for outperformance: $(\hat{\alpha}_i) \times \hat{\pi}_i$

- Test. Split up the sample into two sub-periods: August 2005 - August 2008 and September 2008 - August 2011. Regress Sharpe ratio September 2008 - August 2011 on fund characteristics previous period:

$$\widetilde{SR}_i = \theta_0 + \theta_1 ED_i + \theta_2 MA_i + \theta_3 RV_i + \theta_4 SR_i + \theta_5 (\hat{\pi}_i^+ \times SR_i) + \theta_6 \log AUM_i + \theta_7 \Delta \log AUM_i + \theta_8 AGE_i + \theta_9 MF_i + \theta_{10} PF_i + \theta_{11} HWM_i + \theta_{12} LEV_i + \varepsilon_i .$$

- Only SR_i and $(\hat{\pi}_i^+ \times SR_i)$ are statistically significant at a 95% confidence level. Positive impact.

Conclusion

Conclusion

- We study the relative performance of hedge funds, using a novel tool that characterizes for each fund the peer performance in 3 numbers: $\hat{\pi}_i^0$, $\hat{\pi}_i^+$ and $\hat{\pi}_i^-$.
- Simulation study: relatively accurate, especially for large T
- Application: Relative performance plots, Screening.
- Package CompStrat:
  - ✓ Pair of funds: sharpeTesting;
  - ✓ Universe of funds: sharpeScreening, alphaScreening.
  - ✓ Available from: `www.econ.kuleuven.be/kris.boudt/public`
- Work in progress – comments welcome.

References

- Ardia, D. and Boudt K. 2012. The peer performance of hedge funds. Work in progress. Draft available from SSRN.
- Barras, L., Scaillet, O., Wermers, R., 2010. False discoveries in mutual fund performance: Measuring luck in estimated alphas. Journal of Finance 65, 179–216.
- Ledoit, O.,Wolf, M., 2008. Robust performance hypothesis testing with the Sharpe ratio. Journal of Empirical Finance 15, 850–859.
- Storey, J., 2002. A direct approach to false discovery rates. Journal of the Royal Statistical Society B 64, 479–498.

Appendix

How to estimate π_i^0, π_i^+ and π_i^-?

- Perform pairwise test of equality $H_0 : \alpha_i = \alpha_j$ and then compute the percentage number of observations for which $\hat{\alpha}_i \approx \hat{\alpha}_j$, $\hat{\alpha}_i > \hat{\alpha}_j$ and $\hat{\alpha}_i < \hat{\alpha}_j$

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$$\hat{\alpha}_i - \hat{\alpha}_j = \begin{cases} > 0 & \checkmark \text{ if } \alpha_i > \alpha_j \\ & \text{false positive, otherwise} \end{cases}$$

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$$\hat{\alpha}_i - \hat{\alpha}_j = \begin{cases} > 0 & \checkmark \text{ if } \alpha_i > \alpha_j \\ & \text{false positive, otherwise} \\ \approx 0 & \checkmark \text{ if } \alpha_i = \alpha_j \\ & \text{false negative if } \alpha_i > \alpha_j \\ & \text{false positive if } \alpha_i < \alpha_j \\ < 0 & \checkmark \text{ if } \alpha_i < \alpha_j \\ & \text{false negative, otherwise} \end{cases}$$

Size properties for skewed t data. Function sharpeTesting.


```
library(sn)
control_asy = list( type = 1)
control_bs = list( nBoot = 250, type = 2 )
T = 72
set.seed(1234)
M = 1000
pval_asy = pval_bs = matrix(NA, M, 1)
for (m in 1:M){
  rets = matrix(rst(n = 2*T, shape=3, df=5),T,2)
  tmp = sharpeTesting(x = rets[,1], y = rets[,2], control_asy)
  pval_asy[m] = tmp$pval
  tmp = sharpeTesting(x = rets[,1], y = rets[,2], control_bs)
  pval_bs[m] = tmp$pval
}
```

Size properties for skewed t data ($T = 72$)


```
> mean(pval_asy<0.01) ; mean(pval_bs<0.01)
[1] 0.038
[1] 0.022
> mean(pval_asy<0.05) ; mean(pval_bs<0.05)
[1] 0.099
[1] 0.07
> mean(pval_asy<0.1)  ; mean(pval_bs<0.1)
[1] 0.167
[1] 0.136
```

Size properties for skewed t data ($T = 240$)


```
> mean(pval_asy<0.01) ; mean(pval_bs<0.01)
[1] 0.019
[1] 0.012
> mean(pval_asy<0.05) ; mean(pval_bs<0.05)
[1] 0.082
[1] 0.056
> mean(pval_asy<0.1)  ; mean(pval_bs<0.1)
[1] 0.146
[1] 0.114
```

Simulated pvalues


```
# Generate data
mix = list(p_neg = 0.2, mu_neg = 0   , sd_neg = 1,
           p_zer = 0.7, mu_zer = 0.5 , sd_zer = 1,
           p_pos = 0.1, mu_pos = 1   , sd_pos = 1)
T = 240 ;   N = 1000 ;
N_neg = floor(mix$p_neg * N)
N_pos = floor(mix$p_pos * N)
N_zer = N - N_neg - N_pos
set.seed(1234)
rets_neg = mix$mu_neg + mix$sd_neg * matrix(rnorm(T * N_neg), T, N_
rets_zer = mix$mu_zer + mix$sd_zer * matrix(rnorm(T * N_zer), T, N_
rets_pos = mix$mu_pos + mix$sd_pos * matrix(rnorm(T * N_pos), T, N_
rets = cbind( rets_neg , rets_zer, rets_pos )

# Main function for testing equality of Sharpe ratios on a univers
# Output is $n \times n$ matrix of pvalues (among others)
out = sharpeScreening(rets)
```

Distribution $\hat{\pi}_i^0$

- Suppose no uncertainty in the p-values.
- If n_i^λ is the "true" number of funds with p-value $> \lambda$, this is like n draws from binomial with success rate n^λ/n and hence the estimated proportion:

$$\hat{\pi}_i^0 \sim N\left(\pi_i^0; \frac{1}{n^2(1-\lambda)^2} n \frac{n_i^\lambda}{n} \frac{n - n_i^\lambda}{n}\right),$$

for n large.

Results Monte Carlo Study $\hat{\pi}_i^0$

		$\sigma = 0.5\%$		$\sigma = 1\%$		$\sigma = 2\%$	
	True Value	Bias	RMSE	Bias	RMSE	Bias	RMSE
$T = 72, N = 500, \lambda = 0.7$							
$\mu = 0\%$	0.2	-0.002	0.080	0.009	0.075	0.239	0.349
$\mu = 0.5\%$	0.7	-0.011	0.262	0.000	0.254	0.092	0.235
$\mu = 1\%$	0.1	-0.004	0.042	0.022	0.066	0.257	0.372

- If the 3 distributions are very distinct, no bias. RMSE is largest for group 2 were the true proportion is also the highest. Consistent with variance binomial distribution.
- The more overlap, the more $\hat{\pi}_0$ is upward biased in finite samples.

Sensitivity analysis $\hat{\pi}_i^0$ for $\sigma = 2\%$ ($\lambda = 0.7$)

	True Value	Bias				RMSE			
T ($n_i = 500$)		36	72	108	240	36	72	108	240
$\mu = 0\%$	0.2	0.402	0.239	0.142	0.018	0.511	0.349	0.235	0.078
$\mu = 0.5\%$	0.7	0.132	0.092	0.054	0.009	0.264	0.235	0.222	0.249
$\mu = 1\%$	0.1	0.431	0.257	0.155	0.023	0.543	0.372	0.255	0.068
n_i ($T = 72$)		50	100	500		50	100	500	
$\mu = 0\%$	0.2	0.225	0.231	0.239		0.36	0.350	0.349	
$\mu = 0.5\%$	0.7	0.064	0.073	0.092		0.244	0.237	0.235	
$\mu = 1\%$	0.1	0.243	0.253	0.257		0.379	0.375	0.543	

- Relatively large T is needed to kill the bias (using the asymptotic test).
- Small impact of increasing n.

$T = 72, N = 500, \lambda = 0.7$

		$\sigma = 0.5\%$		$\sigma = 1\%$		$\sigma = 2\%$	
	Pop Value	Bias	RMSE	Bias	RMSE	Bias	RMSE
π_i^+							
$\mu = 0\%$	0	0.029	0.058	0.024	0.048	0.004	0.029
$\mu = 0.5\%$	0.2	0.012	0.242	0.006	0.238	-0.019	0.240
$\mu = 1\%$	0.9	0.003	0.037	-0.013	0.064	**-0.179**	0.297
π_i^-							
$\mu = 0\%$	0.8	0.002	0.070	-0.005	0.078	**-0.164**	0.297
$\mu = 0.5\%$	0.1	0.04	0.211	0.038	0.210	0.021	0.194
$\mu = 1\%$	0	0.014	0.028	0.006	0.014	0.002	0.017

- Overestimation $\hat{\pi}_i^0$ leads to underestimation of $\hat{\pi}_i^-$ and $\hat{\pi}_i^+$

Sensitivity analysis for $\sigma = 2\%$ ($\lambda = 0.7, n_i = 500$)

	True Value	Bias				RMSE			
π_i^+	T	36	72	108	240	36	72	108	240
$\mu = 0\%$	0	0.016	0.004	0.001	0.021	0.072	0.029	0.013	0.041
$\mu = 0.5\%$	0.2	-0.022	-0.019	-0.013	0.001	0.246	0.240	0.239	0.236
$\mu = 1\%$	0.9	-0.32	-0.179	-0.103	-0.013	0.446	0.297	0.203	0.064
π_i^-									
$\mu = 0\%$	0.8	-0.294	-0.164	-0.093	-0.010	0.427	0.297	0.211	0.088
$\mu = 0.5\%$	0.1	0.022	0.021	0.026	0.036	0.203	0.194	0.195	0.208
$\mu = 1\%$	0	0.007	0.002	0.000	0.006	0.043	0.017	0.004	0.014