Time Varying Higher Moments

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R in Finance 2013

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Are (Time Varying) Higher Moments Important?

We know from the body of empirical research:

- Normality can be rejected (Mandelbrot [1963], Fama [1963]).
- There is variation and persistence in volatility (Engle [1982], Bollerslev [1986]).
- Investors want protection against large, infrequent losses (e.g. options volatility smirk).

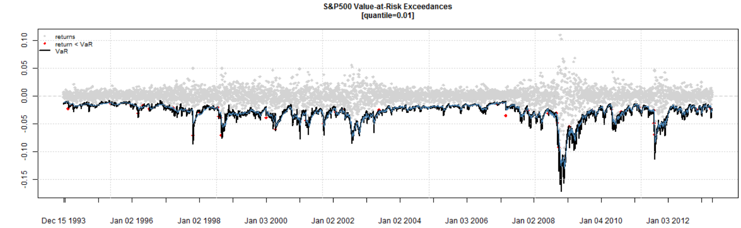
We don't know enough but suspect (see literature review):

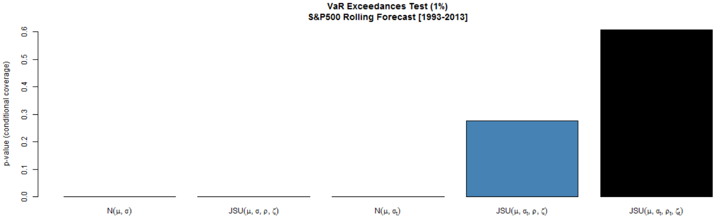
- There is variation in the higher moments.
- There is value in including them for the modelling of risk and in portfolio/trading applications.
- Parameters are well behaved (have $\sqrt{N}$ consistency).

We really don't know:

- What form this variation takes (dynamics, distribution).
- Whether *one* source of randomness is realistic.
- Whether the variation is the result of model misspecification.

Are (Time Varying) Higher Moments Important?





setup: 1-ahead forecasts based on parameters re-estimated every 50 periods & moving window size 2000

data: Yahoo finance.

The Autoregressive Conditional Density Model

Model Representation

$$r_t = \mu_t + \varepsilon_t$$
$$\varepsilon_t = \sigma_t z_t$$
$$z_t \sim \mathrm{D}\left(0, 1, \rho_t, \zeta_t\right)$$
$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$
$$\rho_t = L_\rho + \frac{(U_\rho - L_\rho)}{1 + e^{-\bar{\rho}_t}}$$
$$\zeta_t = L_\zeta + U_\zeta e^{-\upsilon \bar{\zeta}_t}$$

Typical Motion Dynamics

- Quadratic (Hansen [1982]):
  $\bar{\rho}_t = \chi_0 + \chi_1 z_{t-1} + \chi_2 z_{t-1}^2 + \xi_1 \bar{\rho}_{t-1}$
- Piecewise Linear (Jondeau and Rockinger [2003]):
  $\bar{\zeta}_t = \kappa_0 + \kappa_1 |z_{t-1}| I_{z_{t-1} \leqslant 0} + \kappa_2 |z_{t-1}| I_{z_{t-1} > 0} + \psi_1 \bar{\zeta}_{t-1}$

Comments

- One source of randomness (z_t) driving volatility (σ_t), skew (ρ_t) and shape (ζ_t) dynamics.
- Non-Linear bounding transformation of higher order parameters makes this a hard problem to solve confidently.
- Since both skewness and kurtosis are driven by extreme events, this makes their identification with a particular law of motion and (standardized) residuals very hard.
- Higher moments (skewness, kurtosis, etc) jointly determined by skew and shape in a location-scale invariant parameterization of distribution.

Literature Review

Table: Literature Review (Univariate ACD Models)

Authors (by Date)	Data	Distribution	TVHM	Comments
Hansen [1994]	US Excess Holding Yield (M); USD/CHF (W)	Generalised skewed Student	S+K	in-sample inference *(coefficients)*
Harvey and Siddique [1999]	8 equity indices (D, W, M)	Non-central Student	S	in-sample inference *(GMM Test, Persistence, Asymmetry)*
Premaratne and Bera [2000]	NYSE (D)	PIV	S+K	in-sample inference *(parameters not significant for 1991-1996, significant 1987)*
Rockinger and Jondeau [2002]	S&P500, FT 100, Nikkei 225 (W)	Entropy density	S+K	in-sample inference *(little evidence of either, rarity of extremes, low power of tests)*
Jondeau and Rockinger [2003]	6 currencies, 5 stock indices (D)	Generalised skewed Student	S+K	in-sample inference *(solid evidence, wrong conclusion about Harvey+Siddique)*
Bond and Patel [2003]	20 listed UK property companies, 20 US REITS (M)	Generalised skewed Student	S	in-sample inference *(360 obs, convergence problems not surprising, consistency)*
Brannas and Nordman [2003]	NYSE (D)	PIV and LGG	S	*in-sample inference* *(evidence of skewness for LGG but not PIV-incomplete results shown)*
León et al. [2005]	5 exchange rates and 5 stock indices (D)	Gram-Charlier Normal expansion	S+K	in-sample inference *(direct modelling of S+K, solid evidence of both)*
Brooks et al. [2005]	S&P500, FTSE 100, UK and US 10 Yr Bond Indices (D)	Student	K	in-sample inference *(direct modelling of kurtosis, not shape, highly significant)*
Wilhelmsson [2009]	S&P 500 (D)	NIG	S+K	in- and out-of sample *(some evidence, VaR Independence, marginal improvement, but daily roll!!)*
Ergün and Jun [2010b]	S&P 500 (5-min)	Generalised skewed Student	S	in- and out-of sample *(in-sample ACD, out-of-sample ACD only for ES, GARCH for VaR)*
Ergün and Jun [2010a]	Simulation Paper/Best test for judging misspecification	Generalised skewed Student	S+K	GMM Test good size for S but low power. Bad for K in size and power. *HL Test good power for large size and large misspecification.*

Measuring The Cost of GARCH

- Misspecification tests (in-sample, simulation)
  - Non-Parametric Density Test (Hong and Li [2005]) [rugarch::HLTest]
  - GMM Orthogonality Test (Hansen [1982]) [rugarch::GMMTest]
  - BDS Test of iid (Brock and Dechert [1988]) [tseries::bds.test]

- Operational Tests (out-sample/forecast, simulation)
  - VaR Test (Kupiec [1995], Christoffersen [1998], Christoffersen and Pelletier [2004]) [rugarch::VaRTest, rugarch::VaRDurTest]
  - Expected Shortfall Test (McNeil and Frey [2000]) [rugarch::ESTest]
  - Directional Accuracy Test (Pesaran and Timmermann [1992]) [rugarch::DACTest]
  - Loss based comparison Test (Hansen et al. [2003]) [rugarch:::mcs (unexported)]

Simulation Study I

Table: Hong and Li Test

	$M(1,1)$	$M(2,2)$	$M(3,3)$	$M(4,4)$	W
			DGP: GARCH(1,1)-NIG(ρ,ζ)		
Mean[Statistic]	-1.94	-1.74	-1.45	-1.22	-0.88
s.e.[Statistic]	0.31	0.48	0.75	0.99	1.68
%Rejections	0	0	1	3	**8**
			DGP: GARCH(1,1)-NIG(ρ_t,ζ)		
Mean[Statistic]	-1.45	-0.46	1.92	4.82	0.97
s.e.[Statistic]	0.64	1.28	2.54	3.80	1.87
%Rejections	0	7	**47**	**79**	**34**
			DGP: GARCH(1,1)-NIG(ρ,ζ_t)		
Mean[Statistic]	-1.87	-1.45	0.28	2.71	4.43
s.e.[Statistic]	0.36	0.70	1.87	3.19	2.21
%Rejections	0	0	**20**	**56**	**90**
			DGP: GARCH(1,1)-NIG(ρ_t,ζ_t)		
Mean[Statistic]	-1.59	-0.78	2.08	5.93	3.79
s.e.[Statistic]	0.54	1.05	2.49	4.03	2.10
%Rejections	0	3	**50**	**87**	**85**

Notes to table: The table reports the average value of the Hong and Li [2005] statistic from the Monte Carlo experiment using 2000 simulations with 8000 observations each from the GARCH(1,1) model with conditional densities given by NIG(ρ,ζ), NIG(ρ,$\zeta_t[0, 1, 1]$), NIG($\rho_t[1, 0, 1]$,ζ) and NIG($\rho[1, 0, 1]$,$\zeta_t[0, 1, 1]$), where the motion dynamics follow a quadratic model. $M(j, j)$,$j = 1, .., 4$, represents the nonparametric test for misspecification in the conditional moments of the standardized residuals from the fitted GARCH(1,1)-NIG model, and distributed as $N(0, 1)$ under the null of a correctly specified model. The statistic W in column 5 of the table is the Portmanteau type test statistic for general misspecification (using 4 lags) and distributed as $N(0, 1)$ under the null of a correctly specified model.

Simulation Study II

Table: BDS Test

	DGP: GARCH(1,1)-NIG(ρ,ζ)					DGP: GARCH(1,1)-NIG(ρ_t,ζ)					
	$\epsilon/sigma$						$\epsilon/sigma$				
m	**0.5**	**1**	**1.5**	**2**	**2.5**	**m**	**0.5**	**1**	**1.5**	**2**	**2.5**
2	5.3	5.3	5.1	4.9	5.5	**2**	5.37	4.97	5.12	5.22	5.07
3	6.4	4.8	4.7	4.7	3.9	**3**	5.92	5.52	5.17	5.37	5.27
4	9.7	5.3	4.7	4.4	4.1	**4**	8.68	5.72	5.62	5.32	5.27
5	19.2	5.4	5.0	4.5	4.4	**5**	16.11	5.97	5.27	5.57	5.17

	DGP: GARCH(1,1)-NIG(ρ,ζ_t)					DGP: GARCH(1,1)-NIG(ρ_t,ζ_t)					
	$\epsilon/sigma$						$\epsilon/sigma$				
m	**0.5**	**1**	**1.5**	**2**	**2.5**	**m**	**0.5**	**1**	**1.5**	**2**	**2.5**
2	97.63	96.22	92.89	86.09	73.24	**2**	98.8	98.2	96.7	92.7	84.2
3	92.84	90.32	85.58	76.21	62.20	**3**	95.0	94.1	91.1	85.2	76.3
4	82.96	82.71	76.16	66.38	53.48	**4**	87.3	87.6	83.9	77.1	67.5
5	72.73	74.45	67.79	57.61	45.31	**5**	75.4	80.5	76.3	69.3	58.1

Notes to table: The table reports the percentage of rejections for the BDS test of i.i.d. (with embedding dimensions m 2 to 5 and ϵ representing the range of standard deviations of the data) at the 95% confidence level, when applied to the log of the squared standardized residuals of the GARCH-NIG model from simulated data under alternative data generating processes. The Monte Carlo experiment used 2000 simulations with 8000 observations each from the AR(2)-GARCH(1,1) model with conditional densities given by NIG(ρ,ζ), NIG(ρ,$\zeta_t[0, 1, 1]$), NIG($\rho_t[1, 0, 1]$,ζ) and NIG($\rho[1, 0, 1]$,$\zeta_t[0, 1, 1]$), where the motion dynamics follow a quadratic model.

Empirical Study: Dow 10 Dataset

Table: Summary statistics for 10 Dow Stocks

	mean	min	sd	skewness	kurtosis	ARCH-LM(1)	Ljung-Box(1)	Ljung-Box(2)	JB
AA	1.5E-04	-0.28	0.023	-0.31	13.04	0.00	0.01	0.00	0.00
AXP	4.5E-04	-0.30	0.023	-0.28	13.79	0.00	0.00	0.00	0.00
C	1.8E-04	-0.49	0.029	-0.61	41.10	0.00	0.00	0.00	0.00
IBM	3.4E-04	-0.27	0.018	-0.45	16.91	0.00	0.02	0.02	0.00
JPM	3.9E-04	-0.32	0.025	-0.12	16.79	0.00	0.02	0.02	0.00
KO	5.7E-04	-0.29	0.016	-0.55	24.17	0.00	0.11	0.00	0.00
MO	7.1E-04	-0.26	0.018	-0.61	16.43	0.00	0.00	0.00	0.00
PG	5.2E-04	-0.36	0.016	-2.64	71.91	0.00	0.00	0.00	0.00
UTX	5.0E-04	-0.30	0.017	-0.81	19.80	0.00	0.83	0.01	0.00
XOM	5.4E-04	-0.27	0.015	-0.55	23.74	0.00	0.00	0.00	0.00

Notes to table: The Table presents summary statistic for the daily returns of 10 Dow constituents for the period 03-Jan-1984 to 28-Feb-2013, including the mean (mean), minimum (min), standard deviation (sd), skewness (skewness), kurtosis (kurtosis), the p-value of the ARCH-LM test of with 1 lag, the p-value of the test of Ljung-Box for independence using 1 and 2-lags, and the p-value of the normality test of Jarque-Bera.

Can we test for time variation in the higher moments?

While autocorrelation could be extended to measures of autocoskewness and autocokurtosis, it is not clear what the distribution of these measures are and hence how to make any meaningful inference.

Empirical Study: Dow 10 Estimation

Table: ACD parameter estimates for 10 Dow Constituents

	AA	AXP	C	IBM	JPM	KO	MO	PG	UTX	XOM
χ_0	**0.027**	**0.067**	**0.056**	0.009	0.018	**0.097**	-0.012	0.002	0.037	**-0.196**
	0.05	*0.05*	*0.02*	*0.77*	*0.62*	*0.07*	*0.63*	*0.60*	*0.88*	*0.03*
χ_1	**0.129**	**0.176**	**0.158**	**0.134**	**0.181**	**0.120**	-0.051	**0.030**	0.138	0.059
	0.00	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.12*	*0.04*	*0.74*	*0.13*
ξ_1	**0.798**	**0.425**	**0.484**	**0.309**	**0.273**	0.072	**0.413**	**0.933**	0.610	0.039
	0.00	*0.02*	*0.00*	*0.09*	*0.02*	*0.80*	*0.00*	*0.00*	*0.82*	*0.90*
κ_0	0.152	0.062	**0.054**	**0.088**	0.012	0.018	0.000	0.005	**0.097**	0.033
	0.21	*0.14*	*0.02*	*0.07*	*0.50*	*0.12*	*0.88*	*0.34*	*0.10*	*0.27*
κ_1	**0.000**	0.012	**0.018**	**0.035**	**0.019**	**0.023**	**0.032**	**0.014**	0.019	0.030
	0.00	*0.32*	*0.00*	*0.02*	*0.00*	*0.02*	*0.04*	*0.00*	*0.46*	*0.22*
κ_2	0.000	**0.044**	**0.044**	**0.073**	**0.056**	0.008	**0.027**	0.000	**0.081**	0.059
	0.63	*0.04*	*0.00*	*0.00*	*0.00*	*0.73*	*0.01*	*0.86*	*0.05*	*0.16*
ψ_1	**0.925**	**0.953**	**0.962**	**0.941**	**0.975**	**0.984**	**0.988**	**0.995**	**0.931**	**0.956**
	0.00	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*
Log.Lik(ACD)	18510.7	18830.2	18429.6	20571.2	18659.9	21343.8	20554.3	21737.4	20368.8	21415.7
Log.Lik(GARCH)	18495.7	18815.4	18404.7	20553.0	18639.8	21333.8	20450.0	21721.0	20354.1	21411.3
LR_{stat}	**29.96**	**29.54**	**49.90**	**36.24**	**40.30**	**19.97**	**208.52**	**32.68**	**29.37**	**8.73**
p-value	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.12*

Notes to table: The Table presents parameter estimates of an ARMA(1,1)-GARCH(1,1)-NIG(ρ_t, ζ_t) for the daily log returns of 10 Dow constituents for the period 03-Jan-1984 to 28-Feb-2013. The ACD NIG dynamics (ρ_t, ζ_t) were estimated using a quadratic (1,0,1) and piecewise linear (1,1,1) model for the skew and shape parameters respectively. Values in italics represent the p-values from the estimated robust standard errors and the likelihood ratio test. The Log-Likelihood of each model is reported as well as the Log-Likelihood of the restricted GARCH model where the restriction is of constant skew and shape. The Likelihood Ratio statistic under the null of the restricted model is distributed χ^2, with 5 (d.o.f) restrictions representing the dynamic model skew and shape parameters excluding their intercepts. Tested at the 10% level of significance, the GARCH model is rejected in 9 of the 10 securities tested.

data: Yahoo finance.

Empirical Study: Dow 10 Forecast VaR Loss

Table: Model Confidence Set with VaR Loss

	ACD				GARCH			
$[VaR = 1\%]$	GHYP	NIG	JSU	SSTD	GHYP	NIG	JSU	SSTD
AA	0.19	**1.00**	0.85	0.85	0.19	0.19	0.00	0.00
AXP	0.78	**1.00**	0.41	0.41	0.23	0.41	0.23	0.07
C	0.78	**1.00**	0.11	0.11	0.11	0.78	0.05	0.05
IBM	0.04	**1.00**	0.04	0.03	0.04	0.05	0.03	0.03
JPM	0.04	**1.00**	0.03	0.03	0.03	0.03	0.01	0.01
KO	0.00	**1.00**	0.92	0.02	0.02	0.65	0.01	0.00
MO	0.15	0.15	0.15	0.10	0.15	**1.00**	0.04	0.09
PG	**1.00**	0.46	0.00	0.00	0.46	0.46	0.00	0.00
UTX	0.02	**1.00**	0.02	0.02	0.02	0.02	0.02	0.02
XOM	0.02	0.02	0.02	0.02	**1.00**	0.14	0.02	0.02

	ACD				GARCH			
$[VaR = 5\%]$	GHYP	NIG	JSU	SSTD	GHYP	NIG	JSU	SSTD
AA	0.00	**1.00**	0.10	0.00	0.00	0.10	0.00	0.00
AXP	0.66	0.66	**1.00**	0.00	0.00	0.52	0.00	0.00
C	0.00	**1.00**	0.06	0.00	0.02	0.06	0.00	0.00
IBM	0.00	**1.00**	0.00	0.00	0.00	0.01	0.00	0.00
JPM	0.13	**1.00**	0.00	0.00	0.03	0.72	0.00	0.00
KO	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00
MO	0.32	**1.00**	0.32	0.13	0.43	0.43	0.00	0.00
PG	0.92	**1.00**	0.00	0.00	0.00	0.67	0.00	0.00
UTX	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00
XOM	0.20	0.20	0.00	0.00	0.00	**1.00**	0.00	0.00

Notes to table: The Table presents p-values from the MCS test of Hansen et al. [2011] under a 1% and 5% VaR loss function as defined in Lopez [1998], for the 10 Dow constituents under alternative ACD and GARCH dynamics and 4 different conditional distributions. The losses were based on 5353 1-ahead rolling conditional density forecasts, starting in 27-11-1991 and ending in 28-02-2013. The model coefficients were re-estimated every 250 periods using a window size of length 2000.

Multivariate Extensions

Table: Literature Review (Multivariate ACD Models)

Authors (by Date)	Data	Distribution	TVHM	Comments
Jondeau and Rockinger [2009]	4 Equity Indices (D)	Multivariate Skew Student	S+K	in-sample *News Impact Surface demonstration*
Jondeau and Rockinger [2012]	5 Equity Indices (W)	Multivariate Skew Student *(Sahu, Dey and Branco 2003)*	S+K	in- and out-of sample *(Portfolio Application, distributional timing)*
Ghalanos et al. [2013] *IFACD*	14 MSCI equity index iShares (D)	Multivariate Affine GH *(Schmidt et al 2006)*	S+K	in- and out-of sample *(Risk and Portfolio applications)*

The Independent Factor ACD Model (IFACD)

Model Representation

$$\mathbf{r}_t = \mathbf{m}_t + \varepsilon_t \quad t = 1, \ldots, T$$
$$\varepsilon_t = \mathbf{A}\mathbf{f}_t$$
$$\mathbf{A} = \mathbf{\Sigma}^{1/2}\mathbf{U}$$
$$\mathbf{f}_t = \mathbf{H}_t^{1/2}\mathbf{z}_t$$
$$\mathbf{r}_t \left| \mathfrak{F}_{t-1} \sim maGH_N\left(\mathbf{m}_t, \mathbf{\Sigma}_t, \eta_t\right)\right.$$
$$\eta_\mathbf{t} = \left(\eta_{1t}, \ldots, \eta_{Nt}\right)'$$
$$\eta_{it} = \left(\lambda_{it}, \rho_{it}, \zeta_{it}\right)'$$

Analytic Higher Co-moments

$$\boldsymbol{M}_t^3 = \boldsymbol{A}\boldsymbol{M}_{f,t}^3(\boldsymbol{A}' \otimes \boldsymbol{A}'),$$
$$\boldsymbol{M}_t^4 = \boldsymbol{A}\boldsymbol{M}_{f,t}^4(\boldsymbol{A}' \otimes \boldsymbol{A}' \otimes \boldsymbol{A}'),$$

Semi-Analytic Portfolio Density

$$R_t = \boldsymbol{w}_t'\boldsymbol{r}_t = \boldsymbol{w}_t'\boldsymbol{m}_t + (\boldsymbol{w}_t'\boldsymbol{A}\boldsymbol{H}_t^{1/2})\boldsymbol{z}_t$$
$$w_{i,t}r_{i,t} = (w_{i,t}m_{i,t} + \overline{w}_{i,t}z_{i,t}) \sim GH_{\lambda_i}\left(\overline{w}_{i,t}\mu_{i,t} + w_{i,t}m_{i,t}, \left|\overline{w}_{i,t}\right|\delta_{i,t}, \frac{\alpha_{i,t}}{\left|\overline{w}_{i,t}\right|}, \frac{\beta_{i,t}}{\left|\overline{w}_{i,t}\right|}\right)$$

Comments

- Extends the GO-GARCH model of van der Weide [2002] to time varying higher moments.
- Follows model extensions proposed in Chen et al. [2010], Zhang and Chan [2009] and Broda and Paolella [2009].
- Dimensionality reduction possible in PCA (whitening) stage.
- Estimation of U (rotation) by ICA (linear and noiseless assumption).
- VERY large scale estimation possible because of separable representation.
- Only feasible model (AFAIK) to provide analytic time varying higher co-moments.
- Weighted Portfolio Density (and hence d*,p*,q*,r* methods) via FFT.

Empirical Study: MSCI 14 Portfolio Application

Table: Time-varying Higher Co-moments Portfolio with **C**onstant **A**bsolute **R**isk **A**version Utility

	IFACD	CHICAGO	DCC(T)
$\eta = 1$			
$\bar{W}_T$	75.47	79.72	57.21
$\hat{\mu}$	0.0018	0.0018	0.0017
$\hat{\sigma}$	0.0169	0.0170	0.0168
$\sqrt(252)\frac{\hat{\mu}}{\hat{\sigma}}$	1.69	1.71	1.60
LW [stat ; p-value] vs IFACD		[1.248 ; 0.224]	[1.138 ; 0.248]
MCS [p-value]	[0.357]	[1.000]	[0.357]
Log Relative Wealth		0.05	-0.28
$\eta = 5$			
$\bar{W}_T$	94.07	58.60	43.46
$\hat{\mu}$	0.0019	0.0017	0.0016
$\hat{\sigma}$	0.0175	0.0161	0.0159
$\sqrt(252)\frac{\hat{\mu}}{\hat{\sigma}}$	1.72	1.66	1.57
LW [stat ; p-value] vs IFACD		[0.559 ; 0.564]	[1.214 ; 0.228]
MCS [p-value]	[1.000]	[0.054]	[0.044]
Log Relative Wealth		-0.47	-0.77
$\eta = 10$			
$\bar{W}_T$	64.20	39.84	25.33
$\hat{\mu}$	0.0017	0.0015	0.0014
$\hat{\sigma}$	0.0163	0.0154	0.0151
$\sqrt(252)\frac{\hat{\mu}}{\hat{\sigma}}$	1.69	1.58	1.42
LW [stat ; p-value] vs IFACD		[1.301 ; 0.198]	[2.390 ; 0.020]
MCS [p-value]	[1.000]	[0.021]	[0.003]
Log Relative Wealth		-0.48	-0.93
$\eta = 25$			
$\bar{W}_T$	23.83	17.82	11.88
$\hat{\mu}$	0.0013	0.0012	0.0010
$\hat{\sigma}$	0.0145	0.0142	0.0138
$\sqrt(252)\frac{\hat{\mu}}{\hat{\sigma}}$	1.44	1.34	1.20
LW [stat ; p-value] vs IFACD		[1.636 ; 0.107]	[2.697 ; 0.006]
MCS [p-value]	[1.000]	[0.039]	[0.000]
Log Relative Wealth		-0.29	-0.70

The Table reports the out-of-sample performance of the IFACD, CHICAGO* (see Broda and Paolella [2009]), and DCC (T) models from the optimization of the CARA utility approximation using the first 4 co-moment matrices, for 14 MSCI indices from $11/08/2000$ to $28/12/2011$ (2610 days). Starting on $10/08/2000$ ($T = 1$), the last 4 years of data were used to estimate the 3 models, after which the estimates were used to produce rolling forecasts for the next 5 days. The model parameters were re-estimated taking into account new data every 5 days for a total of 522 re-estimations and 2610 out-of-sample forecasts. The performance statistics reported are $\bar{W}_T$ representing terminal wealth of a portfolio with a starting value of 1, the mean ($\hat{\mu}$), average volatility ($\hat{\sigma}$), the annualized (252) risk-return, the statistic and p-values of the Ledoit and Wolf [2008] test for the difference in the Sharpe ratio between the IFACD and other models, the p-values of the MCS procedure of Hansen et al. [2011] using 10,000 bootstrap replications under the range statistic and the relative log difference in terminal wealth between the IFACD and other models. The CARA utility was optimized under 4 different risk aversion levels, from the mild ($\eta = 1$) to very risk averse investor ($\eta = 25$).

data: Yahoo finance.
source: Ghalanos et al. [2013].
CHICAGO: **Conditional **H**eteroscedastic **ICA** **G**eneralized **O**rthogonal model (IFACD without time varying higher moments)*

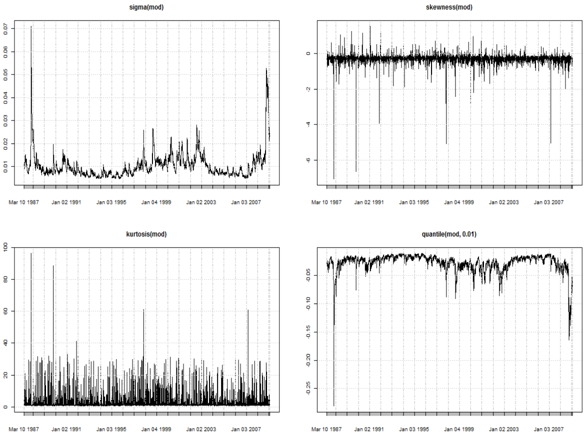
R packages

With the exception of the IFACD model, all other results/models can be replicated using the **rugarch**, **racd**, **rmgarch** and **parma** packages, whose vignettes contain details of the tests/models/etc used here. Additional examples can be found on my blog.

- Univariate GARCH: **rugarch** (CRAN and `http://code.google.com/p/rugarch`)
- Multivariate GARCH: **rmgarch** (CRAN and `http://code.google.com/p/rmgarch`)
- Univariate ACD: **racd** (`http://code.google.com/p/rugarch`) Also see `http://www.unstarched.net/2013/04/22/time-varying-higher-moments-with-the-racd-package/`
- Portfolio Optimization: **parma** (CRAN and `http://code.google.com/p/parma`)
- Semi-parametric distribution: **spd** (CRAN)
- Nonlinear Optimization: **Rsolnp** (CRAN and R-Forge)

racd code demo

```
>
library(racd)
library(rugarch)
data(sp500ret)
spec = acdspec(variance.model=list(model="sGARCH", variance.targeting=TRUE),
distribution.model=list(model="nig",skewOrder=c(1,0,1),
shapeOrder=c(1,1,1),skewmodel="quad",shapemodel="pwl"))
mod = acdfit(spec, sp500ret, solver="msoptim",solver.control=list(restarts=5))
```




```
par(mfrow=c(2,2))
plot(sigma(mod))
plot(skewness(mod))
plot(kurtosis(mod))
plot(quantile(mod, 0.01))
```

Questions (?)

THANKS!

Bibliography I

T. Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327, 1986.

S.A. Bond and K. Patel. The conditional distribution of real estate returns: Are higher moments time varying? *Journal of Real Estate Finance and Economics*, 26(2):319–339, 2003.

K. Brannas and N. Nordman. Conditional skewness modelling for stock returns. *Applied Economics Letters*, 10(11):725–728, 2003.

W.A. Brock and W.D. Dechert. A general class of specification tests: The scalar case. In *Business and Economics Statistics Section of the Proceedings of the American Statistical Society*, pages 70–79, 1988.

S.A. Broda and M.S. Paolella. Chicago: A fast and accurate method for portfolio risk calculation. *Journal of Financial Econometrics*, 7 (4):412, 2009.

C. Brooks, S.P. Burke, S. Heravi, and G. Persand. Autoregressive conditional kurtosis. *Journal of Financial Econometrics*, 3(3):399–421, 2005.

Y. Chen, W. Härdle, and V. Spokoiny. Ghica–risk analysis with gh distributions and independent components. *Journal of Empirical Finance*, 17(2):255–269, 2010.

P. Christoffersen and D. Pelletier. Backtesting value-at-risk: A duration-based approach. *Journal of Financial Econometrics*, 2(1):84–108, 2004.

P.F. Christoffersen. Evaluating interval forecasts. *International Economic Review*, 39(4):841–862, 1998.

R.F. Engle. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica*, 50(4): 987–1007, 1982.

A Ergün and Jongbyung Jun. Conditional skewness, kurtosis, and density specification testing: Moment-based versus nonparametric tests. *Studies in Nonlinear Dynamics and Econometrics*, 14(3):5, 2010a.

A Tolga Ergün and Jongbyung Jun. Time-varying higher-order conditional moments and forecasting intraday var and expected shortfall. *The Quarterly Review of Economics and Finance*, 50(3):264–272, 2010b.

E.F. Fama. Mandelbrot and the stable paretian hypothesis. *Journal of Business*, 36(4):420–429, 1963.

A. Ghalanos, E. Rossi, and G. Urga. Independent factor autoregressive conditional density model. *Econometric Reviews*, forthcoming, 2013.

B.E. Hansen. Autoregressive conditional density estimation. *International Economic Review*, 35(3):705–730, 1994.

L.P. Hansen. Large sample properties of generalized method of moments estimators. *Econometrica*, 50(4):1029–1054, 1982.

P.R. Hansen, A. Lunde, and J.M. Nason. Choosing the best volatility models: The model confidence set approach. *Oxford Bulletin of Economics and Statistics*, 65:839–861, 2003.

P.R. Hansen, A. Lunde, and J.M. Nason. The model confidence set. *Econometrica*, 79(2):453–497, 2011.

Bibliography II

C.R. Harvey and A. Siddique. Autoregressive conditional skewness. *Journal of Financial and Quantitative Analysis*, 34(4):465–487, 1999.

Y. Hong and H. Li. Nonparametric specification testing for continuous-time models with applications to term structure of interest rates. *Review of Financial Studies*, 18(1):37–84, 2005.

E. Jondeau and M. Rockinger. Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements. *Journal of Economic Dynamics and Control*, 27(10):1699–1737, 2003.

E. Jondeau and M. Rockinger. The impact of shocks on higher moments. *Journal of Financial Econometrics*, 7(2):77, 2009.

E. Jondeau and M. Rockinger. On the importance of time variability in higher moments for asset allocation. *Journal of Financial Econometrics*, 10(1):84–123, 2012.

P.H. Kupiec. Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives*, 3(2):73–84, 1995. ISSN 1074-1240.

O. Ledoit and M. Wolf. Robust performance hypothesis testing with the sharpe ratio. *Journal of Empirical Finance*, 15(5):850–859, 2008.

Á. León, G. Rubio, and G. Serna. Autoregresive conditional volatility, skewness and kurtosis. *Quarterly Review of Economics and Finance*, 45(4):599–618, 2005.

Jose Lopez. Methods for evaluating value-at-risk estimates. *Economic Policy Review*, 4(3), 1998.

B. Mandelbrot. The variation of certain speculative prices. *Journal of Business*, 36(4):394–419, 1963.

A.J. McNeil and R. Frey. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3):271–300, 2000.

M.H. Pesaran and A. Timmermann. A simple nonparametric test of predictive performance. *Journal of Business and Economic Statistics*, 10(4):461–465, 1992.

G. Premaratne and A.K. Bera. Modeling asymmetry and excess kurtosis in stock return data. *Illinois Research & Reference Working Paper No. 00-123*, 2000.

M. Rockinger and E. Jondeau. Entropy densities with an application to autoregressive conditional skewness and kurtosis. *Journal of Econometrics*, 106(1):119–142, 2002.

R. van der Weide. Go-garch: a multivariate generalized orthogonal garch model. *Journal of Applied Econometrics*, 17(5):549–564, 2002.

A. Wilhelmsson. Value at risk with time varying variance, skewness and kurtosis–the nig-acd model. *Econometrics Journal*, 12(1):82–104, 2009.

K. Zhang and L. Chan. Efficient factor garch models and factor-dcc models. *Quantitative Finance*, 9(1):71–91, 2009.