The Impact of Computational Error on the Volatility Smile

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Inverting Black-Scholes-Merton

- Objective of option pricing models is to derive an appropriate price.
- This process can be inverted from a known option price to implied volatility.

(Latane & Rendleman, 1976; Schmalensee & Trippi, 1978)

$$c_t^m = S_t N(d_1(S_t, \tau, K, r, \hat{\sigma}_t)) - K e^{-r\tau} N(d_2(S_t, \tau, K, r, \hat{\sigma}_t))$$

$$d_1(S_t, \tau, K, r, \hat{\sigma}_t) = \frac{\ln \frac{S_t}{K} + \left(r + \frac{\hat{\sigma}_t^2}{2}\right)\tau}{\hat{\sigma}_t \sqrt{\tau}}$$

$$d_2 = d_1 - \hat{\sigma}_t \sqrt{\tau}$$

Puzzle of the Volatility Smile

- Options that differ only by strike should have the same implied volatility.
- "To even talk about volatility smiles is schizophrenic." (Mayhew, 1995)
- Research and trading experience uncovers smiles, smirks, and skews – all indications that the model implies multiple volatilities for the same underlying asset.

Source of the Smile

- Previous explanations:
  - Jump return process
  - Stochastic volatility
  - Market frictions
  - Non-normality of returns
  - “Insurance” against market crashes
- **New contribution:**
  - **Computational considerations**

Related Literature and Hypothesis

- Measurement errors in inputs can have significant impact on implied volatility. (Hentschel, 2003)
- Small errors in inputs can lead to large divergence in implied volatility estimate.
- This nonlinearity is an example of sensitive dependence on initial conditions.
- **Therefore, we posit that computational factors contribute significantly to the volatility smile.**

Literature Problems

- Few papers discuss computational matters explicitly.
- Root finding technique and tolerance are rarely mentioned.
- Overwhelming majority of papers on this topic make no mention of how implied volatility is calculated.

Closed-form Approximations

- Several formulas for implied volatility exist.
  - Often require relaxing assumptions or good only in specific cases.
  - Useful in spreadsheet and pedagogical applications.
  - Provide a starting point for iterative techniques.

Iterative Root Finding Techniques

- Five common techniques:
  - Bisection
  - Secant
  - Regula Falsi (False Position)
  - Dekker-Brent (Commonly used by Matlab)
  - Newton-Raphson (Most commonly referenced method)

Theory

- The reflection points can be considered ideal starting points for the Newton-Raphson method. (Manaster & Kohler, 1982)

$$\sigma = \sqrt{\frac{2}{\tau} \cdot \left| \ln\left(\frac{S}{K}\right) + r\tau \right|}$$

- Theorem 1 discusses conditions for quadratic convergence in addition to over- and under-estimation.

Numerical Precision

- Computer models are discrete approximations of theoretically continuous processes.
- Gaussian density cannot be integrated, so further approximations are necessary.
- Results presented here use R (package Rmpfr) for quadruple precision arithmetic (128-bit storage).

Process

- Methodology:
  - Generate Black-Scholes-Merton prices, varying exercise price over a wide range of moneyness.
  - Use these “perfect” prices to estimate implied volatility.
  - Generate graph of implied volatility.

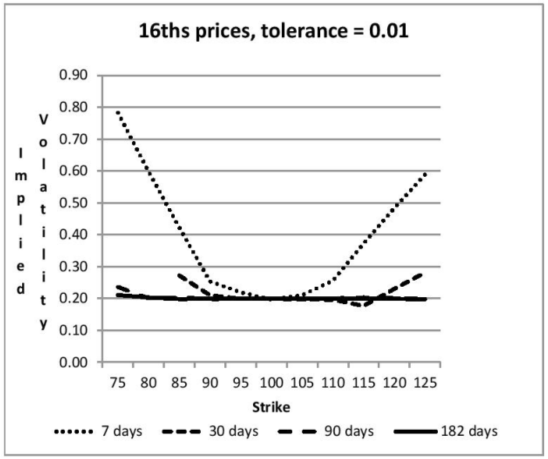
Parameters

- These are the same across all simulations:
  - Spot = $100
  - Strike ranges from $75 to $125 in $5 increments
  - Interest rate = 4%
  - Expiration = 7, 30, 90, and 182 days
  - Volatility = 20%

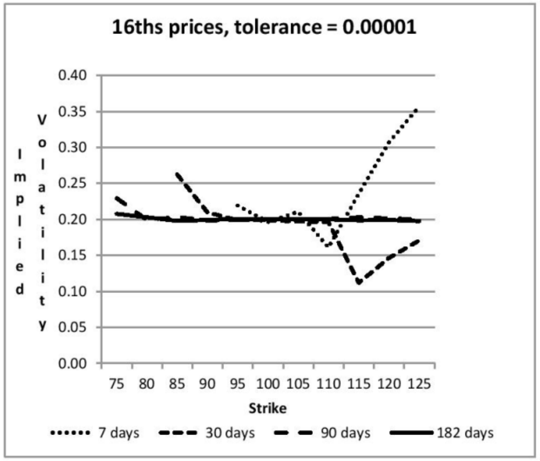
Factors

- Several factors influence the shape of the volatility smile:
  - Numerical precision (tolerance of 0.01, 0.00001, and machine epsilon 2^-52)
  - Quotation unit (“continuous”, penny, sixteenths)
  - Five root finding techniques
  - Initial input/interval

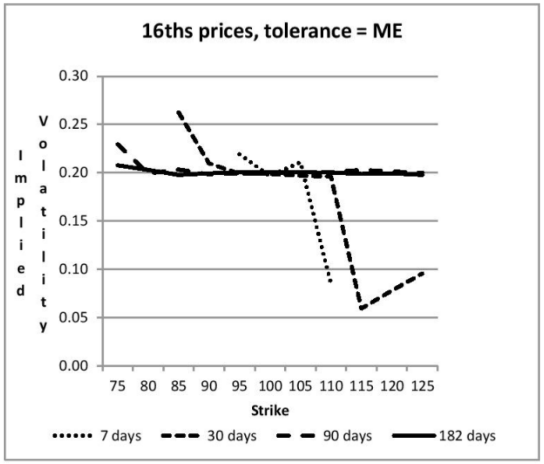
Results



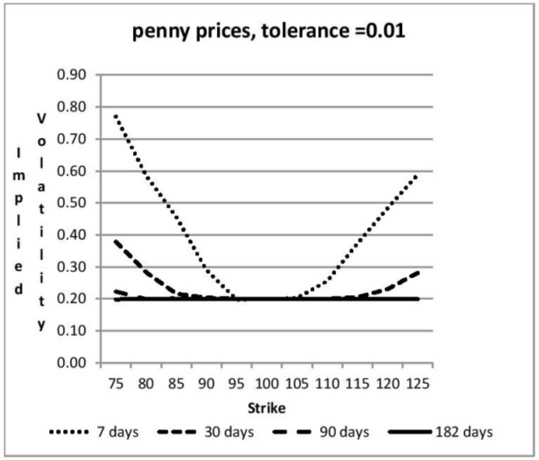
16ths prices, tolerance = 0.01
Implied Volatility
Strike
7 days
30 days
90 days
182 days



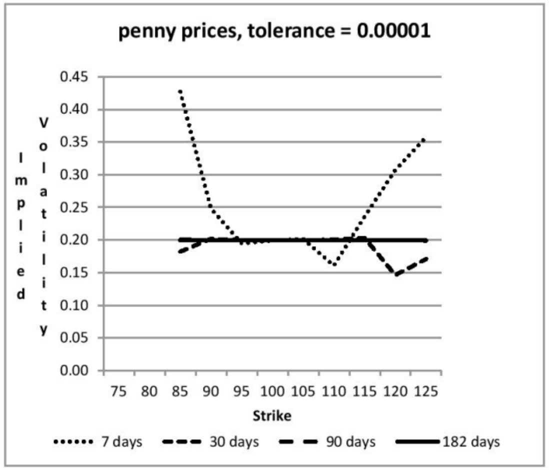
16ths prices, tolerance = 0.00001
Implied Volatility
Strike
7 days
30 days
90 days
182 days



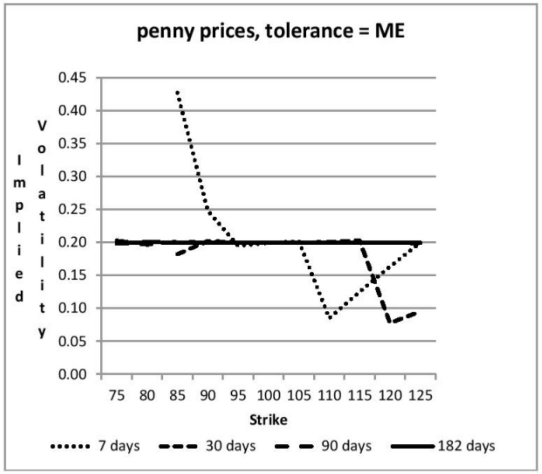
16ths prices, tolerance = ME
Implied Volatility
Strike
7 days
30 days
90 days
182 days



penny prices, tolerance =0.01
Implied Volatility
Strike
7 days
30 days
90 days
182 days



penny prices, tolerance = 0.00001
Implied Volatility
Strike
7 days
30 days
90 days
182 days



penny prices, tolerance = ME
Implied Volatility
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Results: Numerical Precision

- The smile dissipates as tolerance shrinks from penny (blue) to machine epsilon (green)

Results

- Despite full knowledge of the volatility used to generate Black-Scholes-Merton prices, the inversion process creates a wide variety of smiles, skews, and smirks.
- Root finding technique, initial input, quotation rounding, and numerical precision all contribute to the shape of the volatility smile.

Measuring Smiles:Empirical Result

- Assume that the closest to ATM option reveals the true volatility.
- Use that volatility to price other options and calculate sum-of-squared deviations as a measure of the smile.
- Allows comparison of methods.
- Illustrative example of Ebay: 40% of the smile is due to computational factors.

Implications

- We cannot remain cavalier in calculating implied volatility.
- There are apparently no choices that solve the computational problems.
- Other factors likely contribute to smile effects, but possibly cannot be disentangled from computational errors.

We welcome your questions and comments.

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