The Impact of Computational Error on the Volatility Smile

Don M. Chance  
Louisiana State University

Thomas A. Hanson  
Kent State University

Weiping Li  
Oklahoma State University

Jayaram Muthuswamy  
Kent State University

R/Finance 2013:  
Applied Finance with R

18 May 2013
• Objective of option pricing models is to derive an appropriate price.

• This process can be inverted from a known option price to implied volatility.

(Latane & Rendleman, 1976; Schmalensee & Trippi, 1978)

\[
c_t^m = S_t N(d_1(S_t, \tau, K, r, \hat{\sigma}_t)) - K e^{-r\tau} N(d_2(S_t, \tau, K, r, \hat{\sigma}_t))
\]

\[
d_1(S_t, \tau, K, r, \hat{\sigma}_t) = \frac{\ln \frac{S_t}{K} + \left( r + \frac{\hat{\sigma}_t^2}{2} \right) \tau}{\hat{\sigma}_t \sqrt{\tau}}
\]

\[
d_2 = d_1 - \hat{\sigma}_t \sqrt{\tau}
\]
Puzzle of the Volatility Smile

• Options that differ only by strike should have the same implied volatility.

• “To even talk about volatility smiles is schizophrenic.” (Mayhew, 1995)

• Research and trading experience uncovers smiles, smirks, and skews – all indications that the model implies multiple volatilities for the same underlying asset.
Source of the Smile

• Previous explanations:
  – Jump return process
  – Stochastic volatility
  – Market frictions
  – Non-normality of returns
  – “Insurance” against market crashes

• New contribution:
  – Computational considerations
Related Literature and Hypothesis

- Measurement errors in inputs can have significant impact on implied volatility. (Hentschel, 2003)
- Small errors in inputs can lead to large divergence in implied volatility estimate.
- This nonlinearity is an example of sensitive dependence on initial conditions.
- Therefore, we posit that computational factors contribute significantly to the volatility smile.
Literature Problems

- Few papers discuss computational matters explicitly.
- Root finding technique and tolerance are rarely mentioned.
- Overwhelming majority of papers on this topic make no mention of how implied volatility is calculated.
Closed-form Approximations

• Several formulas for implied volatility exist.
  – Often require relaxing assumptions or good only in specific cases.
  – Useful in spreadsheet and pedagogical applications.
  – Provide a starting point for iterative techniques.
Iterative Root Finding Techniques

- Five common techniques:
  - Bisection
  - Secant
  - Regula Falsi (False Position)
  - Dekker-Brent (Commonly used by Matlab)
  - Newton-Raphson (Most commonly referenced method)
The reflection points can be considered ideal starting points for the Newton-Raphson method. (Manaster & Kohler, 1982)

\[ \sigma = \sqrt{\frac{2}{\tau} \cdot \left| \ln \left( \frac{S}{K} \right) + r\tau \right|} \]

Theorem 1 discusses conditions for quadratic convergence in addition to over- and under-estimation.
Numerical Precision

- Computer models are discrete approximations of theoretically continuous processes.
- Gaussian density cannot be integrated, so further approximations are necessary.
- Results presented here use R (package Rmpfr) for quadruple precision arithmetic (128-bit storage).
Process

• Methodology:
  – Generate Black-Scholes-Merton prices, varying exercise price over a wide range of moneyness.
  – Use these “perfect” prices to estimate implied volatility.
  – Generate graph of implied volatility.
Parameters

• These are the same across all simulations:
  – Spot = $100
  – Strike ranges from $75 to $125 in $5 increments
  – Interest rate = 4%
  – Expiration = 7, 30, 90, and 182 days
  – Volatility = 20%
Factors

• Several factors influence the shape of the volatility smile:
  – Numerical precision (tolerance of 0.01, 0.00001, and machine epsilon $2^{-52}$)
  – Quotation unit (“continuous”, penny, sixteenths)
  – Five root finding techniques
  – Initial input/interval
Results
• The smile dissipates as tolerance shrinks from penny (blue) to machine epsilon (green)
Results

• Despite full knowledge of the volatility used to generate Black-Scholes-Merton prices, the inversion process creates a wide variety of smiles, skews, and smirks.

• Root finding technique, initial input, quotation rounding, and numerical precision all contribute to the shape of the volatility smile.
Measuring Smiles: Empirical Result

- Assume that the closest to ATM option reveals the true volatility.
- Use that volatility to price other options and calculate sum-of-squared deviations as a measure of the smile.
- Allows comparison of methods.
- Illustrative example of Ebay: 40% of the smile is due to computational factors.
Implications

• We cannot remain cavalier in calculating implied volatility.
• There are apparently no choices that solve the computational problems.
• Other factors likely contribute to smile effects, but possibly cannot be disentangled from computational errors.
We welcome your questions and comments.

The Impact of Computational Error on the Volatility Smile

Don M. Chance
Thomas A. Hanson
Weiping Li
Jayaram Muthuswamy