Predicting Exchange Rates Out of Sample: Can Economic Fundamentals Beat the Random Walk?

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Joint work with Wei Wang and Ilias Tsiakas

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Bond price and Economic Fundamentals


- Economic fundamentals can predict yield curve.

- Economic fundamentals: more than 100 economic indicators, including industrial production, CPI, money supply, employment rate, …

- Method:

  ![Diagram showing the process of forecasting yield curve](#)
Stock price and Economic Fundamentals


- Economic fundamentals can predict S&P500.

- Economic fundamentals: short-term yield, long-term yield, term spread, default spread, inflation, consumption/wealth, …

- Method: combined forecasts.
Combined forecasts

- $K$ predictive models give $K$ forecasts.
  - Option 1 (simple combination): take the mean, median, or trimmed mean
  - Option 2: take their weighted average, with the weights being determined by the past performance of individual models, or Discounted Mean Squared Error (DMSE).
    \[
    weight_j = \frac{DMSE_j^{-1}}{\sum_{j=1}^{N} DMSE_j^{-1}}
    \]
<table>
<thead>
<tr>
<th>Model</th>
<th>Out-of-sample R-squared (%)</th>
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<tbody>
<tr>
<td>Dividend/price</td>
<td>0.34</td>
</tr>
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<td>Dividend/lagged price</td>
<td>0.26</td>
</tr>
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<td>Earnings/price</td>
<td>0.36</td>
</tr>
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<td>Dividend/earnings</td>
<td>-1.42</td>
</tr>
<tr>
<td>Variance (daily)</td>
<td>-12.97</td>
</tr>
<tr>
<td>Book/market</td>
<td>-2.6</td>
</tr>
<tr>
<td>Net equity issuance</td>
<td>-0.91</td>
</tr>
<tr>
<td>Short-term yield</td>
<td>-2.78</td>
</tr>
<tr>
<td>Long-term yield</td>
<td>-3.09</td>
</tr>
<tr>
<td>Long-term bond return</td>
<td>0.33</td>
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<tr>
<td>Term spread</td>
<td>-2.96</td>
</tr>
<tr>
<td>Default spread</td>
<td>-2.72</td>
</tr>
<tr>
<td>Default spread of returns</td>
<td>-1.1</td>
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<tr>
<td>Inflation</td>
<td>-0.84</td>
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<td>Consumption/wealth</td>
<td>1.44 *</td>
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### Combined forecasts

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<td>Mean</td>
<td>3.58 **</td>
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<td>Median</td>
<td>3.04 **</td>
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<tr>
<td>Trimmed mean</td>
<td>3.51 **</td>
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<td>DMSE, θ=0.1</td>
<td>3.54 **</td>
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<td>DMSE, θ=0.9</td>
<td>3.49 **</td>
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**DMSE:** This forecasts combination is based on individual models’ past performance, measured by Discounted Mean Squared Error (DMSE). θ is the discounting factor.

**Benchmark:** random walk
Whether economic fundamentals can predict other asset prices?

- Look abroad

(Photograph: Guy Parsons)
Foreign exchange rates and Economic Fundamentals - Outline

- Economic fundamentals
- Forecasting method:
  - Individual models
  - “Kitchen-sink” model
  - Combined forecasts
  - Efficient “kitchen-sink” model
- Predictability evaluation:
  - Statistical predictability
  - Portfolio returns
Economic fundamentals

- Random Walk (RW): $x_t = 0$

$$r_{t+1} = \alpha + e_{t+1}$$

$$\hat{r}_{t+1} = \hat{\alpha} = \text{historical average}$$
Economic fundamentals

- Random Walk (RW): \( x_t = 0 \)
- Uncovered Interest Parity (UIP):
  \[
  x_t = x_{1t} = \Delta (\text{interest rate})_t
  \]
  - The difference in interest rates between two countries is equal to the expected change in exchange rates between the countries' currencies.
  
  Otherwise, arbitrage opportunity exists.

- Most studies indicate the violation of this condition.
- Carry trade strategy.
Economic fundamentals

- Random Walk (RW): $x_t = 0$
- Uncovered Interest Parity (UIP):
  \[ x_t = x_{1t} = \Delta (\text{interest rate})_t \]
- Purchasing Power Parity (PPP):
  \[ x_t = x_{2t} = \Delta (\text{price level})_t - s_t \]
  - law of one price.
  - identical goods will have the same price in different markets.
Economic fundamentals

- Random Walk (RW): $x_t = 0$
- Uncovered Interest Parity (UIP):
  \[ x_t = x_{1t} = \Delta \text{(interest rate)}_t \]
- Purchasing Power Parity (PPP):
  \[ x_t = x_{2t} = \Delta \text{(price level)}_t - s_t \]
- Monetary Fundamentals (MF):
  \[ x_t = x_{3t} = \Delta \text{(money supply)}_t - \Delta \text{(national income)}_t - s_t \]
- Taylor Rule (TR):
  \[ x_t = x_{4t} = 1.5 \Delta \text{(inflation)}_t + 0.1 \Delta \text{(output gap)}_t - 0.1 \Delta \text{(price level)}_t - 0.1s_t \]
Model, Return and Econ Fundamentals

- $P_t$: nominal exchange rate (domestic price of 1 foreign currency unit)

- $r_{t+1} = \log(P_{t+1}) - \log(P_t)$ is the foreign exchange rate return

- Different models have different predictor, $x_t$, in the predictive regression

  \[ r_{t+1} = \alpha + \beta x_t + e_{t+1} \]

- Economic fundamentals: $x_t$
Foreign exchange rates and Economic Fundamentals - method

1. Individual models: \( r_{t+1} = \alpha + \beta x_t + e_{t+1} \)

2. “Kitchen sink” regression: include \( x_{1t}, x_{2t}, x_{3t}, x_{4t} \) in a multiple regression

3. Combined forecasts: generate forecasts from individual models.
   - Simple combined forecasts: take the mean, median, or trimmed mean
   - Take their weighted average, with the weights are determined by the past performance of individual models (DMSE).
Data

- Monthly FX data ranging from January 1976 to June 2012 (~ 35 years).
- The 10 most liquid (G10) currencies in the world:
  - Australian dollar
  - Canadian dollar
  - Swiss franc
  - Deutsche mark
  - British pound
  - Japanese yen
  - Norwegian kroner
  - New Zealand dollar
  - Swedish kronor
  - US dollar
- 9 exchange rates.
Out-of-sample forecasts

- The first FX return to be predicted is in January 1986 (using a 10 year estimation window)
- Keep updating estimation window.
Statistical evaluation

- Out-of-sample $R^2$

$$R^2_{os} = 1 - \frac{\sum_{t=1}^{T-1} (\hat{r}_{t+1} - r_{t+1})^2}{\sum_{t=1}^{T-1} (\bar{r}_{t+1} - r_{t+1})^2}.$$ 

$
\hat{r}_{t+1}$ is the model’s forecast, $
\bar{r}_{t+1}$ is the benchmark’s forecast (historical average).

- Positive out-of-sample $R^2$

$\Leftrightarrow$ the lower alternative model’s error
$\Leftrightarrow$ the better the alternative model
## Out-of-sample R square (benchmark: random walk)

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<tr>
<td>Combined - DMSE(0.9)</td>
<td>-0.068</td>
<td>0.060</td>
<td>0.468</td>
<td>-0.591</td>
<td>0.028</td>
<td>0.990</td>
<td>-0.392</td>
<td>-0.023</td>
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<td>Combined - DMSE(1.0)</td>
<td>-0.403</td>
<td>-0.196</td>
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<td>-0.429</td>
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<td>Combined - Median</td>
<td>-0.195</td>
<td>0.071</td>
<td>-0.613</td>
<td>-0.535</td>
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<td>0.563</td>
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<td>0.702</td>
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### Uncovered interest rate parity
-0.968 | 0.528 | -1.806 | -3.337 | -3.387 | -0.417 | -2.736 | -0.851 | -6.376

### Purchasing power parity
-1.831 | -1.308 | -0.964 | -1.380 | 0.490 | -0.110 | -1.336 | -1.550 | -0.443

### Taylor rule
-1.553 | -2.115 | -2.385 | -2.134 | -2.959 | -1.019 | -1.745 | -0.543 | -1.671

### Monetary fundamentals
-1.879 | -1.643 | -0.513 | -2.751 | -0.881 | -1.487 | -2.639 | -1.809 | -0.496

### "Kitchen sink"
Economic evaluation

- Mean-variance strategy

  - Mean-variance strategy: target volatility (annualized) = 10%
  - Covariance estimates: sample covariance
  - We also implement 1/N strategy and momentum strategy
<table>
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<th>Sharpe Ratio</th>
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<th>Volatility (%)</th>
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<td>Combined - DMSE(0.9) (the best one)</td>
<td>11.083</td>
<td>12.761</td>
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“Efficient kitchen-sink” model

- What is the problem with “kitchen-sink” model?

\[ r_{t+1} = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + e_{t+1} \]

- More information leads to bad forecasts??

- Let’s examine \( \beta_1, ..., \beta_4 \) in the predictive regression of each currency
“Efficient kitchen-sink” model

- What is the problem with “kitchen-sink” model?

\[ r_{t+1} = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + e_{t+1} \]

- \( \beta_1, \ldots, \beta_4 \) are inflated.

- This motivates shrinkage estimation.
“Efficient kitchen-sink” model

- Constraint least squares that minimizes

\[
\sum_{t=1}^{T} (r_{t+1} - \alpha - \beta_1 x_{1t} - \beta_2 x_{2t} - \beta_3 x_{3t} - \beta_4 x_{4t})^2
\]

subject to constraints: \( \sum_{j=1}^{4} |\beta_j| < s_1 \) and \( \sum_{j=1}^{4} (\beta_j^2) < s_2 \)

- This is the elastic-net regression.

- Consequence: The estimated regression coefficients \((\beta_1, ..., \beta_4)\) are shrunk towards \(0\).
“Efficient kitchen-sink” model

- More robust and stable compared to traditional ones
- Forecasting error-oriented procedure
- Linear model – consistent with many empirical models in economics and finance
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<td>1.502</td>
<td>1.409</td>
<td>0.329</td>
<td>1.028</td>
<td>1.441</td>
<td>0.056</td>
<td>1.503</td>
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<td>1.829</td>
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Cumulative Wealth:

what if you invested $1 in January 1976?
Cumulative Wealth:

what if you invested $1 in January 1976?
Take-home message..

- It’s all about how to process information.
- Traditional regression is in-sample explanatory power-oriented, not forecasting-oriented.
- Remedies: forecasts combinations; shrinkage estimation
- **R package:** lars, elasticnet, glmnet, grpreg
UNIVERSITY OF NOTRE DAME
Applied and computational mathematics and statistics

THANK YOU!