Regime switches in volatility and correlation of financial institutions

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R/Finance

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Forecasting volatility

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- Standard since Engle (1982) and Bollerslev (1986): Extrapolate the variability in past returns to the future through a GARCH model for the conditional variance h_t:

$$h_t = \omega + \alpha(y_{t-1} - \mu)^2 + \beta h_{t-1}$$

✓ Parameter estimation by maximum likelihood (R packages fGarch, rugarch);

✓ Powerful technique, but using the squared demeaned return as the unique driver of the time-variation in volatility has several drawbacks.

Drawback I

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- Ignores the presence of multiple risk regimes, with rapid transitions (e.g. due to swings in interbank confidence, liquidity)
- Danielsson and Shin (2003):
  - ✓ Exogenous risk: regimes whereby price changes are due to reasons outside the control of market participants;
  - ✓ Endogenous risk: behavior of market players creates additional risk with respect to the uncertainty of fundamental news.

Exogenous and endogenous

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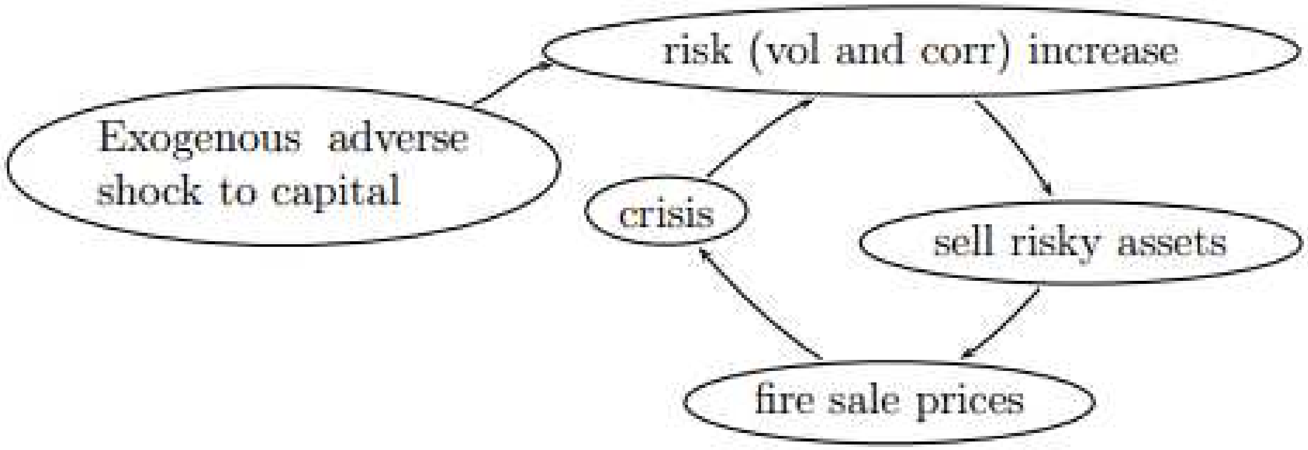
- Example of fire sales in Danielsson, Shin and Zigrand (2011) due a maximum risk constraint.



Multiple regimes

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- Single regime GARCH models, extrapolating the past to the future, are likely to fail when they are perhaps most needed – at the time of a transition between between a low risk and high risk regime.
- We study the design of regime–switching volatility–correlation models for the universe of 15 largest U.S. deposit banks over the period 1994–2011.
  - ✓ Dynamics of the transition probabilities:
    - Proposed solution: Specify them as a function of macro-financial variables: VIX, TED spread, Saint Louis Financial Stability Index.
  - ✓ Quid within–regime dynamics in the volatility (and correlation) parameter?

Quid within–regime dynamics?

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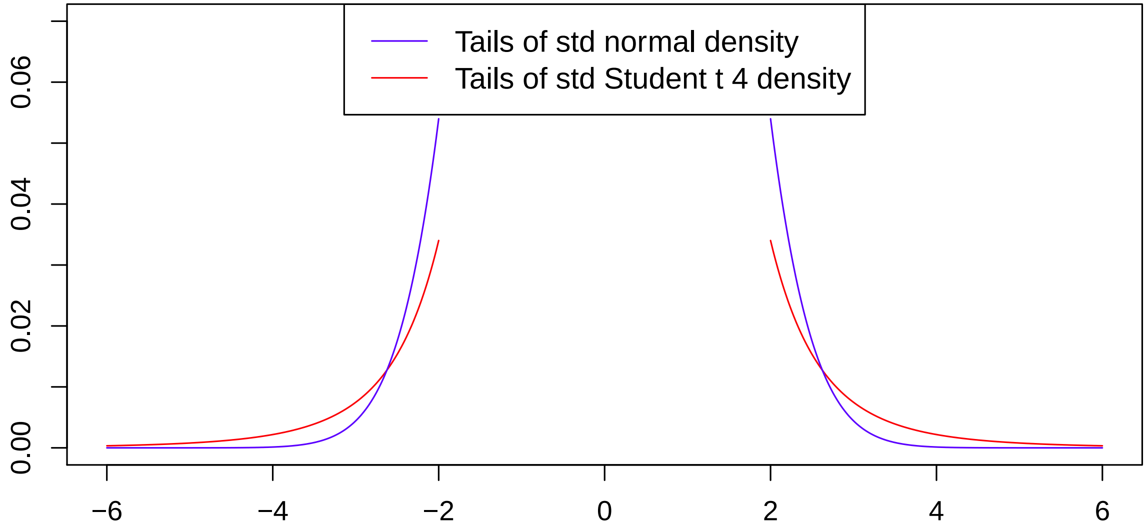
- Since Haas et al (2004) it has become standard to model regime switching GARCH models as:

$$\begin{cases} h_t^I = \omega^I + \alpha^I (y_{t-1} - \mu^I)^2 + \beta^I h_{t-1}^I \\ h_t^{II} = \omega^{II} + \alpha^{II} (y_{t-1} - \mu^{II})^2 + \beta^{II} h_{t-1}^{II} \end{cases}$$

for both Normal and Student t innovations, which is not intuitive:



Drawback II of garch: same volatility response whatever the return distribution

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- An extreme (positive/negative) return is a stronger signal of a volatility increase under the normal distribution than a fat tailed distribution $\Rightarrow$ Different volatility dynamics.
- Example of fat tail realizations: earnings releases.

Subject: [R-SIG-Finance] Modeling sharp drops in volatility

This is less of a programming question and more of a theoretical issue, but hopefully someone can help. How would you suggest going about modeling a sharp drop in volatility using a univariate GARCH model? What I am trying to capture is the overstatement of volatility forecasts that occurs after earnings announcements. As you surely know, the memory of GARCH models incorporates recent (and not so recent) swings in volatility into the variance of the stock price process. As a result, in the periods following an earnings event volatility forecasts will be severely overstated. (...) The effects of such single big moves persist in my process much longer than they should.

Standard approach: same volatility response whatever the return distribution

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- In this paper: The within–regime dynamics in volatility and correlation are driven by the score of the conditional density function: change the parameters in the direction that improve the local likelihood.
  - ✓ This idea was originally proposed by Creal, Koopman and Lucas (2012: GAS models: Generalized Autoregressive Score) and Harvey and Chakravarty (2008: DySco: Dynamic Score models), and is extended here to regime switching models.
  - ✓ Interestingly, the volatility/correlation impact of extreme returns is downweighted under a fat-tailed distribution and hence avoids the overstatement of volatility after a once-off extreme return.

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- Model;
- Results;
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- 2 regimes for mean + volatility and 2 regimes for correlation;
- No within–regime dynamics in the mean;
- Two volatility and correlation regimes, conditional density in each regime is Student t (copula)

$$
\begin{aligned}
f_{t|t-1}(y_t;\theta) &= \prod_{i=1}^{N} f_{it|t-1}(y_{it};\theta_i) \\
&\times c_{t|t-1}(F_{1t|t-1}(y_{1t}),\ldots,F_{Nt|t-1}(y_{Nt});\theta_*).
\end{aligned}
$$

- Across-regime dynamics: macro-financial variables;
- Within–regime dynamics: score.

GAS/DySco

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- Idea: New parameter value: linear combination of the past parameter value and an update, whereby the update is such that it increases the LLH.
- How? The time-variation in the parameter λ_t^k (e.g. variance h_{it}^k, correlation ρ_t^k) is autoregressive and driven by the score

$$\lambda_t^k = o^k + a^k(\lambda_{t-1}^k + S_{t-1}^k \nabla_{t-1}^k) + b^k \lambda_{t-1}^k,$$

where ∇_t^k is the score of the conditional density function:

$$\nabla_t^k = \frac{\partial \log p(y_t | m^k, H_t^k, s_t = k)}{\partial \lambda_t^k}.$$

- To avoid path dependence, we take the scores conditional on knowing the state of the regime.
- Scaling factor: $S_t^k = 1$ (steepest ascent), S_t^k the inverse of the conditional variance (Gauss-Newton updating) or S_t^k the inverse of the conditional standard deviation (Nelson, 1994).

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- The within-regime volatility dynamics are:

$$h_{it}^k = \omega_i^k + \alpha_i^k (1 + 3/\nu_i^k) \frac{\nu_i^k + 1}{[\nu_i^k - 2] + \frac{[y_{it-1} - \mu_i^k]^2}{h_{it-1}^k}} (y_{it-1} - \mu_i^k)^2 + \beta_i^k h_{it-1}^k .$$

- Note:
  - ✓ $\nu = \infty$: RS-GARCH model of Haas et al (JFEC, 2011, no path dependence regime specific vol)
  - ✓ The more fat-tailed the distribution is, the more extreme observations are downweighted.

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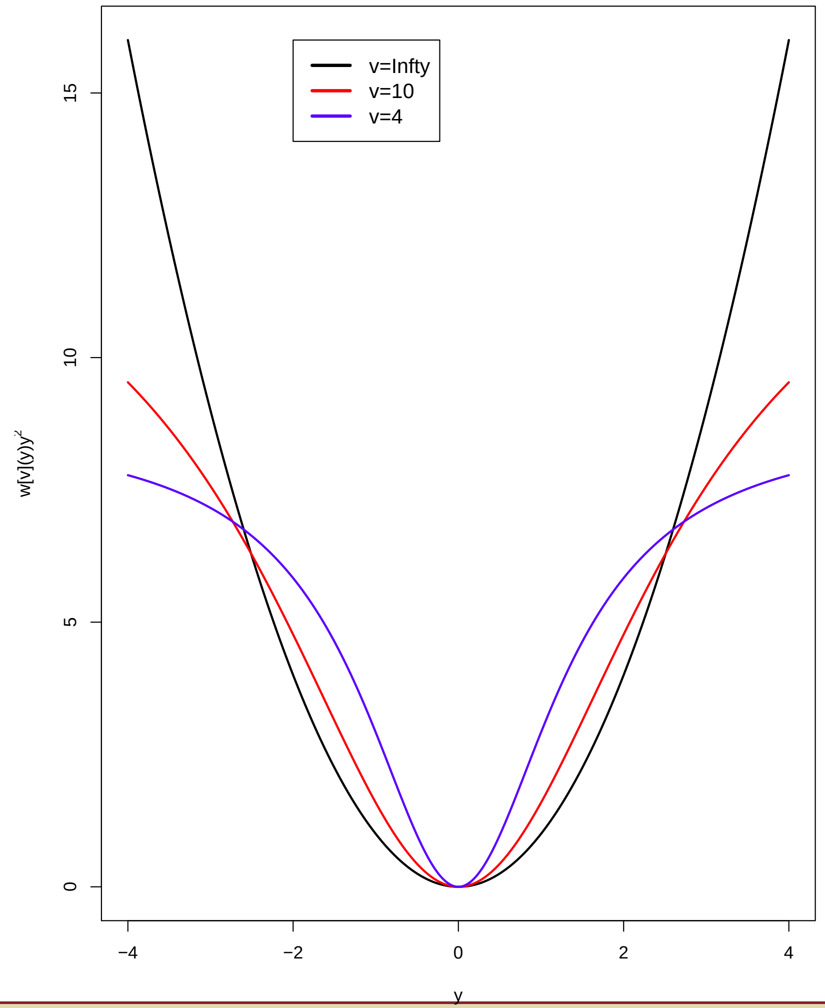
- News impact curve under the Student t score-based conditional variance model



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- A single parameter, assuming equicorrelation:

$$R_t^k = \begin{pmatrix} 1 & \rho_t & \dots & \rho_t \\ \rho_t & 1 & \dots & \rho_t \\ \rho_t & \rho_t & \ddots & \rho_t \\ \rho_t & \rho_t & \dots & 1 \end{pmatrix}$$

Why?

- We focus on a relatively homogenous universe (US deposit banks);
- Inclusion, deletions sector;
- Simplicity, both in terms of analysis, as computational convenience (no matrices calculations needed).
- See Engle and Kelly (2012, JBES) for single regime DECO.

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- To make sure that the estimated correlations are bounded, we specify ρ_t^k as the hyperbolic tangent of an underlying process q_t^k with GAS(1,1) dynamics:

$$\rho_t^k = (\exp(2q_t^k) - 1)/(\exp(2q_t^k) + 1)).$$

$$q_t^k = \omega_*^k + \alpha_*^k (q_{t-1}^k + S_{t-1}^k \nabla_{t-1}^k) + \beta_*^k q_{t-1}^k,$$

where ∇_t^k is the score of the Student t copula density function and S_t^k is the inverse of the conditional standard deviation of the score, and $\alpha_*^k, \beta_*^k > 0$.

- q_t^k is truncated to ensure positive definiteness of the R_t^k matrix requiring $-1/(N-1) < \rho_t^k < 1$.

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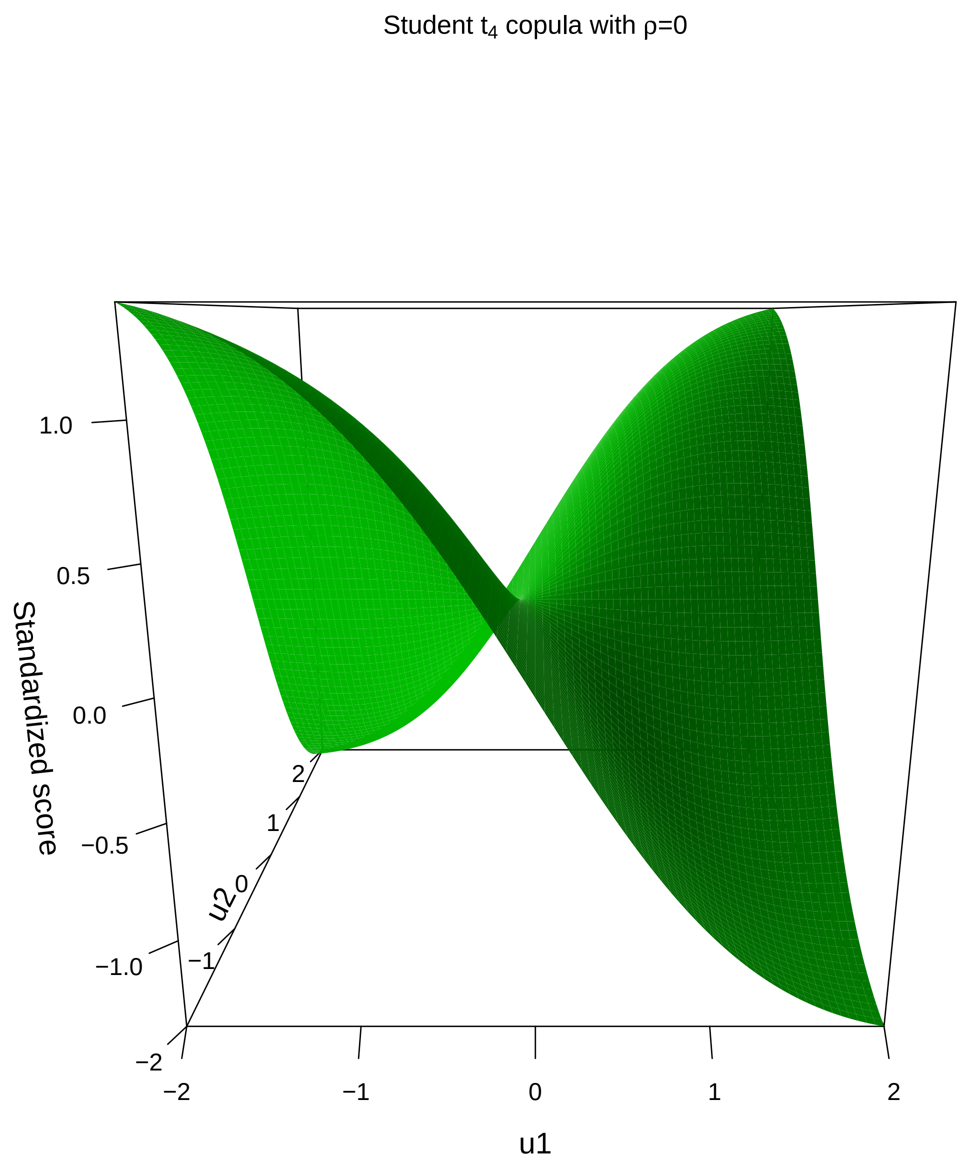
$$S_t^k \nabla_t^k = \underbrace{m_t^k}_{>0} \left[\underbrace{b_t^k}_{>0} \left(\frac{w_t^k}{(N-1)N} \sum_{i=1}^{N} \sum_{j \neq i} \tilde{y}_{it} \tilde{y}_{jt} - \rho_t^k \right) + a_t^k \left(\frac{w_t^k}{N} \sum_{i=1}^{N} \tilde{y}_{it}^2 - 1 \right) \right]$$

The score has three main components:

1. The excess value of the cross-product of weighted devolatilized returns and the conditional correlation: **enforces an increase in the conditional correlation process when the average cross-product of the devolatilized returns exceeds the conditional correlation** ρ_t^k,



- Note the curvature!

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$$S_t^k \nabla_t^k = \underbrace{m_t^k}_{>0} \left[\underbrace{b_t^k}_{>0} \left(\frac{w_t^k}{(N-1)N} \sum_{i=1}^{N} \sum_{j \neq i} \tilde{y}_{it} \tilde{y}_{jt} - \rho_t^k \right) + a_t^k \left(\frac{w_t^k}{N} \sum_{i=1}^{N} \tilde{y}_{it}^2 - 1 \right) \right]$$

$$a_t^k = -\rho_t^k (2 + \rho_t^k (N-2))$$

2. **Adjustment of ρ_t toward 0 in case of high dispersion**, and vice versa. The higher the dispersion, the less informative high values of the cross-products are about increases in correlation. E.g. the correlation signal of $(1, 1)$ is much stronger than $(1/4, 4)$, even though their cross-product is the same.

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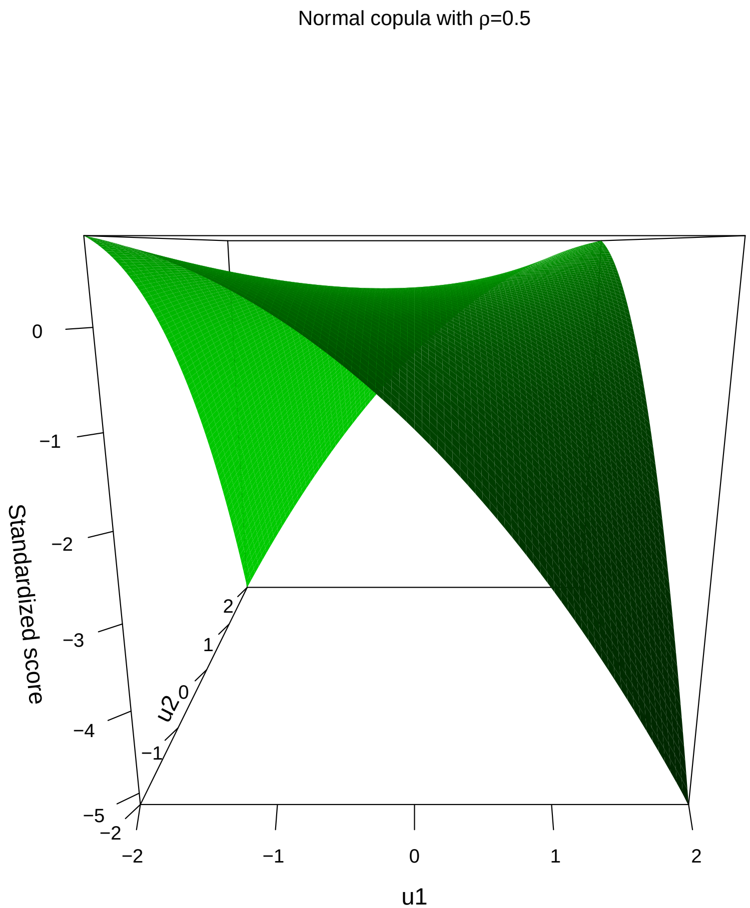
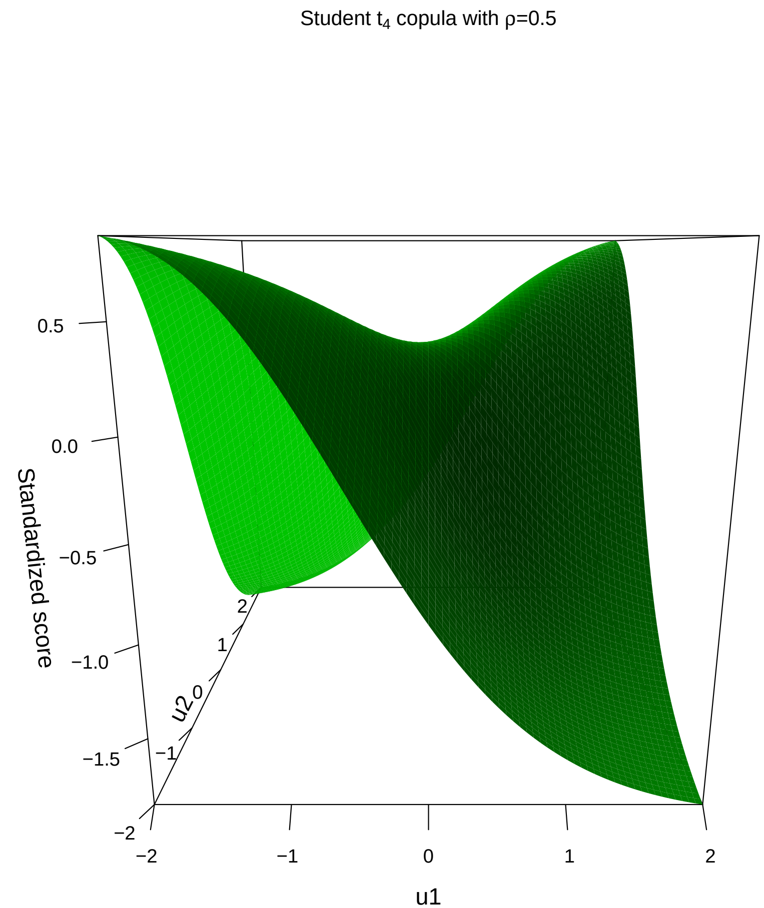
$$S_t^k \nabla_t^k = m_t^k \left[b_t^k \left(\frac{w_t^k}{(N-1)N} \sum_{i=1}^{N} \sum_{j \neq i} \tilde{y}_{it} \tilde{y}_{jt} - \rho_t^k \right) + a_t^k \left(\frac{w_t^k}{N} \sum_{i=1}^{N} \tilde{y}_{it}^2 - 1 \right) \right]$$

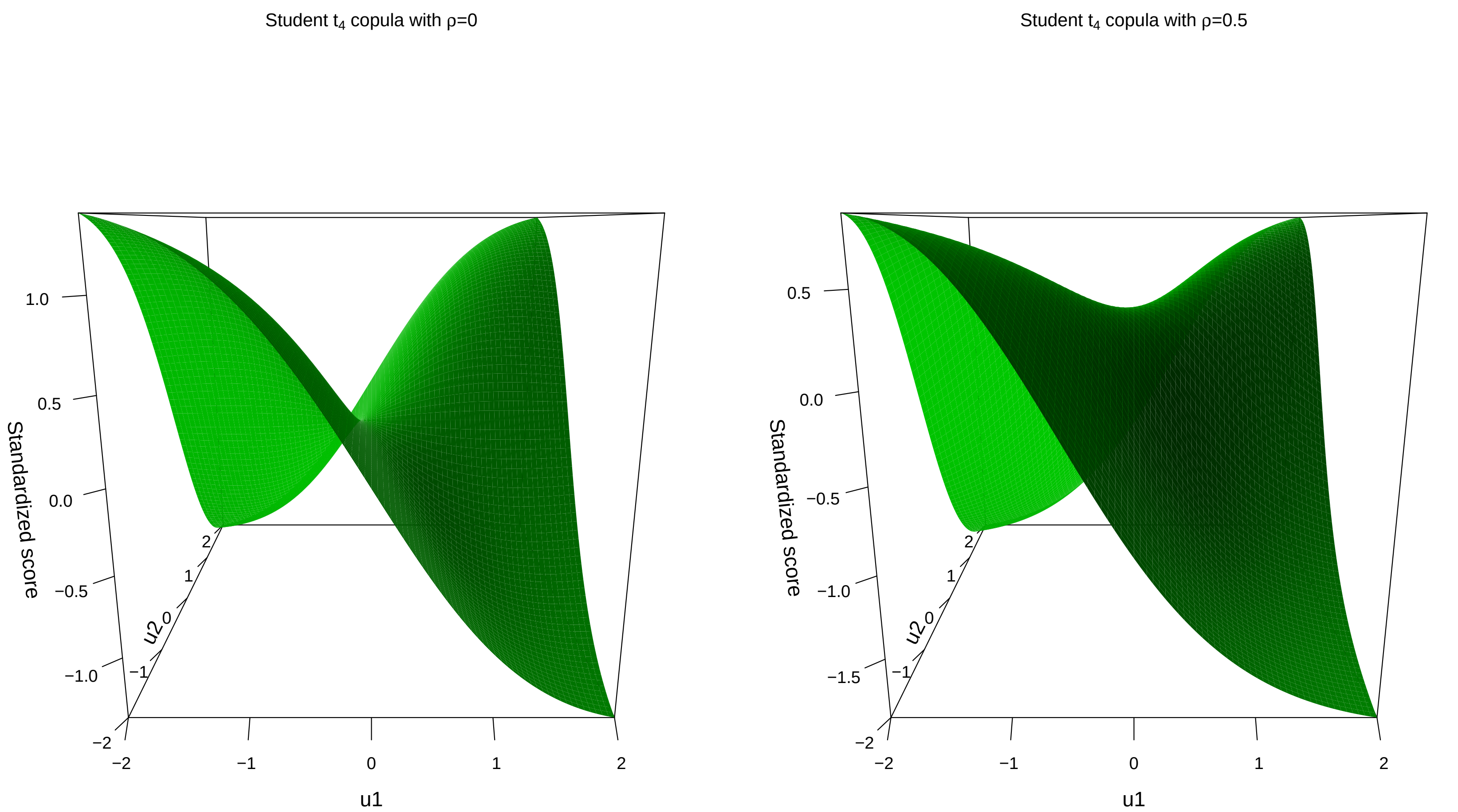
3. The weights applied to the devolatilized returns.

$$w_t^k = \frac{N + {\nu_*}^k}{{\nu_*}^k - 2 + (\tilde{y}_t^k)'(R_t^k)^{-1}(\tilde{y}_t^k)}$$





- The thicker the tails, the more likely it is that abnormally large values of the realized covariance are due to the heavy-tailed feature of the distribution rather than changes in correlation, and therefore the smaller the impact relatively to the Gaussian case.



- Downweighting in function of squared Mahalanobis distance. Correlation coefficient impacts the curvature and which values are considered as extreme.

Model for transition probabilities

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- We assume the states follow a Markov process with the 2×2 dynamic transition matrix $P_{i|t}$. The diagonal elements of this matrix are parameterized using the logit transformation of the time-varying quantities π_{it}^{I} and π_{it}^{II}:

$$
\begin{aligned}
P_{(11)it} &= \exp(\pi_{it}^{I})/[1+\exp(\pi_{it}^{I})]; \\
P_{(22)it} &= \exp(\pi_{it}^{II})/[1+\exp(\pi_{it}^{II})]. \\
\pi_{it}^{I} &= c_i^{I} + d_i^{I} x_{t-1} \\
\pi_{it}^{II} &= c_i^{II} + d_i^{II} x_{t-1},
\end{aligned}
$$

with x_{t-1} the time $t-1$ value of the exogenous variable (VIX, TED spread, Saint Louis Financial Stability Index).

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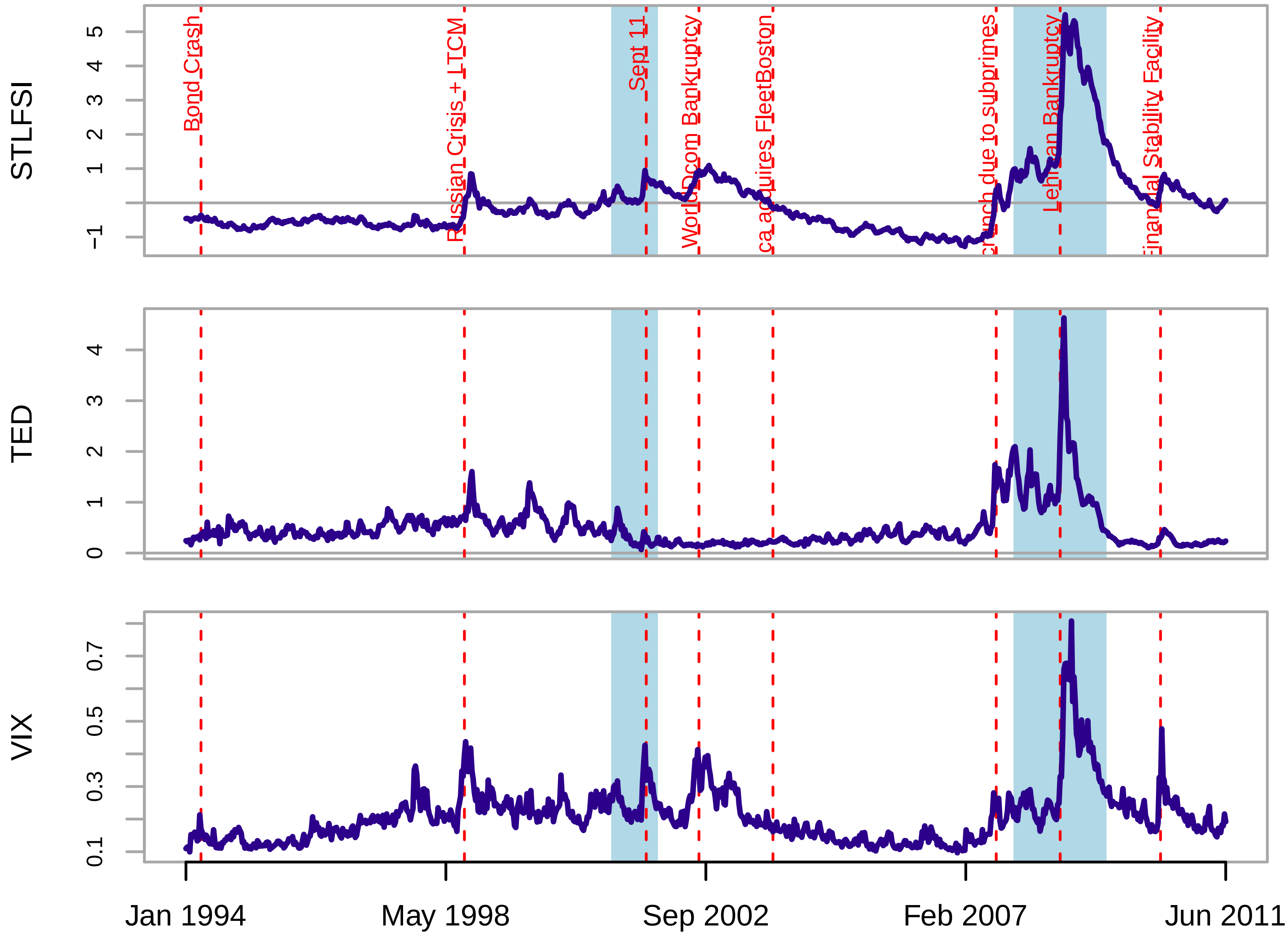
- Used as drivers for changes in the transition probabilities between risk regimes;
  - ✓ Implied Volatility: VIX
  - ✓ Credit risk: TED spread (3-month LIBOR - T-bill)
  - ✓ Saint Louis Financial Stability Index (STLFSI) is defined as the first principal component of eighteen major financial time series capturing some aspect of financial stress (7 interest rates, 6 yield spreads, VIX, Merrill Lynch Bond Market Volatility Index,...).

- Time series of weekly values of the Saint-Louis Financial Stability Index, the TED spread and the VIX.



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- Inference on regime probabilities through the standard Hamilton filter;
- Two–step maximum likelihood (copula assumption);
  - ✓ marginal and copula LLH are tractable, efficient implementation in c++ (Rcpp; Eddelbuettel and François);
  - ✓ still complex, because of multiple local optima (DEoptim to obtain good starting values; Ardia, Mullen, Peterson, Ulrich).

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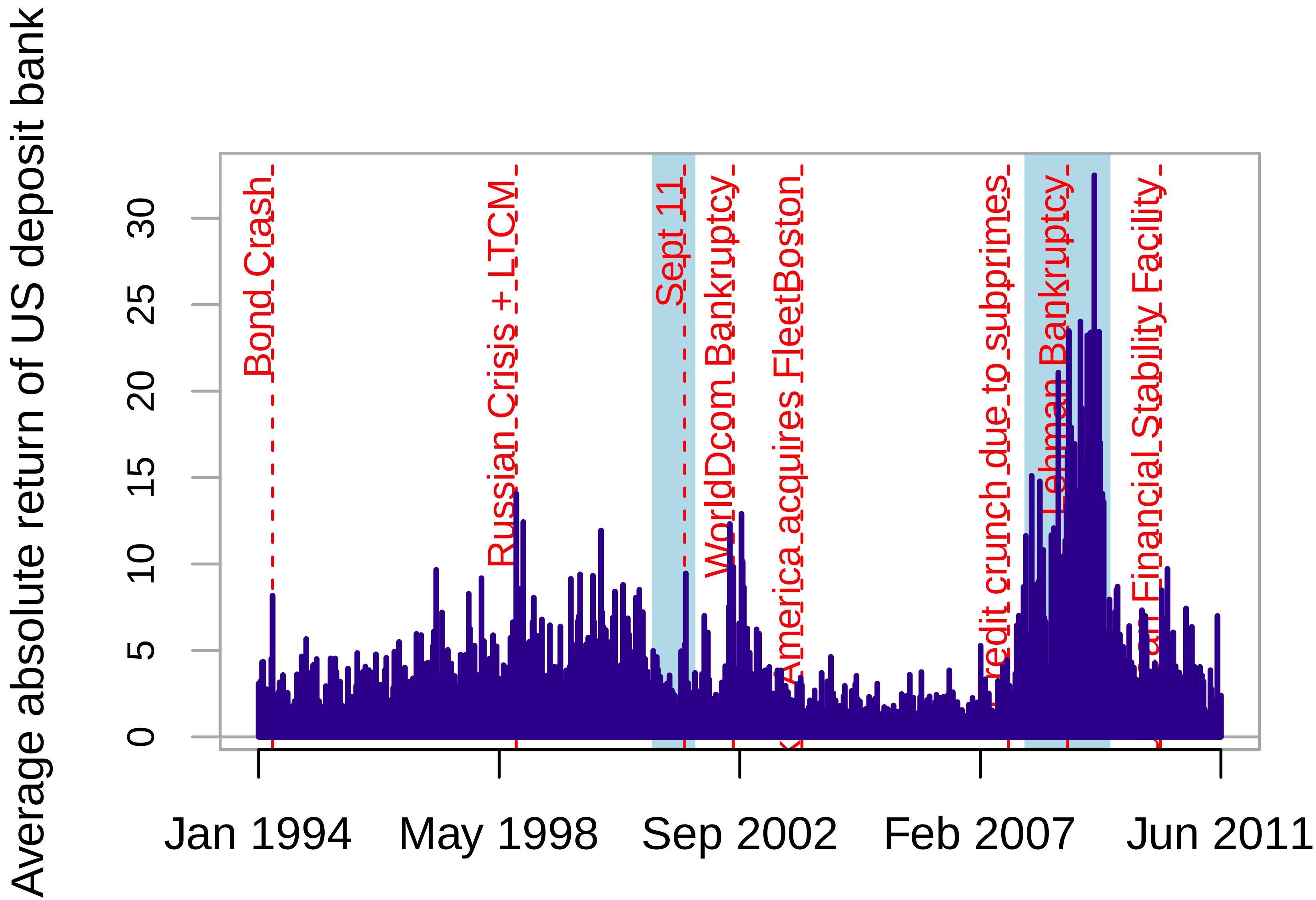
Top 15 largest US deposit banks	First	End
Bank of New York Mellon Corp	2008	2011
Bankamerica Corp	1994	1998
Bank One Corp	1994	2011
Barnett Banks Inc	1994	1997
Capital One Financial Corp	2006	2011
Chemical Banking Corp, Chase Manhattan Corp, JP Morgan Chase & Co	1994	2011
Citicorp	1994	1998
Citigroup	1999	2011
Fifth Third Bancorp	2001	2011
First Union Corp, Wachovia Corp	1994	2008
Fleet Financial Group Inc, Fleet Boston Corp, Fleetboston Financial Corp	1994	2003
Keycorp	1994	2011
Morgan Stanley	2009	2011
National City Corp	1996	2008
Nationsbank Corp, Bankamerica Corp, Bank of America Corp	1994	2011
Norwest Corp	1994	1998
PNC Bank Corp, PNC Financial Services GRP Inc	1994	2011
Regions Financial Corp	2005	2011
Southern National Corp NC, BB&T Corp	2000	2011
Suntrust Banks Inc	1994	2011
US Bancorp	1998	2011
Wells Fargo & Co	1994	2011

- Time series of 1994–2011 weekly values of the mean absolute returns across US deposit bank holding companies.



- 28 model specifications are estimatated for each US deposit bank.
- We first study the average standardized BIC of each volatility model.

R1	R2	BIC	BIC-STLFSI	BIC-TED	BIC-VIX
t-constant		0.965			
g-GAS		0.933			
t-gas		**0.923**			
t-GARCH		0.931			
t-constant	t-constant	0.947	0.893	0.887	<u>0.832</u>
t-gas	t-constant	0.941	0.859	0.83	<u>0.8</u>
t-garch	t-constant	0.94	0.855	0.83	<u>0.818</u>
g-gas	g-gas	0.935	0.885	0.868	<u>0.854</u>
t-gas	t-gas	0.935	**0.852**	**0.825**	<u>0.789</u>
t-garch	t-garch	0.945	0.855	0.828	<u>**0.788**</u>

Distribution of results across banks:

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- Of all models considered, the lowest BIC is always achieved by a double regime volatility model, with time-varying transition probabilities.
- The STLFSI, TED spread and VIX are selected 6, 6, and 10 times respectively.
- For shorter return series, at least one of the regimes tends to be characterized by constant volatility.
- The t-garch model is selected for 8 banks, the t-gas/DySco model for 10 banks.

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Q2: Dynamics in the t-copula?

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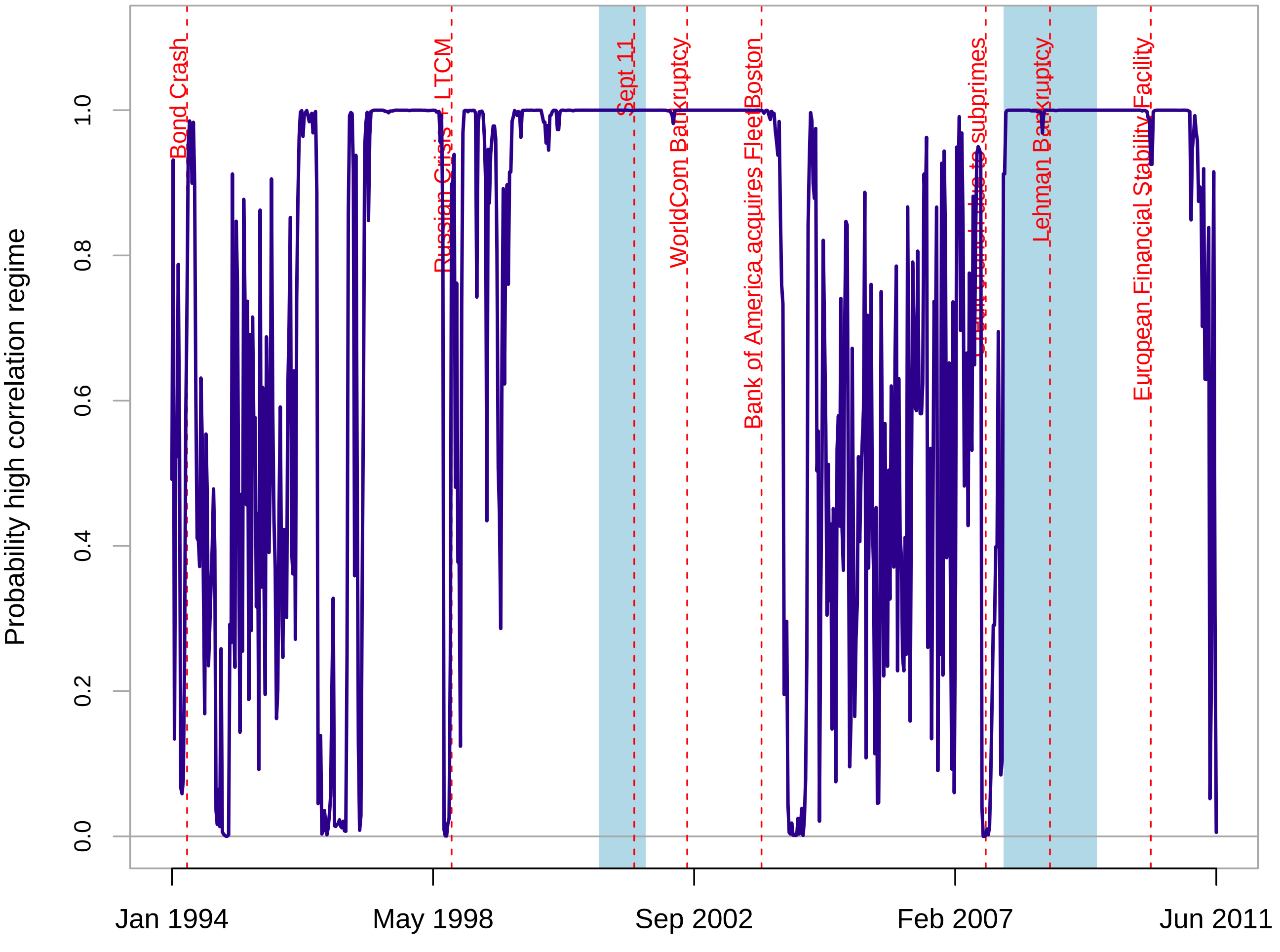
- There does not seem to be much to gain from modeling the within–regime dynamics, which is in support of the regime switching constant correlation model of Pelletier (2006), but with time-varying transition probabilities;
- Best model is a 2-regime model with correlations around 0.42 and 0.75, with time-variation driven by the VIX.

- Time series of predicted probabilities to be in the high correlation regime.



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- Recent literature on exogenous and endogenous risk regimes implies *potential* usefulness of regime switching models;
- We study this question for the volatility and correlation regimes in weekly returns of financial institutions;
- For this, a regime switching volatility–correlation model is proposed;
- Key feature: within–regime dynamics are driven by the score; across–regime dynamics by macroeconomic financial time series;
- Main finding: Strong evidence of regime switches in volatility and correlation, when using time-varying transition probabilities (especially VIX).

References

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