Financial Portfolio Optimization with (O)R
Operations Research & R

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Optimization Modeling & R

Facts about OR/Optimization and R:

- Statisticians love to build optimization models matrix-wise.
- CRAN (Optimization Task View) is focussed on solvers.
- Optimization under Uncertainty is not covered.
- OR people use MatLab/Julia, Python, and/or C++.

Strategy: Simplify and do it - and implement as R packages.

Issue: It is impossible to design a *one fits all* product/package!

Solution: Simple, extensible, lightweight, and *lovable* optimization *modeling* framework.

Optimization modeling gap - Academic Optimization

- Everything is done for one (or two) research paper(s).
- Focus on one specific solution method or one specific solver.
  - The model is created in a solver-readable matrix format.
  - Data and model is mixed (messed) up.
  - Need mathematical proofs to impress non-scientists.
- PhD students are forced to do the implementation.
- If a deadline is missed: you send the project to the next conference or special issue of a journal.

Optimization modeling gap - Real-World

- No one cares about how it is solved, i.e.
  - dirty heuristics are ok, no proofs necessary.
- Data and model should be separated.
  - Different groups and persons are working on it.
- Simplifications are crucial
  - to communicate and maintain the model, and to
  - implement optimization results into your business process.
- If a deadline is missed: you lose money, clients, reputation, your wife & life, and so on...

...the gap is obvious.

Common Issue

Optimization is a side-product *("someone has to do it")*.

Learning from OR - Deterministic Optimization

Reconsider the basic optimization problem:

$$\begin{array}{ll} \text{minimize}_x & f(x, P) \\ \text{subject to} & x \in \mathcal{X}(P) \end{array}$$

- $f(\cdot)$ is a cost function,
- x are decision variables (to be computed), and
- P are (precisely known) parameters.
- $\mathcal{X}$ denotes a set of constraints.

The real-world is uncertain (unfortunately), i.e. $\tilde{P}$!

Why not use $P = \mathbb{E}(\tilde{P})$?

Learning from OR - Deterministic Optimization

Reconsider the basic optimization problem:

$$\begin{array}{ll} \text{maximize } x & f(x, P) \\ \text{subject to} & x \in \mathcal{X}(P) \end{array}$$

- $f(\cdot)$ is a profit function,
- x are decision variables (to be computed), and
- P are (precisely known) parameters.
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The real-world is uncertain (unfortunately), i.e. $\tilde{P}$!

Why not use $P = \mathbb{E}(\tilde{P})$?

Stochastic Optimization

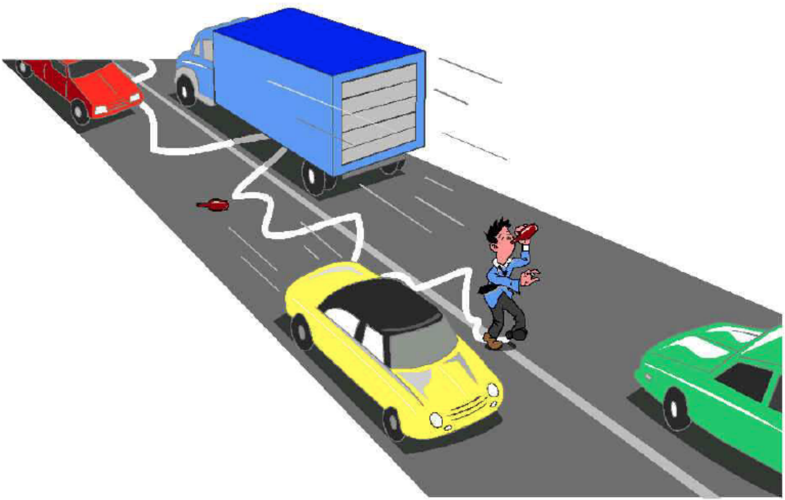
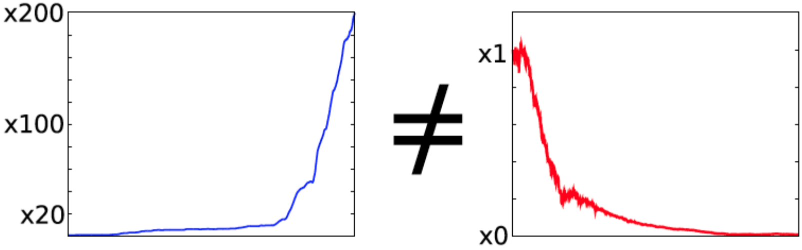


Image Source: Sam L. Savage, Stanford University

$\text{alive}\left(\mathbb{E}\left[\text{position}\right]\right) = \text{true},$ but $\mathbb{E}\left[\text{alive}\left(\text{position}\right)\right] = \text{false}!$

Stochastic Portfolio Optimization

Numerical example [Wiesemann 2010] - simple wealth maximization model with 5 indices (ATX, CAC, DAX, FTSE, SMI):



wealth $(\mathbb{E}\,[\text{stock returns}])$ vs. $\mathbb{E}\,[\text{wealth}\,(\text{stock returns})]$

Modern Portfolio Theory

Calculate an optimal portfolio x given a assets given a vector of expected returns M and a co-variance matrix $\mathbb{C}$ subject to further constraints $\mathcal{X}$, i.e. the well-known Markowitz approach:

$$\begin{array}{ll} \text{minimize } x & x \, \mathbb{C} \, x^T \\ \text{subject to} & x \times M \geq \mu \\ & x \in \mathcal{X}. \end{array}$$

Issues with this approach:

- Uncertainty is just implicitly modeled $\rightarrow$ deterministic!
- QP framework too rigid and specific (OR perspective).
- General extensions (CVaR, ...) are put on top of this base.

Stochastic Programming - Integrating Uncertainty

Stochastic Programming naturally separates the *objective* and *subjective* part of a decision problem.

1. **Optimization model** specifies the event space at each decision stage to integrate **objective** real-world constraints and dynamics, i.e. the event handling (**quantitative** aspects of the solution).
2. **Uncertainty model** is chosen independently from optimization model to reflect **subjective** beliefs of the decision taker (**qualitative** aspects of the solution).

Stochastic Portfolio Optimization

Objective and subjective parts within portfolio optimization:

- $\mathcal{X}$ - set of regulatory & organizational constraints.
- S - asset return uncertainty model.

Flexible to integrate any risk measure (VaR, Omega, ...):

- ℓ_x - loss distribution for some portfolio x, i.e. $\ell_x = \langle x, S \rangle$.

(Bi-criteria) optimization meta-model:

$$\begin{array}{ll} \text{maximize}\ x & \mathit{Return}(\ell_x) \\ \text{minimize}\ x & \mathit{Risk}(\ell_x) \\ \text{subject to} & x \in \mathcal{X} \end{array}$$

Optimization in R - matrix-based (modopt)

```
# maximize: 2*x1 + x2;
# subject to: x1+x2 <= 5;
# subject to: x1 <= 3;
# x1 >= 0, x2 >= 0

f <- c(2, 1)
A <- matrix(c(1, 1, 1, 0), nrow=2, byrow=TRUE)
b <- c(5, 3)

solution <- linprog(-f, A, b)
print(solution$x)
```

Optimization in R - matrix-based (modopt)

```
# Markowitz minimization

H <- covariance
f <- rep(0, assets)
Aeq <- rep(1, assets)
beq <- 1
lb <- rep(0, assets)
ub <- rep(1, assets)

solution <- quadprog(H, f, NULL, NULL,
    Aeq, beq, lb, ub)
portfolio <- solution$x
print(portfolio)
```

Optimization Modeling in R

The ingredients of every optimization problem are:

- Parameters: Input.
- Variables: Output.
- Objective function(s): Minimization, Maximization.
- Constraints.

Provide functions for exactly these (and only these) ingredients within R.

Optimization Modeling in R (modopt)

```
model()

x1 <- 0; x2 <- 0
variable(x1, lb=0)
variable(x2, lb=0)

maximize("2*x1 + x2")
subject_to("x1+x2 <= 5")
subject_to("x1 <= 3")

optimization()
```

Optimization Modeling in R with `modopt`

Light-weight: Just a handful of functions to add objective function(s), constraints, parameters, to initialize (optimization) variables, and to define (optimization variable) sets.

- `model()` to create a new optimization model.
- `optimization()` to execute the current model.

The design of the package is heavily influenced by the standalone optimization modeling languages AMPL and ZIMPL as well as CVX and YalMip for MatLab.

Optimization modeling in R with `modopt`

Advantages of this approach:

- Readable and teachable optimization models.
  - Model libraries (OR people love these).
- Clear separation of data, model, and optimization.
- Domain-specific modeling shortcuts.
  - Simplified portfolio optimization modeling.
- Modeling simplifications for special optimization purposes.
  - Multi-stage Stochastic Programming.

Simplified portfolio optimization modeling

Strategy: Simplify and do it (and extend after simplification).

`modopt.portfolio` - minimum additional input:

- Preferred: Discrete representation of return scenarios.
- Alternative: Markowitz-style mean vector M and covariance matrix $\mathbb{C}$.

Convert each input into the other (e.g. in case of $M/\mathbb{C}$ sample scenarios) to allow for any optimization remodeling.

Simplified portfolio optimization modeling

Loss distribution-based risk measures (OR)

```
sd(loss)
mad(loss)
cvar(loss, alpha)
var(loss, alpha)
ginimd(loss)
minimax-young(loss)
omega(loss, threshold)
avgdd(loss)
...
```

Mean/Covariance-based risk measures

```
markowitz(loss)
...
```

Stochastic Portfolio Optimization:

Basic Markowitz in `modopt.portfolio`:

```
minimize("markowitz(loss)")
minimize("subject to mean(loss) >= 0.02")
```

CVaR minimization with Markowitz constraint:

```
minimize("minimize cvar(loss, 0.05)")
subject_to("sd(loss) <= 0.02")
subject_to("mean(loss) >= 0.01")
```

Conversion example:

```
minimize minimax-young(loss)
subject to mean(loss) >= 0.02
```

Stochastic Portfolio Optimization - Conversion Example

```
set("Asset", 1:dim(scenario_set)[2])
set("Scenario", 1:dim(scenario_set)[1])
parameter(scenario_set, "Asset, Scenario")
parameter(scenario_prob, "Scenario")

variable(loss, "Scenario")
variable(portfolio, "Asset")

subject_to("sum('portfolio[Asset] *
    scenario[Asset, Scenario]', 'Asset')
    == loss[Scenario]", "Scenario")
```

Simplified portfolio optimization modeling

Specific additions of `modopt.portfolio.minimax-young` are

```
variable(gamma)
variable(scenario_downside, "Scenario", lb=0)

subject_to("scenario_downside[Scenario] - gamma +
  loss[Scenario] >= 0", "Scenario")

maximize("gamma")
```

`optimization()` will calculate the optimal portfolio but also provide the optimization model within R for further modification.

Simplified portfolio optimization modeling

Other objective functions & constraints:

```
track(sp500)
cardinality(10)
...
```

Once a clear idea of how to reformulate a new objective and/or constraints, one may add this converter to `modopt.portfolio`.

Multi-stage decision optimization under uncertainty

Design goal: Modeling language independent of

- optimization modeling approach:
  - Expectation-based convex multi-stage stochastic programming,
  - Worst-case optimization,
  - ...
- underlying solution technique:
  - Tree-based deterministic equivalent formulation.
  - Primal/dual linear decision rules, upper/lower bounds.
  - ...

A simple multi-stage model

Stylized multi-stage stochastic programming example from [Heitsch el al. 2006]:

$$
\begin{array}{lll}
\text{minimize} & \mathbb{E}\left(\sum_{t=1}^{T} V_t x_t \right) & \\
\text{subject to} & s_t - s_{t-1} = x_t & \forall t = 2, \ldots, T \\
& s_1 = 0, s_T = a, & \\
& x_t \geq 0, s_t \geq 0. &
\end{array}
$$

A simple multi-stage model in R (modopt.multistage)

```
parameter(a)
variable(x, lb=0); variable(s, lb=0)

maximize(id="objective", "E(x)")
subject_to(id="non_anticitpativity", "s - s(-1) = x")
subject_to(id="root_stage", "s = 0")
subject_to(id="terminal_stage", "s = a")

deterministic("T", a)
stochastic("0..T", x, s, "objective")
stochastic("1..T", "non_anticipativity")
stochastic("0", "root_stage")
stochastic("T", "terminal_stage")
```

Contact

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