FX Pricing: Regulatory Requirements & The Challenge of Ultimate Drill-down

Thomas P. Harte

Morgan Stanley[1]
New York, NY

R in Finance 2013

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SR Letter 11-7
Attachment

Board of Governors of the Federal Reserve System
Office of the Comptroller of the Currency

April 4, 2011

SUPERVISORY GUIDANCE ON
MODEL RISK MANAGEMENT

CONTENTS

I. INTRODUCTION

Banks rely heavily on quantitative analysis and models in most aspects of financial decision making.[1] They routinely use models for a broad range of activities, including underwriting credits; valuing exposures, instruments, and positions; measuring risk; managing and safeguarding client assets; determining capital and reserve adequacy; and many other activities. In recent years, banks have applied models to more complex products and with more ambitious scope, such as enterprise-wide risk measurement, while the markets in which they are used have also broadened and changed. Changes in regulation have spurred some of the recent developments, particularly the U.S. regulatory capital rules for market, credit, and operational risk based on the framework developed by the Basel Committee on Banking Supervision. Even apart from these regulatory considerations, however, banks have been increasing the use of data-driven, quantitative decision-making tools for a number of years.

The expanding use of models in all aspects of banking reflects the extent to which models can improve business decisions, but models also come with costs. There is the direct cost of devoting resources to develop and implement models properly. There are also the potential indirect costs of relying on models, such as the possible adverse consequences (including financial loss) of decisions based on models that are incorrect or misused. Those consequences should be addressed by active management of model risk.

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Business | Financial Markets

posted: 2 months ago

JPMorgan Tells SEC New VaR Model Didn't Require Prior Disclosure As Whale Impact Fades

JPMorgan Chase, facing criticism that it misled investors about a change to a risk model as trades backfired last year, told U.S. regulators that the bank wasn't obligated to disclose the move until May.

Bloomberg reports that while there was an 'interim change' to the lender's so-called value-at-risk model during the first three months of 2012, that adjustment had been reversed by the time the company filed its quarterly report in May, then-Chief Financial Officer Douglas Braunstein told the Securities and Exchange Commission in a December 3rd letter that was released Wednesday.

'As a result, the firm believes there was no model change within the meaning of' securities-disclosure laws, he wrote.

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- **Let's see what "document" means for something that's simple**...

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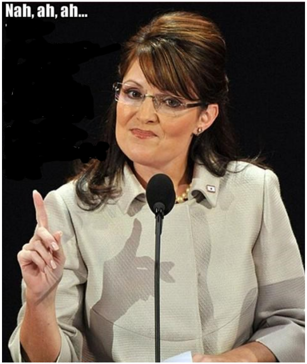

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- Do all desks price the same way? Why not?

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- That's a pretty tall order

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$$\boxed{\text{USDCCY} = \frac{\text{CCY}}{\text{USD}} = \frac{\text{Units of CCY}}{1 \times \text{Unit of USD}}} \tag{1}$$

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2. Invest an equivalent amount of the non-domestic currency CCY today in the foreign country at the foreign interest rate r_f for time T:

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   Using Equation2, i.e. we take valuation and/or P&L in USD—hence “domestic”:

$$\underbrace{S_0}_{\text{CCYUSD}}, \text{ where CCYUSD} = \frac{\text{USD}}{\text{CCY}} = \frac{\text{Units of USD}}{1 \times \text{Unit of CCY}}$$

3. Strike a forward contract to exchange a notional amount of CCY $1/S_0 \times e^{r_f T}$ for USD $F_{0,T}/S_0 \times e^{r_f T}$ at T:

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  What you receive at T: $\uparrow N^{\text{C}}$ (in units of CCY)
  What you pay at T: $\downarrow N^{\text{C}} \times K$ (in units of USD)
- Let's say you unwind the position immediately at maturity, the net cash flow in USD is:

$$C^{\$,\text{Long}}_{T,\text{Realized}} = N^{\text{C}} \times \underbrace{(S_T - K)}_{\text{CCYUSD}} \tag{11}$$

- Let's say you were short the same position
  What you receive at T: $\uparrow N^{\text{C}} \times K$ (in units of USD)
  What you pay at T: $\downarrow N^{\text{C}}$ (in units of CCY)

$$C^{\$,\text{Short}}_{T,\text{Realized}} = N^{\text{C}} \times \underbrace{(K - S_T)}_{\text{CCYUSD}} \tag{12}$$

Convention for Longs & Shorts

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- It should be clear that

$$C^{\$,\textsc{Long}}_{T,\textsc{Realized}} = -C^{\$,\textsc{Short}}_{T,\textsc{Realized}} \tag{13}$$

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(16)

Convention for Longs & Shorts

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$$C^{\$,\text{LONG}}_{T,\text{REALIZED}} = -C^{\$,\text{SHORT}}_{T,\text{REALIZED}} \tag{13}$$

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$$\tag{16}$$

The cash flow in USD that occurs at maturity T is:

$$\boxed{C^{\$}_{T,\text{REALIZED}} = \pm N^{\mathrm{C}} \times \underbrace{(S_T - K)}_{\text{CCYUSD}}} \tag{17}$$

The Expected Future Spot Price

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- The NPV at t of the expected cash flow at maturity T is simply this cash flow discounted to present value by the domestic risk-free rate for tenor $T - t$, viz. $r_{d,t,T-t}$
  This is the fair value $V_{t,T-t}$ intrinsic to the contract

The Fair Value of a FX Forward Contract

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&= \pm N^{\text{C}} \times e^{-r_d(T-t)} \times \underbrace{\left(F_{t,T-t} - K\right)}_{\text{CCYUSD}}
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What if we wished to value the same contract in the foreign currency?

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What if we wished to value the same contract in the foreign currency? Repeating the argument <u>mutatis mutandis</u> we arrive at:

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$$V^{C}_{t,T-t} = \mp N^{\$} \times e^{-r_f(T-t)} \times \underbrace{(F_{t,T-t} - K)}_{\text{USDCCY}} \tag{20}$$

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But is it?

No Free Lunch

No Free Lunch

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&= \mp N^{\mathrm{C}} \times e^{-r_f(T-t)} \times K\left(\frac{1}{S_t e^{(r_d - r_f)(T-t)}} - \frac{1}{K}\right)
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&= \mp N^{\mathrm{C}} \times e^{-r_f(T-t)} \times K\left(\frac{K - S_t e^{(r_d-r_f)(T-t)}}{K S_t e^{(r_d-r_f)(T-t)}}\right)
\end{aligned}
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Phew!

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Phew!

If this were not the case you could make money by parking your cash in a foreign bank

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Phew!

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Perhaps in Cyprus?

Expressions for the Fair Value (CCYUSD)

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- $S_t, F_{t,T-t}, K$ quoted as CCYUSD

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&= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (S_t + f_{t,T-t} - K) \quad \text{from Equation (8)} && (24)
\end{aligned}
$$

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$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (\mathbb{E}_t\left[S_T\right] - K) \quad \text{from Equation (??)} \tag{22}$$

$$= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (F_{t,T-t} - K) \quad \text{from Equation (19)} \tag{23}$$

$$= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (S_t + f_{t,T-t} - K) \quad \text{from Equation (8)} \tag{24}$$

$$= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (S_t e^{(r_d - r_f)(T-t)} - K) \quad \text{from Equation (5)} \tag{25}$$

Expressions for the Fair Value (CCYUSD)

- $S_t, F_{t,T-t}, K$ quoted as CCYUSD

$$
\begin{aligned}
V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) &= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (\mathbb{E}_t\left[S_T\right] - K) \quad \text{from Equation (\textbf{??})} && (22)\\
&= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (F_{t,T-t} - K) \quad \text{from Equation (19)} && (23)\\
&= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (S_t + f_{t,T-t} - K) \quad \text{from Equation (8)} && (24)\\
&= \pm N^{\mathrm{C}} \times e^{-r_d(T-t)} \times (S_t e^{(r_d - r_f)(T-t)} - K) \quad \text{from Equation (5)} && (25)\\
&= \pm N^{\mathrm{C}} \times \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right) && (26)\\
&\qquad \text{by rearranging terms in Equation (25)}
\end{aligned}
$$

- The dependencies are expressed most explicitly as:

Expressions for the Fair Value (CCYUSD)

- $S_t, F_{t,T-t}, K$ quoted as CCYUSD

$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{C} \times e^{-r_d(T-t)} \times (\mathbb{E}_t\left[S_T\right] - K) \quad \text{from Equation (??)} \tag{22}$$

$$= \pm N^{C} \times e^{-r_d(T-t)} \times (F_{t,T-t} - K) \quad \text{from Equation (19)} \tag{23}$$

$$= \pm N^{C} \times e^{-r_d(T-t)} \times (S_t + f_{t,T-t} - K) \quad \text{from Equation (8)} \tag{24}$$

$$= \pm N^{C} \times e^{-r_d(T-t)} \times (S_t e^{(r_d - r_f)(T-t)} - K) \quad \text{from Equation (5)} \tag{25}$$

$$= \pm N^{C} \times \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right) \tag{26}$$

by rearranging terms in Equation (25)

- The dependencies are expressed most explicitly as:

$$\boxed{V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{C} \times \Big(\underbrace{S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}}_{\text{CCYUSD}}\Big)} \tag{27}$$

Sensitivity to FX spot (CCYUSD)

Sensitivity to FX spot (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial S_t}\Delta S \\
&= \frac{\partial}{\partial S_t}\left(\pm N^{\mathrm{C}} \times \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right)\right)\Delta S \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S \qquad (28)
\end{aligned}
$$

Sensitivity to FX spot (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial S_t}\Delta S \\
&= \frac{\partial}{\partial S_t}\left(\pm N^{\mathrm{C}} \times \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right)\right)\Delta S \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S
\end{aligned} \tag{28}
$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

Sensitivity to FX spot (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_{t}}{\partial S_t}\Delta S \\
&= \frac{\partial}{\partial S_t}\Big(\pm N^{\mathrm{C}} \times \big(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\big)\Big)\Delta S \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S
\end{aligned} \tag{28}
$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$
\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t) \\
&= \pm N^{\mathrm{C}} \times \Big(\big((S_t + \Delta S)e^{-r_f(T-t)} - K e^{-r_d(T-t)}\big) - \big(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\big)\Big) \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S
\end{aligned} \tag{29}
$$

Sensitivity to FX spot (CCYUSD)

$$\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial S_t}\Delta S \\
&= \frac{\partial}{\partial S_t}\left(\pm N^{\mathrm{C}} \times \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right)\right)\Delta S \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S \qquad (28)
\end{aligned}$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t) \\
&= \pm N^{\mathrm{C}} \times \left(\left((S_t + \Delta S)e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right) - \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right)\right) \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S \qquad (29)
\end{aligned}$$

where $V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t)$ means perturb only S_t in $V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f)$

Sensitivity to FX spot (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial S_t}\Delta S \\
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Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$
\begin{aligned}
\Delta V^{\$}_{t,\textsc{Spot}} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t) \\
&= \pm N^{\mathrm{C}} \times \left(\left((S_t + \Delta S)e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right) - \left(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\right)\right) \\
&= \pm N^{\mathrm{C}} \times e^{-r_f(T-t)}\Delta S
\end{aligned} \tag{29}
$$

where $V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t)$ means perturb only S_t in $V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f)$

From both approaches we have:

Sensitivity to FX spot (CCYUSD)

$$\begin{aligned}
\Delta V^{\$}_{t,\text{SPOT}} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial S_t}\Delta S \\
&= \frac{\partial}{\partial S_t}\left(\pm N^{\text{C}} \times \left(S_t e^{-r_f(T-t)} - Ke^{-r_d(T-t)}\right)\right)\Delta S \\
&= \pm N^{\text{C}} \times e^{-r_f(T-t)}\Delta S
\end{aligned} \tag{28}$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$\begin{aligned}
\Delta V^{\$}_{t,\text{SPOT}} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t) \\
&= \pm N^{\text{C}} \times \left(\left((S_t + \Delta S)e^{-r_f(T-t)} - Ke^{-r_d(T-t)}\right) - \left(S_t e^{-r_f(T-t)} - Ke^{-r_d(T-t)}\right)\right) \\
&= \pm N^{\text{C}} \times e^{-r_f(T-t)}\Delta S
\end{aligned} \tag{29}$$

where $V^{\$}_{t,T-t}(S_t + \Delta S) - V^{\$}_{t,T-t}(S_t)$ means perturb only S_t in $V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f)$

From both approaches we have:

$$\boxed{\Delta V^{\$}_{t,\text{SPOT}} = \pm N^{\text{C}} \times e^{-r_f(T-t)}\Delta S} \tag{30}$$

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,r_d} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_d}\Delta r \\
&= \frac{\partial}{\partial r_d}\Big(\pm N^{\mathrm{C}} \times \big(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\big)\Big)\Delta r \\
&= \pm N^{\mathrm{C}} \times K(T-t)e^{-r_d(T-t)}\Delta r \\
&\approx \pm N^{\mathrm{C}} \times K(T-t)\big(1 - r_d(T-t)\big)\Delta r \\
&= \pm N^{\mathrm{C}} \times \Big(K(T-t)\Delta r - \underbrace{r_d(T-t)^2\Delta r}_{\approx 0}\Big) \\
&\approx \pm N^{\mathrm{C}} \times K(T-t)\Delta r
\end{aligned}
\tag{31}
$$

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

$$\begin{aligned} \Delta V_{t,r_d}^{\$} &\stackrel{\text{def}}{=} \frac{\partial V_t^{\$}}{\partial r_d} \Delta r \\ &\approx \pm N^{\mathrm{c}} \times K(T-t)\Delta r \end{aligned} \tag{31}$$

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

$$\begin{aligned} \Delta V^{\$}_{t,r_d} &\overset{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_d}\Delta r \\ &\approx \pm N^{\text{c}} \times K(T-t)\Delta r \end{aligned} \tag{31}$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,r_d} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_d} \Delta r \\
&\approx \pm N^{c} \times K(T-t)\Delta r
\end{aligned} \tag{31}
$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

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&\approx \pm N^{c} \times K(T-t)\Delta r
\end{aligned} \tag{31}
$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$
\begin{aligned}
\Delta V^{\$}_{t,r_d} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(r_d + \Delta r) - V^{\$}_{t,T-t}(r_d) \\
&\approx \pm N^{c} \times K(T-t)\Delta r
\end{aligned} \tag{32}
$$

From both approaches we have:

Sensitivity to Domestic Spot Zero-Rate (CCYUSD)

$$\begin{aligned}\Delta V^{\$}_{t,r_d} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_d}\Delta r \\ &\approx \pm N^{\text{c}} \times K(T-t)\Delta r \end{aligned} \quad (31)$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$\begin{aligned}\Delta V^{\$}_{t,r_d} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(r_d + \Delta r) - V^{\$}_{t,T-t}(r_d) \\ &\approx \pm N^{\text{c}} \times K(T-t)\Delta r \end{aligned} \quad (32)$$

From both approaches we have:

$$\boxed{\Delta V^{\$}_{t,r_d} = \pm N^{\text{c}} \times K(T-t)\Delta r} \quad (33)$$

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,r_f} &\overset{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_f}\Delta r \\
&= \frac{\partial}{\partial r_f}\Big(\pm N^{C} \times \big(S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}\big)\Big)\Delta r \\
&= \pm N^{C} \times \Big(-S_t(T-t)e^{-r_f(T-t)}\Delta r\Big) \\
&\approx \pm N^{C} \times \Big(-S_t(T-t)\big(1 - r_f(T-t)\big)\Delta r\Big) \\
&= \pm N^{C} \times \Big(-S_t(T-t)\Delta r - \underbrace{r_f(T-t)^2\Delta r}_{\approx 0}\Big) \\
&\approx \pm N^{C} \times -S_t(T-t)\Delta r
\end{aligned}
$$

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,r_f} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_f}\Delta r \\
&\approx \pm N^{\mathrm{C}} \times -S_t(T-t)\Delta r
\end{aligned}
$$

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

$$\Delta V^{\$}_{t,r_f} \stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_f} \Delta r$$
$$\approx \pm N^{\text{c}} \times -S_t (T - t) \Delta r$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,r_f} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial r_f}\Delta r \\
&\approx \pm N^{\mathrm{c}} \times -S_t(T-t)\Delta r
\end{aligned}
$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

$$\begin{aligned}\Delta V^{\$}_{t,r_f} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_{t}}{\partial r_f}\Delta r \\ &\approx \pm N^{c} \times -S_t(T-t)\Delta r\end{aligned}$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$\begin{aligned}\Delta V^{\$}_{t,r_f} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(r_f+\Delta r) - V^{\$}_{t,T-t}(r_f) \\ &\approx \pm N^{c} \times -S_t\Big(\Delta r(T-t)\Big)\end{aligned}$$

From both approaches we have:

Sensitivity to Foreign Spot Zero-Rate (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,r_f} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_{t}}{\partial r_f}\Delta r \\
&\approx \pm N^{\text{c}} \times -S_t(T-t)\Delta r
\end{aligned}
$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$
\begin{aligned}
\Delta V^{\$}_{t,r_f} &\stackrel{\text{def}}{=} V^{\$}_{t,T-t}(r_f+\Delta r) - V^{\$}_{t,T-t}(r_f) \\
&\approx \pm N^{\text{c}} \times -S_t\Big(\Delta r(T-t)\Big)
\end{aligned}
$$

From both approaches we have:

$$
\boxed{\Delta V^{\$}_{t,r_f} = \pm N^{\text{c}} \times -S_t(T-t)\Delta r} \tag{34}
$$

Sensitivity to Tenor (CCYUSD)

Sensitivity to Tenor (CCYUSD)

$$
\begin{aligned}
\Delta V^{\$}_{t,\tau} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial \tau}\Delta\tau \\
&= \frac{\partial}{\partial \tau}\left(\pm N^{C} \times \left(S_t e^{-r_f \tau} - K e^{-r_d \tau}\right)\right)\Delta\tau \\
&= \pm N^{C} \times \left(-r_f S_t e^{-r_f \tau} + r_d K e^{-r_d \tau}\right)\Delta\tau \\
&\approx \pm N^{C} \times \left(-r_f S_t (1 - r_f \tau) + r_d K (1 - r_d \tau)\right)\Delta\tau \\
&= \pm N^{C} \times \left(-r_f S_t \Delta\tau + \underbrace{r_f^2 S_t \tau \Delta\tau}_{\approx 0} + r_d K \Delta\tau - \underbrace{r_d^2 K \tau \Delta\tau}_{\approx 0}\right) \\
&\approx \pm N^{C} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau\right)
\end{aligned}
$$

Sensitivity to Tenor (CCYUSD)

$$
\begin{aligned}
\Delta V_{t,\tau}^{\$} &\stackrel{\text{def}}{=} \frac{\partial V_t^{\$}}{\partial \tau} \Delta\tau \\
&\approx \pm N^{c} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau \right)
\end{aligned}
$$

Sensitivity to Tenor (CCYUSD)

$$\begin{aligned}
\Delta V^{\$}_{t,\tau} &\stackrel{\text{def}}{=} \frac{\partial V^{\$}_t}{\partial \tau}\Delta\tau \\
&\approx \pm N^{\mathrm{c}} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau\right)
\end{aligned}$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

Sensitivity to Tenor (CCYUSD)

$$
\begin{aligned}
\Delta V_{t,\tau}^{\$} &\stackrel{\text{def}}{=} \frac{\partial V_t^{\$}}{\partial \tau}\Delta\tau \\
&\approx \pm N^{\mathrm{c}} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau\right)
\end{aligned}
$$

Alternatively, we could perturb $V_{t,T-t}^{\$}$ directly:

Sensitivity to Tenor (CCYUSD)

$$\Delta V^{\$}_{t,\tau} \overset{\text{def}}{=} \frac{\partial V^{\$}_{t}}{\partial \tau} \Delta\tau$$

$$\approx \pm N^{c} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau \right)$$

Alternatively, we could perturb $V^{\$}_{t,T-t}$ directly:

$$\Delta V^{\$}_{t,\tau} \overset{\text{def}}{=} V^{\$}_{t,\tau}(\tau + \Delta\tau) - V^{\$}_{t,\tau}(\tau)$$

$$\approx \pm N^{c} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau \right)$$

From both approaches we have:

Sensitivity to Tenor (CCYUSD)

$$\begin{aligned}
\Delta V_{t,\tau}^{\$} &\stackrel{\text{def}}{=} \frac{\partial V_t^{\$}}{\partial \tau}\Delta\tau \\
&\approx \pm N^{c} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau\right)
\end{aligned}$$

Alternatively, we could perturb $V_{t,T-t}^{\$}$ directly:

$$\begin{aligned}
\Delta V_{t,\tau}^{\$} &\stackrel{\text{def}}{=} V_{t,\tau}^{\$}(\tau + \Delta\tau) - V_{t,\tau}^{\$}(\tau) \\
&\approx \pm N^{c} \times \left(-r_f S_t \Delta\tau + r_d K \Delta\tau\right)
\end{aligned}$$

From both approaches we have:

$$\boxed{\Delta V_{t,\tau}^{\$} = \pm N^{c} \times \left(-r_f S_t + r_d K\right)\Delta\tau} \tag{35}$$

The FX Forward Contract (CCYUSD)

The FX Forward Contract (CCYUSD)

$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{\mathrm{C}} \times \Big(\underbrace{S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}}_{\text{CCYUSD}}\Big)$$

The FX Forward Contract (CCYUSD)

$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{\mathrm{C}} \times \Bigl(\underbrace{S_t e^{-r_f(T-t)} - Ke^{-r_d(T-t)}}_{\text{CCYUSD}}\Bigr)$$

$$\Delta V^{\$}_{t,\text{Spot}} = \pm N^{\mathrm{C}} \times e^{-r_f(T-t)} \Delta S$$

The FX Forward Contract (CCYUSD)

$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{\text{C}} \times \Big(\underbrace{S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}}_{\text{CCYUSD}}\Big)$$

$$\Delta V^{\$}_{t,\text{Spot}} = \pm N^{\text{C}} \times e^{-r_f(T-t)} \Delta S$$

$$\Delta V^{\$}_{t,r_d} = \pm N^{\text{C}} \times K(T-t)\Delta r$$

The FX Forward Contract (CCYUSD)

$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{C} \times \Big(\underbrace{S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}}_{\text{CCYUSD}} \Big)$$

$$\Delta V^{\$}_{t,\text{Spot}} = \pm N^{C} \times e^{-r_f(T-t)} \Delta S$$

$$\Delta V^{\$}_{t,r_d} = \pm N^{C} \times K(T-t)\Delta r$$

$$\Delta V^{\$}_{t,r_f} = \pm N^{C} \times -S_t(T-t)\Delta r$$

The FX Forward Contract (CCYUSD)

$$V^{\$}_{t,T-t} \Leftrightarrow V^{\$}(T-t, S_t, K, r_d, r_f) = \pm N^{\text{C}} \times \Big(\underbrace{S_t e^{-r_f(T-t)} - K e^{-r_d(T-t)}}_{\text{CCYUSD}}\Big)$$

$$\Delta V^{\$}_{t,\text{Spot}} = \pm N^{\text{C}} \times e^{-r_f(T-t)} \Delta S$$

$$\Delta V^{\$}_{t,r_d} = \pm N^{\text{C}} \times K(T-t)\Delta r$$

$$\Delta V^{\$}_{t,r_f} = \pm N^{\text{C}} \times -S_t(T-t)\Delta r$$

$$\Delta V^{\$}_{t,\tau} = \pm N^{\text{C}} \times \Big(-r_f S_t + r_d K\Big)\Delta\tau$$

The FX Forward Contract (USDCCY)

The FX Forward Contract (USDCCY)

$$V^C_{t,T-t} \Leftrightarrow V^C(T-t, S_t, K, r_d, r_f) = \mp N^{\$} \times \Big(\underbrace{S_t e^{-r_d(T-t)} - K e^{-r_f(T-t)}}_{\text{USDCCY}}\Big)$$

The FX Forward Contract (USDCCY)

$$V^{C}_{t,T-t} \Leftrightarrow V^{C}(T-t, S_t, K, r_d, r_f) = \mp N^{\$} \times \Big(\underbrace{S_t e^{-r_d(T-t)} - K e^{-r_f(T-t)}}_{\text{USDCCY}} \Big)$$

$$\Delta V^{C}_{t,\textsc{Spot}} = \mp N^{\$} \times e^{-r_d(T-t)} \Delta S$$

The FX Forward Contract (USDCCY)

$$V^C_{t,T-t} \Leftrightarrow V^C(T-t, S_t, K, r_d, r_f) = \mp N^{\$} \times \Big(\underbrace{S_t e^{-r_d(T-t)} - K e^{-r_f(T-t)}}_{\text{USDCCY}}\Big)$$

$$\Delta V^C_{t,\textsc{Spot}} = \mp N^{\$} \times e^{-r_d(T-t)} \Delta S$$

$$\Delta V^C_{t,r_d} = \mp N^{\$} \times -S_t(T-t)\Delta r$$

The FX Forward Contract (USDCCY)

$$V^C_{t,T-t} \Leftrightarrow V^C(T-t, S_t, K, r_d, r_f) = \mp N^{\$} \times \Big(\underbrace{S_t e^{-r_d(T-t)} - K e^{-r_f(T-t)}}_{\text{USDCCY}}\Big)$$

$$\Delta V^C_{t,\textsc{Spot}} = \mp N^{\$} \times e^{-r_d(T-t)} \Delta S$$

$$\Delta V^C_{t,r_d} = \mp N^{\$} \times -S_t(T-t)\Delta r$$

$$\Delta V^C_{t,r_f} = \mp N^{\$} \times K(T-t)\Delta r$$

The FX Forward Contract (USDCCY)

$$V_{t,T-t}^{C} \Leftrightarrow V^{C}(T-t, S_t, K, r_d, r_f) = \mp N^{\$} \times \Big(\underbrace{S_t e^{-r_d(T-t)} - K e^{-r_f(T-t)}}_{\text{USDCCY}}\Big)$$

$$\Delta V_{t,\text{Spot}}^{C} = \mp N^{\$} \times e^{-r_d(T-t)} \Delta S$$

$$\Delta V_{t,r_d}^{C} = \mp N^{\$} \times -S_t(T-t)\Delta r$$

$$\Delta V_{t,r_f}^{C} = \mp N^{\$} \times K(T-t)\Delta r$$

$$\Delta V_{t,\tau}^{C} = \mp N^{\$} \times \Big(-r_d S_t + r_f K\Big)\Delta \tau$$

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- We're done!

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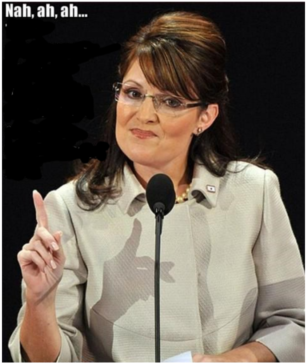
- **Nah, ah, ah!**



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- Does any of this stuff actually work?
- That's a pretty tall order

Perturbing Spot: Directionality

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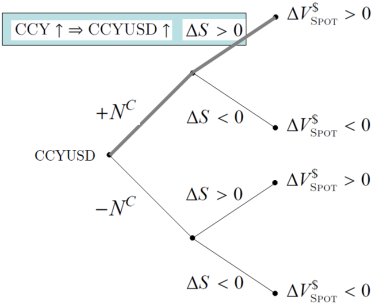

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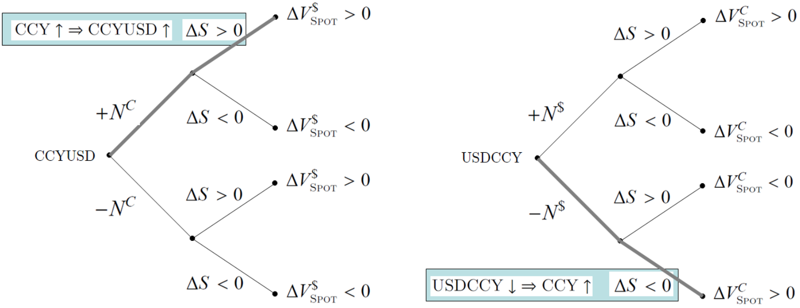

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That's my new portmanteau: arbitrage opportunity $\Leftrightarrow$ arbortunity

Implied Non-domestic Zero Rates (CCYUSD)

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