Twinkle Twinkle Little STAR: Smooth Transition AR Models in R.

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Outline

Market States and Cycles

- Secular Cycles
- Structural Shifts
- Shocks/Crashes

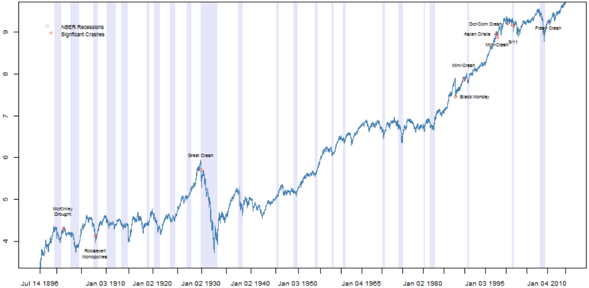


Figure: History of the Dow

Table: DJIA Monthly Return Statistics

	mean	sd	min	max	sum
[NBER=1]	-0.00975	0.072952	-0.36674	0.298862	-3.47971
[NBER=0]	0.00924	0.045305	-0.26417	0.337761	9.720649

Observed and Unobserved Switching in R

Markov Switching (Unobserved)

- MSwM (Sanchez-Espigares and Lopez-Moreno [2014])
- depmixS4 (Visser and Speekenbrink [2010])
- fMarkovSwitching (Perlin [2008])

Threshold Autoregressive (Observed)

- TSA (Chan and Ripley [2012])
- tsDyn (Antonio et al. [2009])
- RSTAR (useR 2008) [vaporware]

Selected Literature Review

During the past twelve years many economic series have undergone what appears to be a permanent change in level. Carmichael [1928]

Selected Literature Review (Models)

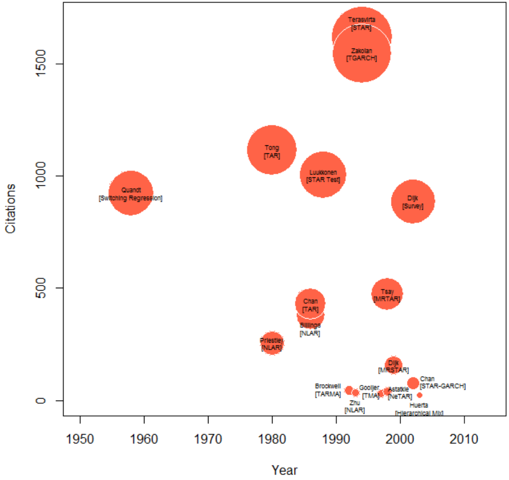


Figure: Selected Publications (sized by no. of citations)

Table: Selected Threshold AR Applications

Author(s)	**model/contribution**
Carmichael [1928]	Arctangent Transform
Quandt [1958]	Switching Regression
Tong and Lim [1980]	TAR
Priestley [1980]	NLAR
Billings and Voon [1986]	NLAR
Chan and Tong [1986]	TAR
Luukkonen et al. [1988]	STAR Test
Brockwell et al. [1992]	TARMA
Zhu and Billings [1993]	NLAR
Teräsvirta [1994]	STAR
Zakoian [1994]	TGARCH
Astatkie et al. [1997]	NeTAR
Gooijer [1998]	TMA
Tsay [1998]	MRTAR
van Dijk and Franses [1999]	MRSTAR
Chan and McAleer [2002]	STAR-GARCH
van Dijk et al. [2002]	Survey
Chan and McAleer [2003]	STAR-GARCH
Huerta et al. [2003]	Hierarchical Mixture

Selected Literature Review (Applications)

Table: Selected Threshold AR Applications

Author(s)	**model**	**study**	**type**
Teräsvirta and Anderson [1992]	STAR	log production (13 countries and Europe)	E
Pesaran and Potter [1997]	(Endogenous Delay) TAR	US GNP	E
Clements and Krolzig [1998]	SETAR and MSAR	US GNP	E
Filardo and Gordon [1998]	MSAR (w/th latent probit model)	US Business Cycle durations	E
Peel and Speight [1998]	SETAR	GDP (5 industrialized economies)	E
van Dijk and Franses [1999]	MRSTAR	US Employment and GNP	E
Kapetanios [2003]	(Endogenous Delay) TAR	US GNP	E
Enders et al. [2007]	D-TAR	US GDP	E
Deschamps [2008]	STAR and MSAR	US Employment	E
Chinn et al. [2013]	STECM	US Employment and GDP (Okun's Law)	E
Pfann et al. [1996]	SETAR with heteroscedastic dynamics	US Term Structure	I
Tsay [1998]	MRTAR	US Term Structure	I
Gospodinov [2005]	TAR-GARCH	US Term Structure	I
Maki [2006]	STAR	Japan Term Structure	I
Cao and Tsay [1992]	TAR	Volatility	S
Zakoian [1994]	TGARCH	Volatility	S
Domian and Louton [1997]	TAR	Stock Returns and Industrial Production	S
citeTsay1998	MTAR	S&P 500 Futures Arb	S
Martens et al. [2009]	SP[Z]-DAXRL	S&P 500 futures volatility	S

Key: *E*: Economic Output, *I*: Interest Rates, *S*: Stock Market

Model Representation-TAR

- 2-state TAR model (Tong and Lim [1980]):

$$y_t = \phi'_1 y_t^{(p)} \mathbf{I}_{z_{t-d \leqslant c}} + \phi'_2 y_t^{(p)} \mathbf{I}_{z_{t-d > c}} + \varepsilon_t$$
$$y_t^{(p)} = \left(1, \tilde{y}_t^{(p)}\right)', \tilde{y}_t^{(p)} = (y_{t-1}, \ldots, y_{t-p})'$$
$$\phi_i = (\phi_{i0}, \phi_{i1}, \ldots, \phi_{ip})'$$
$$\varepsilon_t \sim ID\,(0, \sigma)$$

- Rich dynamics, limit cycles, asymmetric behavior and jumps
- Abrupt switch between states

Model Representation-STAR

- 2-state STAR model (Franses and van Dijk [2000]):

$$y_t = \phi'_1 y_t^{(p)} \left(F\left(z_{t-d}; \gamma, \alpha, c\right)\right) + \phi'_2 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma, \alpha, c\right)\right) + \varepsilon_t$$
$$y_t^{(p)} = \left(1, \tilde{y}_t^{(p)}\right)', \tilde{y}_t^{(p)} = \left(y_{t-1}, \ldots, y_{t-p}\right)'$$
$$\phi_i = \left(\phi_{i0}, \phi_{i1}, \ldots, \phi_{ip}\right)'$$
$$\alpha = \left(\alpha_1, \ldots, \alpha_k\right)'$$
$$\varepsilon_t \sim ID\left(0, \sigma\right)$$
$$i = 1, 2(states)$$

- State Transition function:

$$\text{(Logistic):} F\left(z_{t-d}; \gamma, \alpha, c\right) = \left(1 + \exp\left\{-\gamma\left(\alpha' z_{t-d} - c\right)\right\}\right)^{-1}, \gamma > 0$$
$$\text{(Exponential):} F\left(z_{t-d}; \gamma, \alpha, c\right) = \left(1 - \exp\left\{-\gamma\left(\alpha' z_{t-d} - c\right)^2\right\}\right), \gamma > 0$$

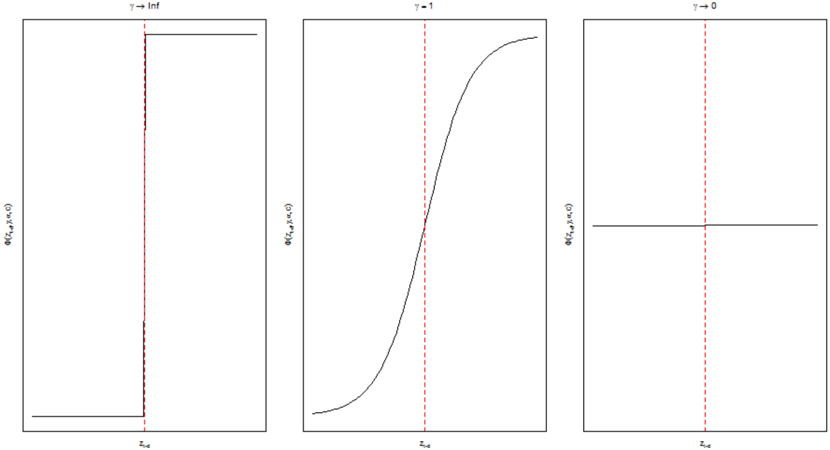
- State switching variable(s):

$$z_{t-d} = \left(z_{1t-d}, \ldots, z_{jt-d}\right)', j = 1, \ldots, k$$

- Identification restriction $\alpha_1 = 1$

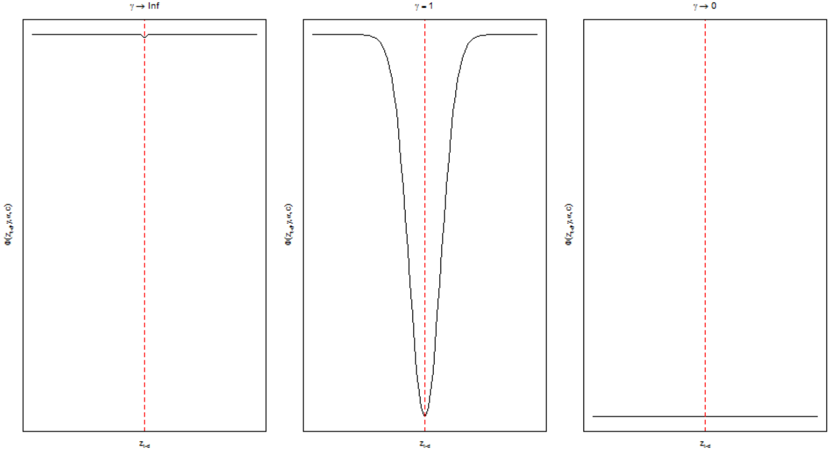
Transition Function (Logistic)

LSTAR Model



Transition Function (Exponential)

ESTAR Model



Model Extensions-AR State Dynamics

- Subsume γ and introduce AR dynamics[1]:

$$F\left(z_{t-d};\alpha,c,\beta\right) = \left(1+\exp\left\{-\pi_t\right\}\right)^{-1}$$
$$\pi_t = c + \alpha' z_{t-d} + \beta' \pi_t^{(q)}$$
$$\pi_t^{(q)} = \left(\pi_{t-1},\ldots,\pi_{t-q}\right)'$$

- Recursion Initialization:

$$\pi_0 = \frac{c+\alpha'\bar{z}}{1-\beta'\mathbf{1}}$$
$$\bar{z} = \left(E\left[z_1\right],...,E\left[z_k\right]\right)'$$

- Stationarity constraint: $\left|\sum_{i=1}^{q}\beta_i\right| < 1$
- Equivalence with standard representation:

$$c = \gamma c$$
$$\alpha' = \gamma(1,\alpha_2,\ldots,\alpha_j)', j = 1,\ldots,k$$
$$\beta = 0$$

[1] As in the dynamic binary response model of Kauppi and Saikkonen [2008].

Model Extensions-(MA)(X) Dynamics

- The STARMAX Model:

$$
\begin{aligned}
y_t &= \left(\phi'_1 y_t^{(p)} + \xi'_1 x_t + \psi'_1 e_t^{(q)}\right)\left(F\left(z_{t-d};\alpha,c,\beta\right)\right) \\
&\quad + \left(\phi'_2 y_t^{(p)} + \xi'_2 x_t + \psi'_2 e_t^{(q)}\right)\left(1 - F\left(z_{t-d};\alpha,c,\beta\right)\right) + \varepsilon_t \\
\varepsilon_t^{(q)} &= \left(\varepsilon_{t-1},\ldots,\varepsilon_{t-q}\right)' \\
\psi'_i &= \left(\psi_{i1},\ldots,\psi_{iq}\right)' \\
x_t &= \left(x_1,\ldots,x_l\right)' \\
\xi'_1 &= \left(\xi_{i1},\ldots,\xi_{il}\right)'
\end{aligned}
$$

Model Extensions-Gaussian Mixture

Consider the STARMAX 2-state model:

$$
\begin{aligned}
y_t &= \left(\phi_1' y_t^{(p)} + \xi_1' x_t + \psi_1' e_t^{(q)}\right)\left(F\left(z_{t-d};\alpha,c,\beta\right)\right) \\
&\quad + \left(\phi_2' y_t^{(p)} + \xi_2' x_t + \psi_2' e_t^{(q)}\right)\left(1 - F\left(z_{t-d};\alpha,c,\beta\right)\right) + \varepsilon_t \\
\varepsilon_t &= y_t - (\mu_{1t}) p_t - (\mu_{2t})\left(1 - p_t\right), d > 0
\end{aligned}
$$

Add and subtract $y_t p_t$, and re-arrange:

$$
\begin{aligned}
\varepsilon_t &= +y_t p_t - (\mu_{1t}) p_t + y_t - y_t p_t - (\mu_{2t})\left(1 - p_t\right) \\
\varepsilon_t &= +y_t p_t - (\mu_{1t}) p_t + y_t\left(1 - p_t\right) - (\mu_{2t})\left(1 - p_t\right) \\
\varepsilon_t &= (y_t - \mu_{1t}) p_t + (y_t - \mu_{2t})\left(1 - p_t\right) \\
\varepsilon_t &= (\varepsilon_{1,t}) p_t + (\varepsilon_{2,t})\left(1 - p_t\right) \\
\varepsilon_{1,t} &\sim N\left(0, \sigma_1^2\right) \qquad \varepsilon_{2,t} \sim N\left(0, \sigma_2^2\right) \\
\varepsilon_t &\sim N\left(0, \sigma_1^2 p_t + \sigma_2^2\left(1 - p_t\right)\right)
\end{aligned}
$$

- Can be thought of as restricted STARMAX-STGARCH model with common state dynamics (with ARCH=GARCH=0).

Model Extensions-Multiple States

- van Dijk and Franses [1999] propose the following 4-state model :

$$
\begin{aligned}
y_t = & \left[{\phi'}_1 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma_1, \alpha, c\right)\right) + {\phi'}_2 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma_1, \alpha, c\right)\right)\right] \left(1 - F\left(z_{t-d}; \gamma_2, b, d\right)\right) \\
& + \left[{\phi'}_3 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma_1, \alpha, c\right)\right) + {\phi'}_4 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma_1, \alpha, c\right)\right)\right] F\left(z_{t-d}; \gamma_2, b, d\right) + \varepsilon_t
\end{aligned}
$$

- Effectively 2 unique states modelled and one interaction:

$$
\begin{aligned}
\mu_1 &= {\phi'}_1 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma_1, \alpha, c\right) - F\left(z_{t-d}; \gamma_2, b, d\right) + F\left(z_{t-d}; \gamma_1, \alpha, c\right) F\left(z_{t-d}; \gamma_2, b, d\right)\right) \\
\mu_2 &= {\phi'}_2 y_t^{(p)} \left(1 - F\left(z_{t-d}; \gamma_1, \alpha, c\right) - F\left(z_{t-d}; \gamma_2, b, d\right) + F\left(z_{t-d}; \gamma_1, \alpha, c\right) F\left(z_{t-d}; \gamma_2, b, d\right)\right) \\
\mu_3 &= {\phi'}_3 y_t^{(p)} \left(F\left(z_{t-d}; \gamma_2, b, d\right) - F\left(z_{t-d}; \gamma_1, \alpha, c\right) F\left(z_{t-d}; \gamma_2, b, d\right)\right) \\
\mu_4 &= {\phi'}_4 y_t^{(p)} \left(F\left(z_{t-d}; \gamma_2, b, d\right) - F\left(z_{t-d}; \gamma_1, \alpha, c\right) F\left(z_{t-d}; \gamma_2, b, d\right)\right)
\end{aligned}
$$

- Interaction used in modelling Time Varying (TV) STAR model.

Model Extensions-Multiple States (cont'd)

- Alternative representation follows multinomial regression paradigm:

$$y_t = \sum_{i=1}^{s} \left[\left(\phi'_i y_t^{(p)} + \xi'_i x_t + \psi'_i e_t^{(q)} \right) F_i \left(z_{t-d}; \alpha_i, c_i, \beta_i \right) \right] + \varepsilon_t$$

- s-1 distinct states modelled

$$F_i \left(z_{t-d}; \alpha_i, c_i, \beta_i \right) = \frac{e^{\pi_{i,t}}}{1 + \sum_{i=1}^{s-1} e^{\pi_{i,t}}}$$

$$F_s \left(z_{t-d}; \alpha_i, c_i, \beta_i \right) = \frac{1}{1 + \sum_{i=1}^{s-1} e^{\pi_{i,t}}}$$

- $\sum_{i=1}^{s} F_i \left(\ldots \right) = 1$

Implementation

The twinkle package

```
>require(devtools)
>install_bitbucket("twinkle","alexiosg")
# depends on rugarch
```

- (D)(ST)(AR)(MA)(X) with static, mixture or GARCH variance
- Multiple states (max. 4)
- Specification, Estimation, Filtering, Forecasting and Simulation
- S4 classes and methods
- Enhanced methods (quantile, pit, states)
- Estimation/forecast and simulation in C for speed.
- Fully documented with vignette
- Large testing suite with examples
- GIRF (coming soon)
- No tests yet...

Use R-SIG-FINANCE to report bugs or ask questions!

Model Specification

```
>starspec
(mean.model=list(states=2, include.intercept=c(1,1), arOrder=c(1, 1),
maOrder=c(0, 0), matype="linear", statevar=c("y", "s"), s=NULL, ylags=1,
xreg=NULL, statear=FALSE,  yfun=NULL, transform="log"),
variance.model=list(dynamic=FALSE, model="sGARCH", garchOrder=c(1, 1),
submodel=NULL, vreg=NULL, variance.targeting=FALSE),
distribution.model="norm", start.pars=list(), fixed.pars=list(),
fixed.prob=NULL, ...)
```

- custom y-transformation function ('yfun')
- MA part can be inside ('state') or outside ('linear')
- variance: 'static' (default), 'mixture' or one of 3 GARCH models (vanilla, gjr or exponential)
- distributions: same as in rugarch ([skew]norm,[skew]std,[skew]ged,jsu,nig,ghyp,ghst)
- Methods on **STARspec** object include *setbounds*, *setstart* and *setfixed*

Estimation

```
>starfit
(spec, data, out.sample=0, solver="optim", solver.control=list(),
fit.control=list(stationarity=0, fixed.se=0, rec.init="all"),
cluster=NULL, n=25, ...)
```

- Maximum likelihood estimation
- Main solver 'BFGS' (unconstrained). Bound constraints use logistic transformation
- 2 strategies:
  - random search multi-start ('msoptim')
  - cycling between non-state and state parameters ('strategy')

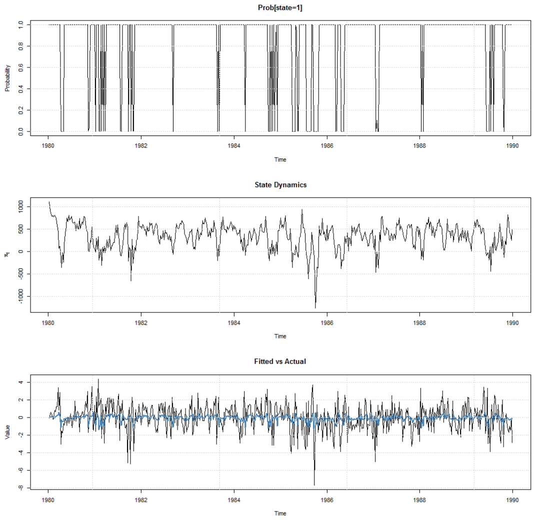
Estimation-Dutch Gilder example

```
>library(twinkle)
>library(quantmod);
>data(forex)
# State variable as in Franses and van Dijk (2000)
>fx = na.locf(forex, fromLast = TRUE)
>fx = fx[which(weekdays(index(forex))=="Wednesday"),4]
>fx = ROC(fx, na.pad=FALSE)*100
fun = function(x){
  x = as.numeric(x)
  N = length(x)
  if(N<4){
    y = abs(x)
  } else{
    y = runMean(abs(x), n=4)
    y[1:3] = c(abs(x[1]), mean(abs(x[1:2])),
    mean(abs(x[1:3])))
  }
  return(y)
}
>spec=starspec(mean.model=list(states=2,statevar="y",
+statear=TRUE,yfun=fun, include.intercept=c(0,1),
+arOrder=c(1,1),ylags=1))
>control=list(maxit=10000,reltol=1e-12,trace=1,
+method="BFGS",parsearch=TRUE)
>mod = starfit(spec, fx[1:521], solver='strategy',
+n=6, solver.control=control)
```

```
> mod

Optimal Parameters (Robust Standard Errors)
------------------------------------
             Estimate  Std. Error   t value Pr(>|t|)
s1.phi1       0.18259    0.054069    3.3769 0.000733
s2.phi0      -0.69411    0.187701   -3.6980 0.000217
s2.phi1      -0.16054    0.075440   -2.1281 0.033327
s1.c       1174.30851    8.562348  137.1480 0.000000
s1.alpha1 -623.18683    4.557827 -136.7289 0.000000
s1.beta      -0.22237    0.002187 -101.6743 0.000000
sigma         1.53464    0.067537   22.7229 0.000000

LogLikelihood : -962.4082
Akaike       3.7213
Bayes        3.7785
Shibata      3.7210
Hannan-Quinn 3.7437
r.squared         :  0.0486
r.squared (adj)   :  0.0356
RSS               :  1227.013
skewness (res)    :  -0.46385
ex.kurtosis (res) :  1.00692

AR roots
         Moduli1
state_1 5.476806
state_2 6.228787
```

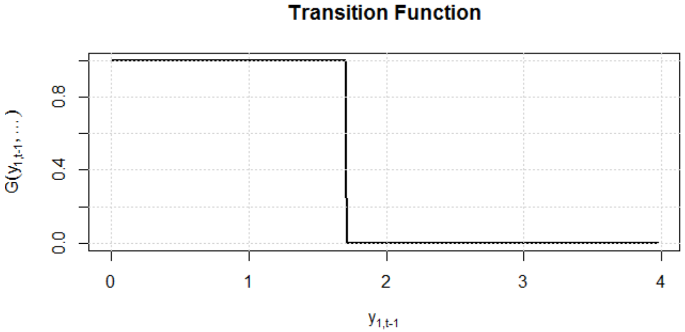
Estimation-Dutch Gilder example (cont'd)

```
>plot(mod)
```



Estimation-Dutch Gilder example (cont'd)

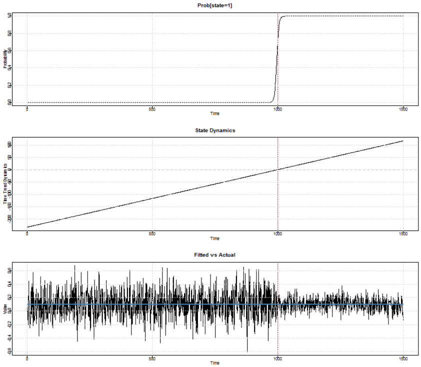
```
>trans2fun2d(mod, colidx = 1)
```



Estimation-2 states (mixture) example

```
>set.seed(25)
>gmix = xts(c(rnorm(1000, 0.1, 0.2),
+rnorm(500, 0.1, 0.1)), as.Date(1:1500))
>ttrend = xts(seq(0, 1, length.out=1500), index(gmix))
spec = starspec(mean.model=list(states=2,
+include.intercept=c(1,1), arOrder=c(0,0),
+statevar="s", s=ttrend), variance.model=list(
+dynamic=TRUE, model="mixture"))
solver.control=list(maxit=17000, reltol=1e-12,
+trace=1, method="BFGS")
mod = starfit(spec, data=gmix, solver="strategy",
+solver.control=solver.control, n=6)
```

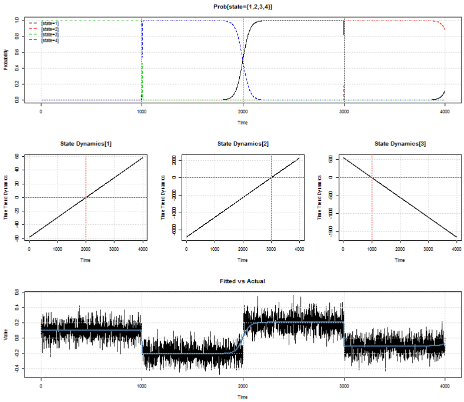
```
>round(mod@fit$robust.matcoef, 4)
           Estimate  Std. Error   t value Pr(>|t|)
s1.phi0      0.1014      0.0040   25.5403        0
s2.phi0      0.1010      0.0055   18.2999        0
s1.c      -229.2983      0.2874 -797.7274        0
s1.alpha1  345.5743      0.4258  811.5344        0
s1.sigma     0.0941      0.0029   31.9533        0
s2.sigma     0.2006      0.0044   45.8985        0
```



Estimation-4 states example

```
set.seed(77)
>mix4=xts(c(rnorm(1000, 0.1, 0.1),
+rnorm(1000, -0.2, 0.1),
+rnorm(1000,  0.2, 0.1),
+rnorm(1000, -0.1, 0.1)),
+as.Date(1:4000))
>ttrend=xts(seq(0, 1, length.out=4000),
+index(mix4))
>spec=starspec(mean.model=list(states=4,
+include.intercept=c(1,1,1,1),
arOrder=c(0,0,0,0), statevar="s", ylags=1,
+s=ttrend))
solver.control=list(maxit=10000, reltol=1e-14,
+trace=1,method="BFGS")
>mod=starfit(spec, data=mix4, solver="strategy",
+solver.control=solver.control, n=15)
```

```
> round(mod@fit$robust.matcoef, 4)
            Estimate Std. Error    t value Pr(>|t|)
s1.phi0       0.2058     0.0039    52.8805        0
s2.phi0      -0.1027     0.0036   -28.7900        0
s3.phi0       0.1024     0.0031    33.3015        0
s4.phi0      -0.2039     0.0033   -62.5670        0
s1.c        -57.8622     0.1887  -306.6501        0
s1.alpha1   115.8724     0.7737   149.7634        0
s2.c      -6807.9119     1.5574 -4371.3689        0
s2.alpha1  9115.6159     2.0940  4353.1041        0
s3.c        551.7943     0.1744  3163.8852        0
s3.alpha1 -2208.0543     0.6974 -3166.3001        0
sigma         0.1009     0.0016    62.4083        0
```



Forecasting

Consider a general nonlinear first order autoregressive model:

$$y_t = F(y_{t-1};\theta) + \varepsilon_t$$

- 1-step ahead: $\hat{y}_{t+1|t} = E[y_{t+1} | \Im_t] = F(y_t;\theta)$
- h-step ahead[2]: $g(y_{t+h} | \Im_t) = \int_{-\infty}^{\infty} g(y_{t+h} | y_{t+h-1}) g(y_{t+h-1} | \Im_t) dy_{t+h-1}$
- Nonlinear relationship: $E[F(.)] \neq F(E[.])$
- Start at h=2[3]: $\hat{y}_{t+2|t} = \frac{1}{T}\sum_{i=1}^{T} F(\hat{y}_{t+1|t} + \varepsilon_i;\theta)$
- Recursively estimate for each $h > 2$ using quadrature integration or monte carlo summation

---

[2] This is based on the Chapman-Kolmogorov relation:

$$g(y_{t+h} | \Im_t) = \int_{-\infty}^{\infty} g(y_{t+h} | y_{t+h-1}) g(y_{t+h-1} | \Im_t) dy_{t+h-1}$$

which leads to the h-step ahead equation after taking conditional expectations from both sides.

[3] In the case of a GARCH model this should be:

$$\hat{y}_{t+2|t} = \frac{1}{T}\sum_{i=1}^{T} F\left(\hat{y}_{t+1|t} + z_i \hat{\sigma}_{t+2|t};\theta\right)$$

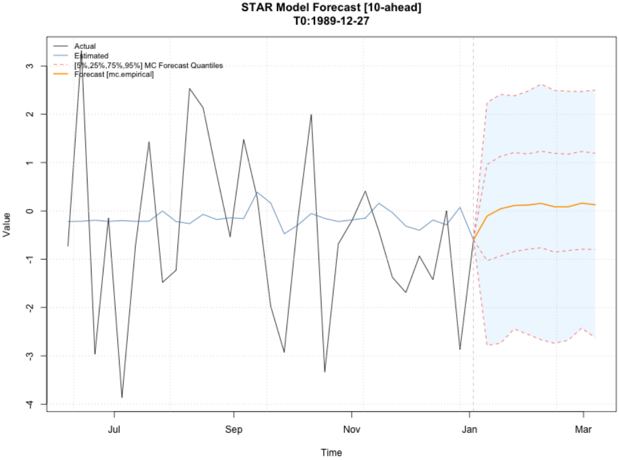
Forecasting (cont'd)

```
>starforecast
(fitORspec, data=NULL, n.ahead=1, n.roll=0, out.sample=0,
external.forecasts = list(xregfor=NULL, vregfor=NULL, sfor=NULL,
probfor=NULL), method=c("an.parametric","an.kernel", "mc.empirical",
"mc.parametric", "mc.kernel"), mc.sims=NULL, ...)
```

- Multiple dispatch methods (**STARfit** and **STARspec** with fixed parameters)
- Choice of integral evaluation for $h > 1$ (quadrature and monte carlo)
- Choice of error distribution (parametric, empirical and kernel)
- Rolling h-ahead forecasts (in combination with out.sample option)

Forecasting-Dutch Gilder Example (cont'd)

```
>forc=starforecast(mod, n.ahead=10, method="mc.empirical", mc.sims=4000)
>plot(forc)
```



Rolling Estimation and Forecasting

```
>rollstar
function (spec, data, n.ahead=1, forecast.length=500, n.start=NULL,
refit.every=25, refit.window=c("recursive", "moving"),
window.size=NULL, solver="msoptim", fit.control=list(),
solver.control=list(), calculate.VaR=TRUE,
VaR.alpha=c(0.01, 0.05), cluster=NULL, keep.coef=TRUE, ...)
```

- Support for parallel evaluation of estimation windows
- Quick extractor methods for rolling quantiles (VaR) and PIT
- Forecast evaluation tests from rugarch: VaRTest, ESTest, HLTest, BerkowitzTest, GMMTest, and mcs
- **resume** method for resubmitting non-converged windows

Rolling Estimation and Forecasting-Dutch Gilder Example (cont'd)

```
>library(parallel)
>cl=makePSOCKcluster(15)
>clusterEvalQ(cl, library(quantmod))
>roll = rollstar(spec, data=dx[1:521],
forecast.length=100,
refit.every=5, refit.window="recursive",
solver="strategy", cluster = cl)
```

```
>show(roll)
*-------------------------------------*
*              STAR Roll              *
*-------------------------------------*
No.Refits : 20
Refit Horizon : 5
No.Forecasts : 100
states       : 2
statevar      : y
statear       : FALSE
variance      : static
distribution  : norm

Forecast Density
                Mu  Sigma Prob[State=1] Prob[State=2] Realized
1988-02-03 -0.2025 1.5513      0.6657        0.3343   1.0335
1988-02-10  0.4348 1.5513      0.0444        0.9556   0.1317
1988-02-17  0.3203 1.5513      0.0115        0.9885   0.9173
1988-02-24  0.3522 1.5513      0.0035        0.9965  -0.7069
1988-03-02  0.0682 1.5513      0.0044        0.9956  -0.1052
1988-03-09 -0.1126 1.5446      0.9993        0.0007  -1.4572

........................
                Mu  Sigma Prob[State=1] Prob[State=2] Realized
1989-11-22  0.0596 1.5399      0.0003        0.9997  -1.3736
1989-11-29 -0.3152 1.5343      0.0009        0.9991  -1.6905
1989-12-06 -0.5405 1.5343      0.0121        0.9879  -0.9338
1989-12-13 -0.4006 1.5343      0.0300        0.9700  -1.4249
1989-12-20 -0.3619 1.5343      0.1556        0.8444   0.0000
1989-12-27 -0.1384 1.5343      0.0159        0.9841  -2.8651

Elapsed: 32.86328 secs
```

Additional Methods

- Filtering: **starfilter**
- Simulation: **starsim**, **starpath**
- Standard Extractors: **residuals**, **fitted**, **coef**, **likelihood**, **infocriteria**, **score**, **vcov**, **modelmatrix**
- Special Extractors: **quantile**, **pit**, **states**, **sigma**
- Inference: **plot**, **show**

Background[4]

- What drives aggregate market volatility?
- Excess volatility and clustering
- Volatility and the business cycle (Schwert [1989], Paye [2012], Christiansen et al. [2012])

[4] This is joint work with Eduardo Rossi (Department of Economics and Management, University of Pavia)

Realized Volatility Across the Business Cycle

Figure: S&P 500 Monthly Realized Volatility

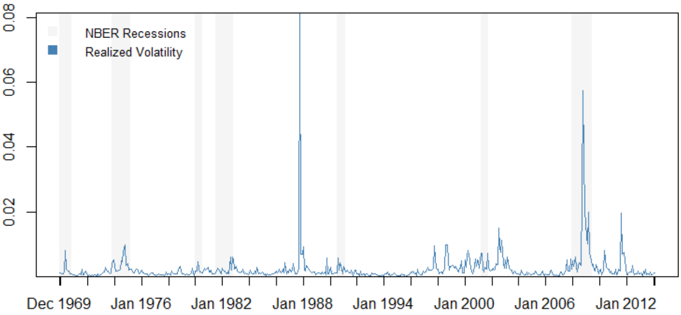


Table: S&P 500 Monthly Realized Volatility Statistics

	mean	sd	min	max
[NBER=1]	0.00489	0.00776	0.00079	0.05730
[NBER=0]	0.00196	0.00426	0.00021	0.08138

Realized Volatility Models

- Multiplicative Error Model (MEM) of Engle and Gallo [2006]

$$y_t = \mu_t \varepsilon_t, \varepsilon_t \sim \Gamma(\phi, \phi)$$
$$\mu_t = \omega + \alpha\mu_{t-1} + \beta y_{t-1}$$

- Heterogeneous AR Model of Corsi [2009]

$$\log RV_{t+1d}^{(d)} = c + \beta^{(d)} \log RV_t^{(d)} + \beta^{(w)} \log RV_t^{(w)} + \beta^{(m)} \log RV_t^{(m)} + \varepsilon_{t+1d}^{(d)}$$

- Realized GARCH model of Hansen et al. [2012][5]

[5] See www.unstarched.net/2014/01/02/the-realized-garch-model/

Setup-Dataset

- S&P 500 monthly realized variance
- Logarithmic transformation of realized variance[6]
- Optimal Forecast under log transformation (Granger and Newbold [1976]):

$$y^{opt}_{t+h|t} = \exp\left(\log y_{t+h|t} + \frac{1}{2}\sigma^2_{\log y}(h)\right)$$

- In-sample period: Apr-1967 to Feb-2014
- Out-of-sample period: Oct-1995-Feb-2014 (recursive window with base Apr-1967)
- 11 Economic and Market explanatory factors:

Id	Variable	Description
x_1	$\%\Delta CPI^3_{t-1}$	3 month % change in inflation (CPI)
x_2	$\%\Delta IP^3_{t-1}$	3 month % change in industrial production (IP)
x_3	$\%\Delta NFP^1_{t-1}$	1 month % changes in non-farm payrolls (NFP)
x_4	$\%\Delta MDU^1_{t-1}$	1 month % changes in median duration of unemployment (MDU)
x_5	$\%\Delta SPX^1_{t-1}$	1 month % change in the S&P500 return (SPX)
x_6	$T^{10y}_{t-1} - T^{3m}_{t-1}$	Term Spread 10Y and 3Month
x_7	$T^{10y}_{t-1} - AAA_{t-1}$	Spread 10Y and Moody's AAA Corporate
x_8	$AAA_{t-1} - BAA_{t-1}$	Spread Moody's AAA and BAA Corporate
x_9	$NAPM_{t-1}$	PMI Composite Index (NAPM)
x_{10}	$BEARBULL_{t-1}$	month-end ratio of bearish to bullish consensus (Investors Intelligence)
x_{11}	$NYHILO_{t-1}$	NYSE News Highs to Lows as % of Total Issues traded

[6] See Cao and Tsay [1992] and Gonçalves and Meddahi [2011] for an alternative based on the Box-Cox transform.

Setup-Models

- Model1: HAR model of Corsi [2009]:

$$\mathsf{LRV}_t^{1M} = \phi_{1,0} + \xi_{1,1}\mathsf{LRV}_{t-1}^{1M} + \xi_{1,2}\mathsf{LRV}_{t-1}^{3M} + \xi_{1,3}\mathsf{LRV}_{t-1}^{6M} + \xi_{1,4}\mathsf{LRV}_{t-1}^{12M} + \varepsilon_t$$

- Model2: HAR(MA)(X)

$$\begin{aligned}\mathsf{LRV}_t^{1M} = \phi_{1,0} &+ \xi_{1,1}\mathsf{LRV}_{t-1}^{1M} + \xi_{1,2}\mathsf{LRV}_{t-1}^{3M} + \xi_{1,3}\mathsf{LRV}_{t-1}^{6M} + \xi_{1,4}\mathsf{LRV}_{t-1}^{12M} \\ &+ \sum_{j=1}^{11} \alpha_j x_{j,t-1} + \psi_{1,1}\varepsilon_{t-1} + \varepsilon_t\end{aligned}$$

- Model3: 2-state (X) Smooth Transition HAR

$$\mathsf{LRV}_t^{1M} = \sum_{j=1}^{2} F_j\left(\pi_t; c, a\right)\left(\phi_{j,0} + \xi_{j,1}\mathsf{LRV}_{t-1}^{1M} + \xi_{j,2}\mathsf{LRV}_{t-1}^{3M} + \xi_{j,3}\mathsf{LRV}_{t-1}^{6M} + \xi_{j,4}\mathsf{LRV}_{t-1}^{12M}\right) + \varepsilon_t$$

$$F_1 = \frac{1}{1 + e^{-\pi_t}}, \qquad F_2 = 1 - F_1$$

$$\pi_t = c + \sum_{j=1}^{11} \alpha_j x_{j,t-1}$$

- Model4: 2-state (Self-Exciting) Smooth Transition HAR

$$\mathsf{LRV}_t^{1M} = \sum_{j=1}^{2} F_j\left(\mathsf{LRV}_{t-1}^{1M}; c, a\right)\left(\phi_{j,0} + \xi_{j,1}\mathsf{LRV}_{t-1}^{1M} + \xi_{j,2}\mathsf{LRV}_{t-1}^{3M} + \xi_{j,3}\mathsf{LRV}_{t-1}^{6M} + \xi_{j,4}\mathsf{LRV}_{t-1}^{12M}\right) + \varepsilon_t$$

- Model5: 2-state (X) Smooth Transition HAR(MA)
- Model6: 2-state (X) Smooth Transition HAR(Normal-Mixture)
- Model7: MEM model of Engle and Gallo [2006] on volatility (QML based)

Results-In Sample

Table: S&P 500 realized variance model (in-sample)

	HAR	HARMAX	SE-STHAR	X-STHAR	X-STHARMA	X-STHAR-NM
Log-Variance Dynamics						
$\phi_{1,0}$	-1.355***	-1.488 ***	2.000***	-2.072 ***	-2.440***	-2.106 ***
$\psi_{1,1}$		-0.145			0.366***	
$\xi_{1,1}$	0.406***	0.491 ***	-0.906***	0.308 ***	0.053	0.334 ***
$\xi_{1,2}$	0.232**	0.132	8.005***	0.235 **	0.314***	0.220 **
$\xi_{1,3}$	0.091	0.093	-24.186***	0.080	0.159	0.069
$\xi_{1,4}$	0.069	0.058	17.310***	0.087	0.130	0.080
$\phi_{2,0}$			-1.404***	-0.280	0.492	-0.349
$\psi_{2,1}$					-0.827***	
$\xi_{2,1}$			0.395***	0.151	1.201***	0.097
$\xi_{2,2}$			0.222**	0.343	-0.408*	0.351
$\xi_{2,3}$			0.105	-0.179	-0.033	-0.083
$\xi_{2,4}$			0.069	0.587 ***	0.268***	0.526 **
α_1		0.201				
α_2		-0.540				
α_3		5.123				
α_4		0.723				
α_5		-3.589 ***				
α_6		-0.041 **				
α_7		-0.024				
α_8		-0.037				
α_9		-0.003				
α_{10}		-0.133 *				
α_{11}		-0.525				
State Dynamics						
c			47.124***	570.201 ***	186.114***	351.067 ***
α_1			12.786***	-99.607 ***	12.776***	5.564 ***
α_2				155.425 ***	28.142***	46.392 ***
α_3				-71.974 ***	-31.480***	-21.970 ***
α_4				-20.375 ***	-0.387	-4.925 ***
α_5				520.141 ***	224.050***	340.672 ***
α_6				-83.107 ***	-40.516***	-122.273 ***
α_7				-109.205 ***	-90.230***	-153.698 ***
α_8				-146.412 ***	-18.889***	-25.688 ***
α_9				395.537 ***	182.134***	266.380 ***
α_{10}				441.498 ***	147.223***	178.393 ***
α_{11}				875.194 ***	321.606***	508.072 ***
σ_1	0.590***	0.564 ***	0.581***	0.537 ***	0.523***	0.473 ***
σ_2						0.705 ***
LogLik	-473.245	-449.260	-465.145	-422.886	-409.371	-407.884
AIC	1.805	1.760	1.801	1.679	1.636	1.627
BIC	1.853	1.905	1.906	1.865	1.8373	1.820
R.squared	0.522	0.564	0.537	0.605	0.6245	0.601
R.squared (adj)	0.517	0.548	0.525	0.587	0.6059	0.582
Res.Sum.Squares	184.814	168.850	179.261	152.884	145.2956	154.442
Res.Skewness	1.043	1.052	1.061	0.897	0.88501	0.858
Res.Kurtosis (ex)	3.763	4.364	4.005	3.499	3.7558	3.508

***,** and * denote significance at the 1%,5% and 10% levels respectively, based on robust standard errors.

Results-Out of Sample

Table: S&P 500 Realized Variance Forecast Tests

Panel A: Forecast Error Statistics

	HAR	HARMAX	HARMEMX	SE-STHAR	X-STHAR	X-STHARMA	X-STHAR-NM
RMSE	0.00049	0.00028	0.00031	0.00056	0.00034	0.00031	0.00025
MAE	0.00162	0.00156	0.00189	0.00169	0.00170	0.00174	0.00174
MedAE	0.00069	0.00062	0.00082	0.00069	0.00063	0.00063	0.00063
Skewness	7.11177	6.79254	3.51048	6.97993	6.00732	3.51368	5.37995
Stdev	0.004226	0.004110	0.004137	0.004555	0.004452	0.004799	0.004498

Panel B: Mincer-Zarnowitz Regression

	HAR	HARMAX	HARMEMX	SE-STHAR	X-STHAR	X-STHARMA	X-STHAR-NM
(Intercept)	-0.00023	0.00067*	0.00061*	-0.00010	0.00069*	0.00129***	0.0008**
β	1.26282***	0.86645***	0.73746***	1.24941***	0.87857***	0.66227***	0.80489***
R.squared (adj)	0.4103	0.4358	0.4790	0.3064	0.3325	0.3005	0.3319
Prob(Intercept=0,β=1)	0.0064	0.0704	0.0000	0.0220	0.1710	0.000	0.0238

Panel C: Mincer-Zarnowitz High-Low State Regression (X-STHAR States)

	HAR	HARMAX	HARMEMX	SE-STHAR	X-STHAR	X-STHARMA	X-STHAR-NM
(Intercept)	6.78E-04	0.0012150***	0.00114***	8.85E-04	0.00099*	0.001826***	0.00106**
β_H	1.268***	0.8545***	0.73713***	1.2713***	0.86537***	0.64525***	0.79626***
β_L	0.668***	0.5012**	0.42836***	0.5960**	0.65926**	0.27917***	0.63596**
R.squared (adj)	0.4310	0.4400	0.4868	0.3330	0.3309	0.3019	0.3291
Prob(Intercept=0,β_L=1,β_H =1)	0.0002	0.0451	0.0000	0.0004	0.2474	1.42E-06	0.0520

Panel D: MCS Test

	HAR	HARMAX	HARMEMX	SE-STHAR	X-STHAR	X-STHARMA	X-STHAR-NM
Loss1:	0.54	1.00	0.29	0.95	0.28	0.27	0.27
Loss2:	0.56	1.00	0.01	0.56	0.56	0.56	0.37
Loss3:	0.05	0.08	0.00	0.08	0.08	0.08	1.00
Loss4:	0.22	1.00	0.00	0.18	0.73	0.86	0.18
Loss5:	0.06	0.85	0.00	0.06	0.59	1.00	0.06
Loss6:	0.10	0.82	0.00	0.10	0.58	1.00	0.10
Loss7:	0.31	1.00	0.01	0.31	0.94	0.95	0.41

Loss[1]: $\left(R_{t+1} - \hat{R}_{t+1}\right)^2$

Loss[2]: $\left|R_{t+1} - \hat{R}_{t+1}\right|$

Loss[3]: $\left|R_{t+1} - \hat{R}_{t+1}\right| * I_{\hat{S}_{t+1}=L} + (R_{t+1} - \hat{R}_{t+1})^2 * I_{\hat{S}_{t+1}=H}$

Loss[4]: $\left|R_{t+1} - \hat{R}_{t+1}\right| * I_{R_{t+1}<0.005} + (R_{t+1} - \hat{R}_{t+1})^2 * I_{R_{t+1}\geq 0.005}$

Loss[5]: $\left|R_{t+1} - \hat{R}_{t+1}\right| * I_{R_{t+1}<0.007} + (R_{t+1} - \hat{R}_{t+1})^2 * I_{R_{t+1}\geq 0.007}$

Loss[6]: $\left|R_{t+1} - \hat{R}_{t+1}\right| * I_{R_{t+1}<0.01} + (R_{t+1} - \hat{R}_{t+1})^2 * I_{R_{t+1}\geq 0.01}$

Loss[7]: $\left|R_{t+1} - \hat{R}_{t+1}\right| * I_{NBER=0} + (R_{t+1} - \hat{R}_{t+1})^2 * I_{NBER=1}$

Results-Out of Sample (cont'd)

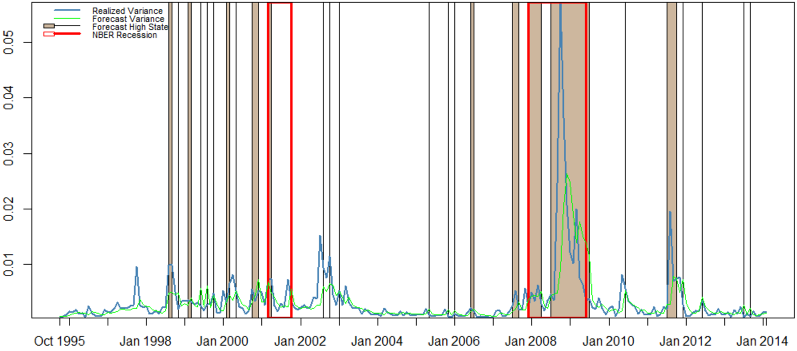


Figure: S&P 500 Forecast Variance and States

Final Thoughts

- Powerful modelling tool
- Accessible interface
- Possible future extensions: STARFIMA, Smooth Transition VAR, ECM
- State variable transformations, basis functions and separation

Thanks/Q&A

- blog: `http://www.unstarched.net`
- *current* development repository [b]: `https://bitbucket.org/alexiosg`

Package	CRAN	R-Forge	Bitbucket	Description
rugarch	✓		✓	Univariate GARCH
rmgarch	✓		✓	Multivariate GARCH
racd			✓	Higher Moment Dynamics
twinkle			✓	STAR
parma	✓	✓		Portfolio Optimization
spd	✓			Semi-Parametric Distribution
Rsolnp	✓	✓		Nonlinear Solver

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