New ideas for network analysis

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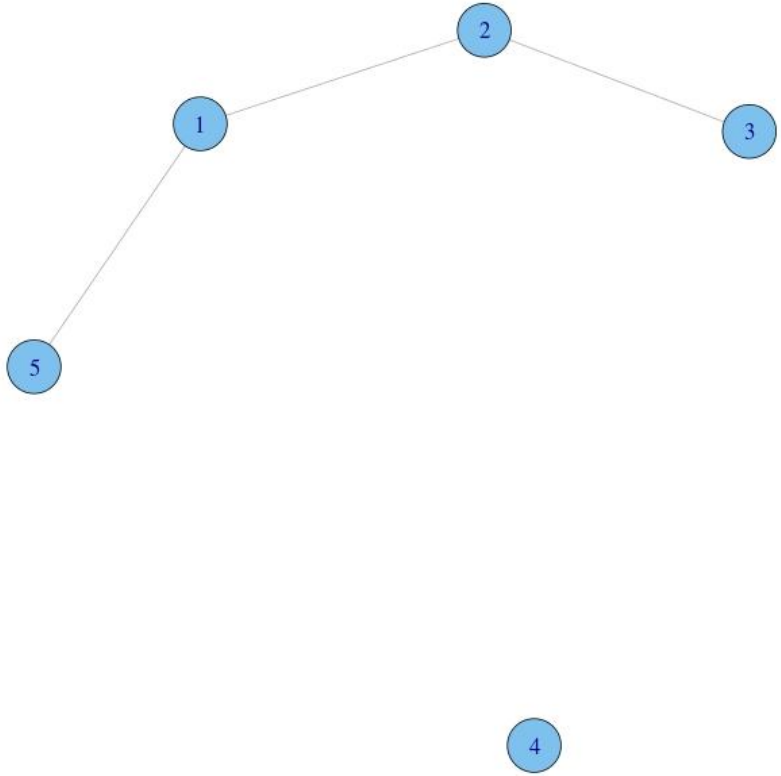
http://goo.gl/ZbJEUO

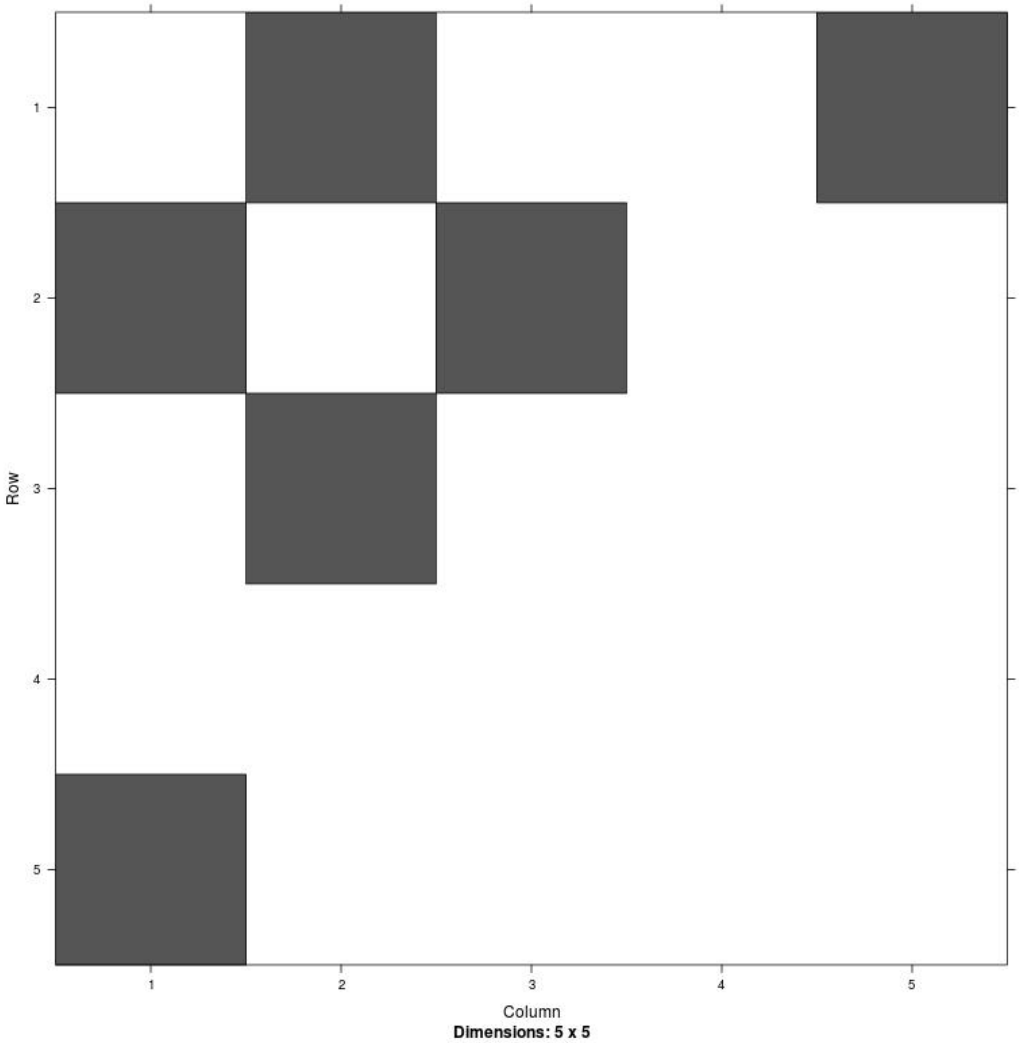
The work of:

Baglama, Benzi, Boito, Estrada, Fenu, Golub, Kleinberg, Meurant, Reichel, Rodriguez, and others...

A simple undirected network



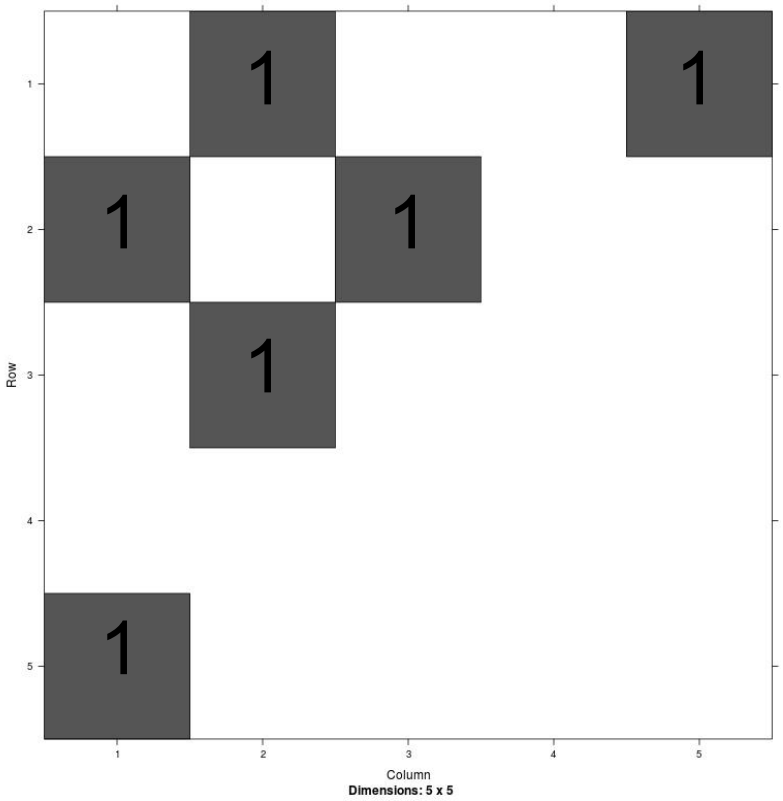
Represented as an adjacency matrix **A**



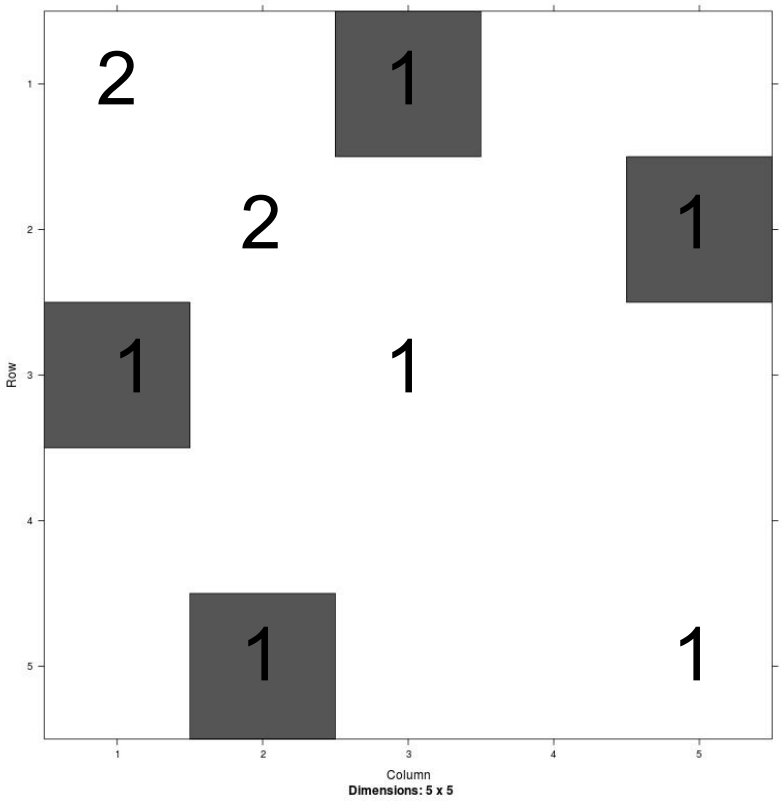
$$[A^m]_{i,j}$$

Counts the number of paths of length m between nodes i and j

A


1
1
1
1
1
1
Row
Column
Dimensions: 5 x 5

A^2


2
1
2
1
1
1
1
1
Row
Column
Dimensions: 5 x 5

$$[A + A^2 + A^3 + \ldots]_{i,j}$$

Counts paths of all lengths between nodes i and j

(may be unbounded)

Maybe long paths are not as important as short ones?

De-emphasize long paths by weighting them

$$w_0 I + w_1 A + w_2 A^2 + w_3 A^3 + \ldots$$

When $w_j = 1/j!$, then

$$w_0 I + w_1 A + w_2 A^2 + w_3 A^3 + \ldots$$

$$= \exp(A)$$

Some useful measurements of networks

for functions f that are nonnegative, nondecreasing on the spectrum of a symmetric matrix A, for example f(A) = exp(A)

Importance of a node in the network

f-subgraph centrality

$$[f(A)]_{i,i}$$

Ease of travel between two nodes

f-subgraph communicability

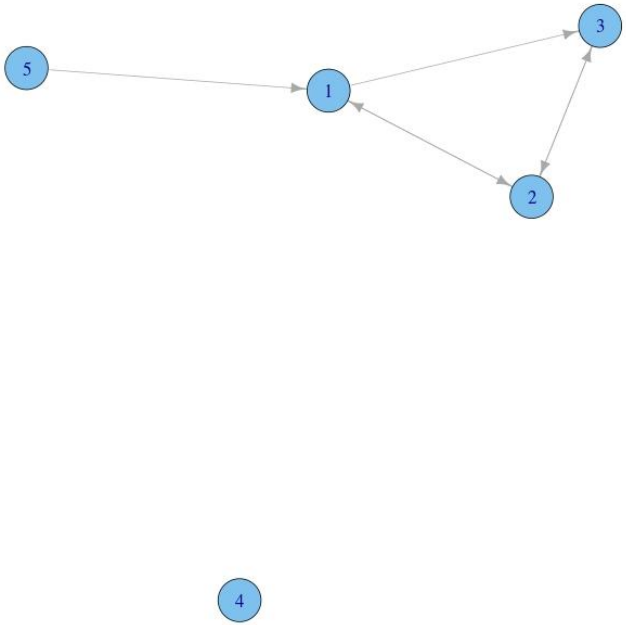
$$[f(A)]_{i,j}$$

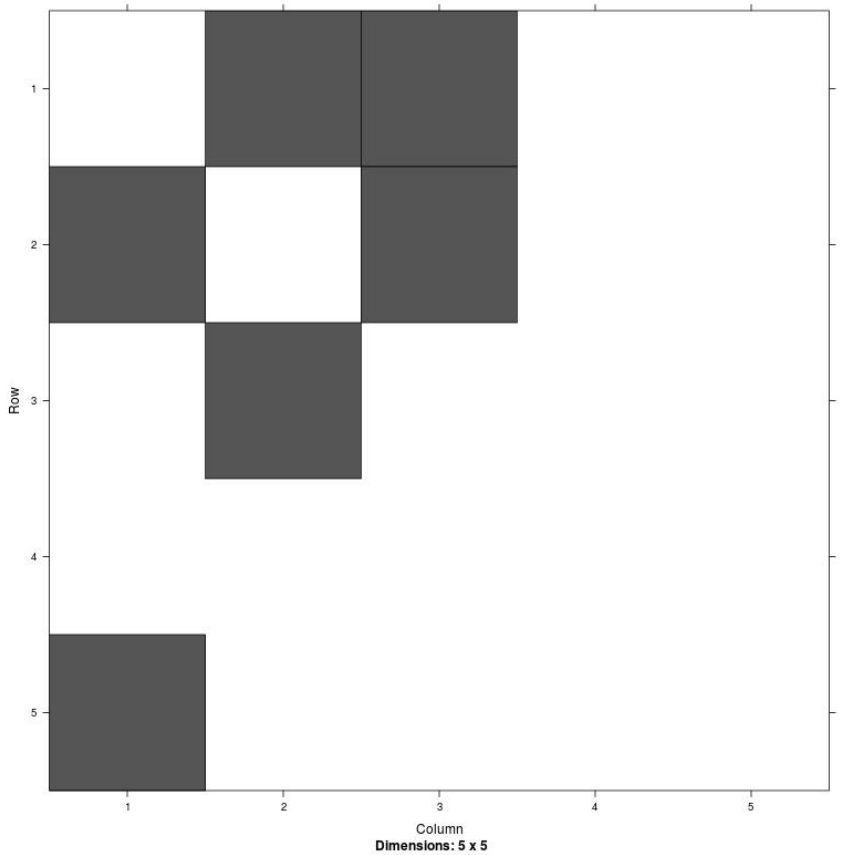
Starting nodes giving greatest network coverage

f-subgraph starting convenience

$$e_i^T f(A) c$$

Directed networks ➔ Nonsymmetric matrices





In that case we can work with

$$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$$

Related to the HITS hub/authority approach

In that case, we compute

- f-subgraph hub centrality
- f-subgraph authority centrality
- f-subgraph hub communicability
- f-subgraph authority communicability
- ...

Computing this stuff

When f(A) = exp(A) we can use

expm

from the R's superb Matrix package

Benzi and Boito:

Use orthogonal polys from quadrature rules (Golub)

What if we only care about the top m important nodes?

IRL methods

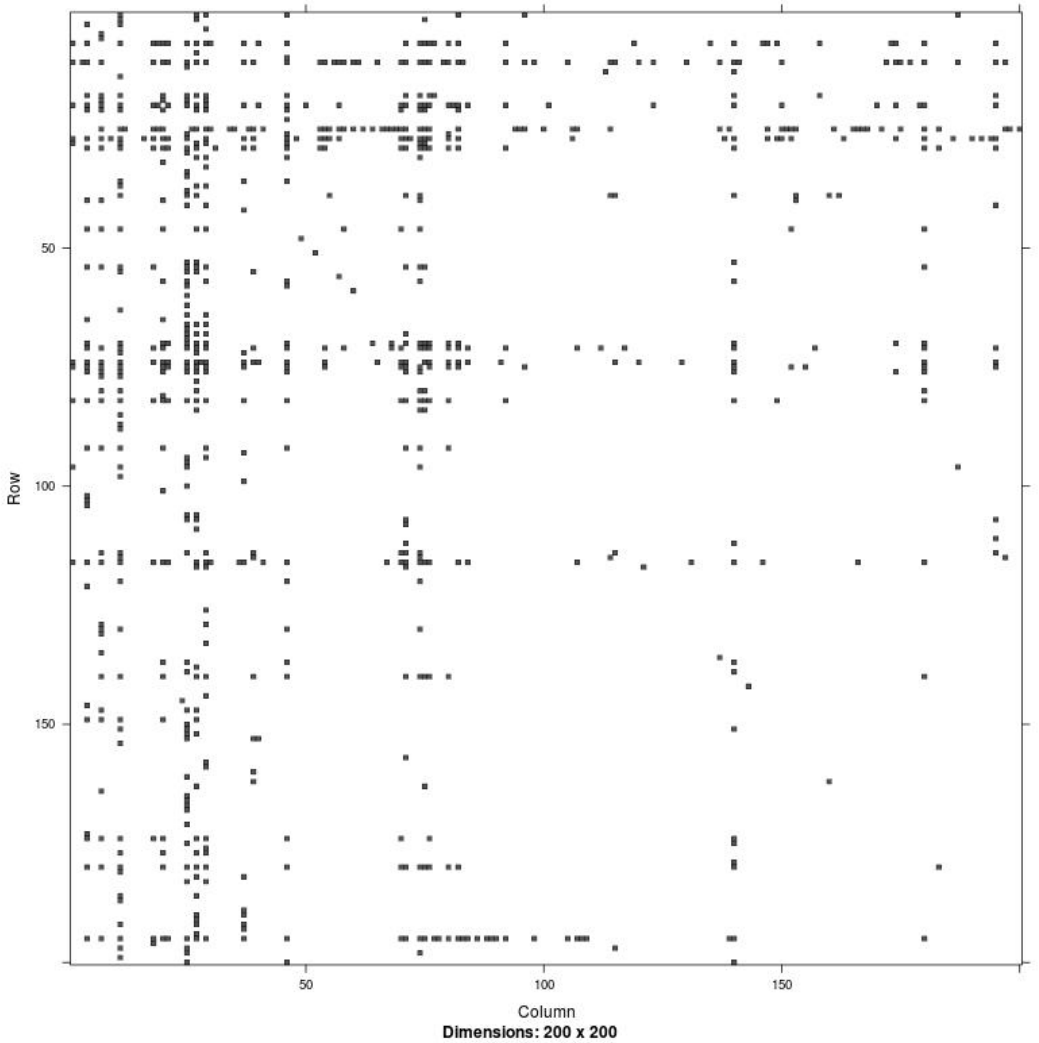
project into a subspace *guaranteed* to contain the most important nodes

Demo

bitcoin transaction network (directed)

Ivan Brugere (UIC!!)

http://compbio.cs.uic.edu/data/bitcoin/



6.3 M x 6.3 M adjacency matrix

16.3 M nodes

Compute top 5 hub centralities

```
> load("bitcoin_from_to_graph.rdata")
> t1 <- proc.time()
> x  <- topm(B,q=2,tol=0.1,m_b=5)

> proc.time() - t1
   user  system elapsed
 86.970  24.350 111.605
```

Small problem comparison

Leading 1000 x 1000 submatrix of bitcoin data

```
> t1 <- proc.time()
> ex <- diag(expm(X) + expm(-X))/2
> proc.time() - t1

   user  system elapsed
151.080   0.220 151.552

> i <- order(ex,decreasing=TRUE)

> i[1:5]

[1] 11 25 27 29 74
```

```
> t1  <- proc.time()
> top <-topm(X,type="cent")
> proc.time() - t1

 user  system elapsed
0.555   0.010   0.565



> top$hubs

[1] 11 25 27 29 74
```

Plan

New IRL package will contain at least

- **lRLBA (truncated SVD)**
- **lRBLB (windowed truncated SVD)**
- **lRBLeigs (symmetric variant)***
- **topm (network bounds)**
- **others...**

Succeeds current lRLBA package

Some new research in process...

https://github.com/bwlewis/IRL

These slides are available here:

http://goo.gl/ZbJEUO