Portfolio Optimization: Utility Functions, Computational Methods, and an Application to Equities

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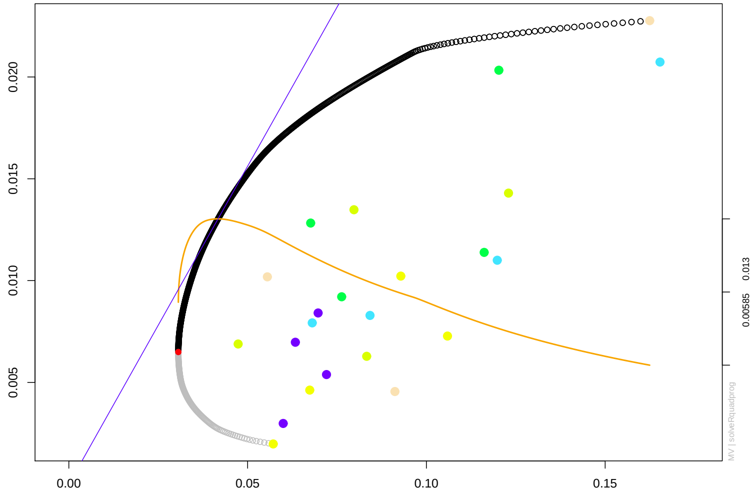
University of Michigan students' "Value 40 Fund" vs. SPDR S&P 500

	FUND	SPY	Spread
2013	62%	32%	30%
2012	35%	16%	19%
2011	-17%	2%	-19%
2010	46%	15%	31%
2009	111%	26%	84%
2008	-45%	-37%	-8%
2007	21%	5%	16%
2006	55%	16%	39%
2005	35%	5%	30%
2004**	9%	3%	6%
	with dividends reinvested		
	** starts in Dec. '04		

Subsets of stocks in the Value 40 Fund in May 2014

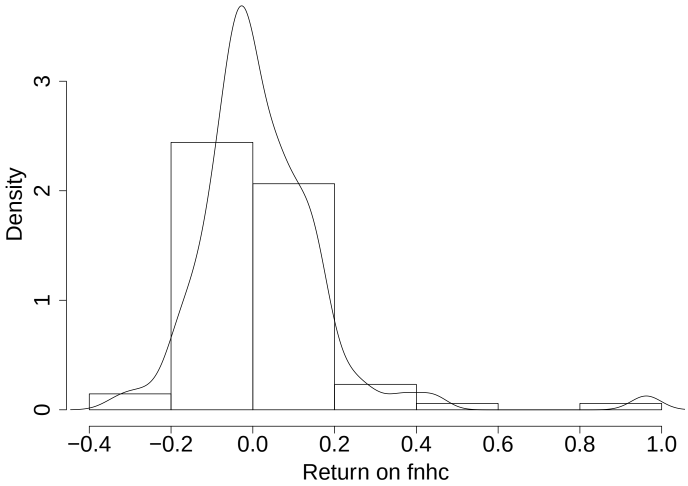
- Yahoo Finance has data on adjusted closing prices from December 1999 through April 2014 for 27 of the 40.
- Real after-tax returns are positive over that period for 23 of the 27.
- On Markowitz's mean-variance efficient frontier for a portfolio of risky assets with a long-only constraint, implemented by **fPortfolio**, we find that
  - the minimum variance portfolio gives positive weights to 16 of the 23
  - the tangency portfolio gives positive weights to 14 of the 23
- Anderson-Darling normality tests, as implemented by **nortest**, reject the Gaussian hypothesis for 18 of the 23.

Markowitz portfolio frontiers



Distribution of returns for Federated National Holding Co.

Right-skewed and leptokurtic



Preferences over higher moments

Prudence: preference for higher third moment (skewness)
Temperance: preference for lower fourth moment (kurtosis)
Edginess: preference for higher fifth moment

"Prudence, temperance, edginess, and, more generally, risk apportionment of any degree are the consequences of the natural idea that the sensitivity to detrimental changes should decrease with initial wealth" (Denuit and Rey 2010)

Utility functions

Boundedness favored by Arrow (1965) and Samuelson (1977).

Sensitivity to detrimental changes decreases with wealth if derivatives alternate in sign: $+ -$, $+$, $-$, ...

A simple bounded utility function whose derivatives with respect to wealth w alternate in sign:

$$U(w) = \frac{w}{c+w}, \quad c > 0$$

$$U'(w) > 0, \;\; U''(w) < 0, \;\; U'''(w) > 0, \;\; U''''(w) < 0, \ldots$$

Utility as a function of gross return R given initial wealth w_0:

$$U = \frac{w_0 R}{c + w_0 R} = \frac{R}{c/w_0 + R}$$

Maximizing expected utility

- Gross return R on portfolio is a function of asset weights
- Utility is a function of R and thus asset weights
- Expected utility depends on the distribution of future returns, approximated by the distribution of past returns
- Expected utility is maximized by choosing asset weights, using a differential evolution algorithm (Hagströmer and Binner 2009) implemented in **DEoptim**.
- Result when, for example, $c/w_0 = 1/2$:

stock	ppc	fnhc	wrb	aet	vsec
weight	.2151	.2717	.1028	.0547	.3557

References

Kenneth J. Arrow. *Aspects of Risk-Bearing*. Yrhö Jahnssonin Säätiö, Helsinki, 1965.

Michel Denuit and Beatrice Rey. Prudence, temperance, edginess, and risk apportionment as decreasing sensitivity to detrimental changes. *Mathematical Social Sciences*, 60(2):137–43, September 2010.

Björn Hagströmer and Jane M. Binner. Stock portfolio selection with full-scale optimization and differential evolution. *Applied Financial Economics*, 19(19–21):1559–71, October–November 2009.

Paul A. Samuelson. St. Petersburg paradoxes: Defanged, dissected, and historically described. *Journal of Economic Literature*, 15(1):24–55, March 1977.