fractional view

Your Analytics Department

Ostfalia
Hochschule für angewandte Wissenschaften

Package ILMTS

Simulate & Detect Intermediate and Long Memory Processes

R in Finance, May 16/17, 2014

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Fractional Brownian Motion

Cauchy-Formula:

$$\underbrace{\int_0^t \cdots \int_0^{s''}}_{\text{n times}} \left[\int_0^{s'} f(s)ds \right] ds' = \frac{1}{\Gamma(n)} \int_0^t (t-s)^{n-1} f(s)ds$$

theoccasionalinformationist.com

Fractional Brownian Motion:

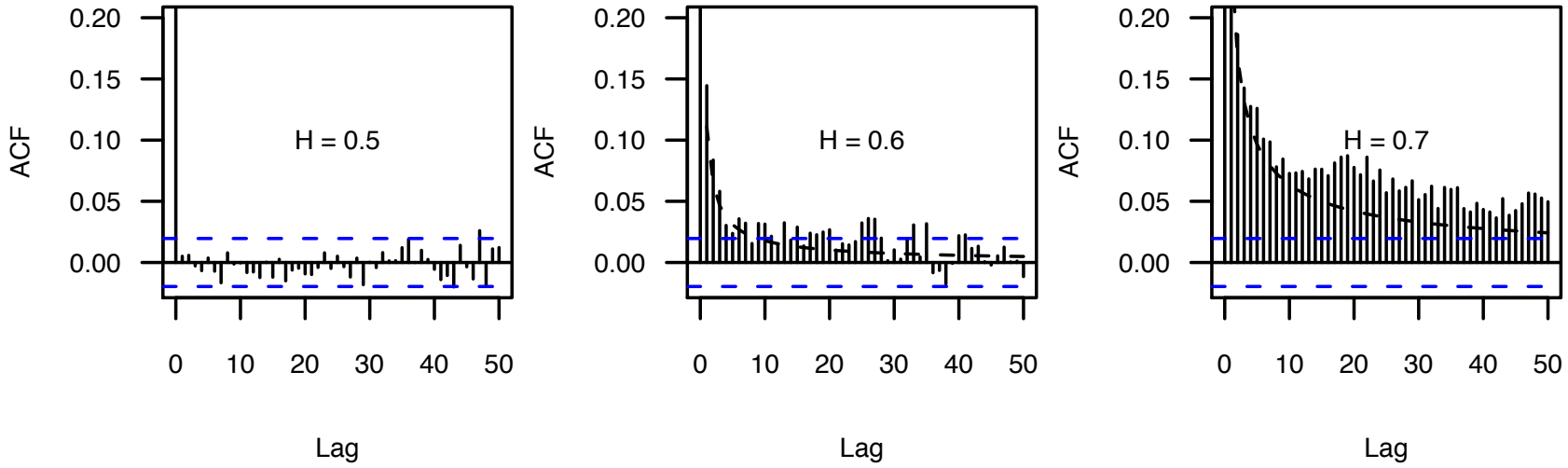
$$B_H(t) = C \int_0^t (t-s)^{H-1/2} dB(s)$$

Hurst Exponent

Brownian Motion

Long Memory: H > 1/2

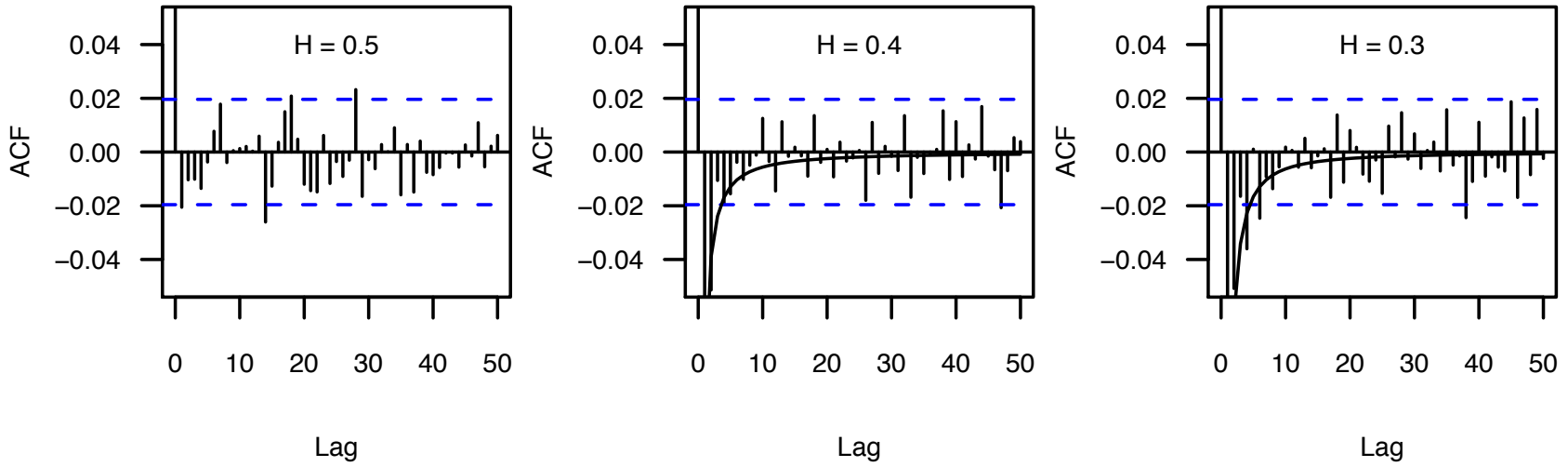
ACF, large k $\rho(k) \sim H(H-1/2)|k|^{2(H-1)}$



- Autocorrelations decay very slowly
- Autocorrelations are **not** summable: $\sum_{0}^{\infty} \rho(k) = \infty$

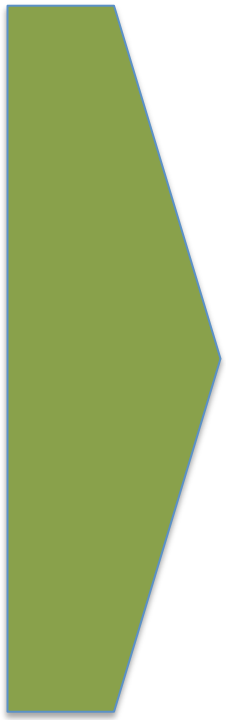
Intermediate Memory: H < 1/2

ACF, large k $\rho(k) \sim H(H-1/2)|k|^{2(H-1)}$



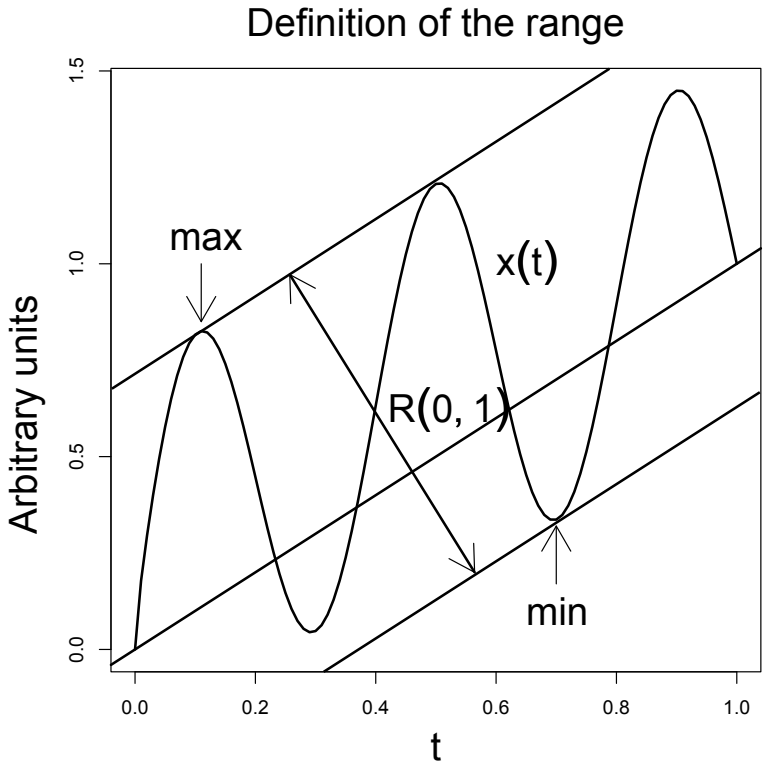
- Autocorrelations decay very slowly
- Autocorrelations **are** summable: $\sum_{0}^{\infty} \rho(k) < \infty$

Opportunities



- Trading Opportunites
- Risk Management
- Funds Management
- …

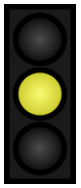
Estimation & Testing: Range / Standard Deviation



Concept to determine H:

Regression of log Q_k gives H.

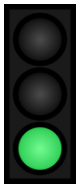
$$Q_k = R_k / S_k \sim k^H$$



Concept for formal test:

For H = 1/2, test statistics has Feller Distribution and Expectation:

$$\mathrm{E}\left[\frac{Q_k}{\sqrt{k}}\right] = \sqrt{\frac{\pi}{2}}$$



Estimation & Testing: Range / Standard Deviation

What can go wrong?

$$\frac{R}{S}$$

Range

- Outliers
- Structural Breaks

Standard Deviation

- (short range) Autocorrelation
- Non-Stationarity

Estimation & Testing: GPH-Method

Spectral Density (Fourier Transform of Autocorrelations)

$$f(\lambda) = \frac{\sigma^2}{2\pi} \sum_{k=-\infty}^{\infty} \rho(k) \mathrm{e}^{ik\lambda}$$

Very small λ:

$$f(\lambda) \sim \frac{\sigma^2}{2\pi} \lambda^{1-2H}$$

Geweke & Porter Hudak:

$$\log f(\lambda) \approx (1 - 2H) * \log(\lambda) + \dots$$

Replace with periodogram

Estimation & Testing: GPH

What can go wrong?

- Choice of Frequencies
- Short Range Autocorrelations
- Efficiency

ILMTS: Methods

- Estimation: Maximum Likelihood
  - Exact: Cholesky Decomposition, Durbin-Levinson, State Space
  - Approximate: Haslett-Raftery, Whittle Estimation, …
- Estimation: Other Methods
  - Scaled Windowed Variance
  - Detrended Fluctuation Analysis
  - R/S Method and Extensions
  - GPH Method and Extensions
- Simulation
  - Cholesky Decomposition
  - Fractional Brownian Motion/ Fractional Gaussian Noise
  - Fast Fourier Transform (Davies & Harte)
  - Aggregation

Contact

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