Data Mining with Markowitz Portfolio Optimization in Higher Dimensions

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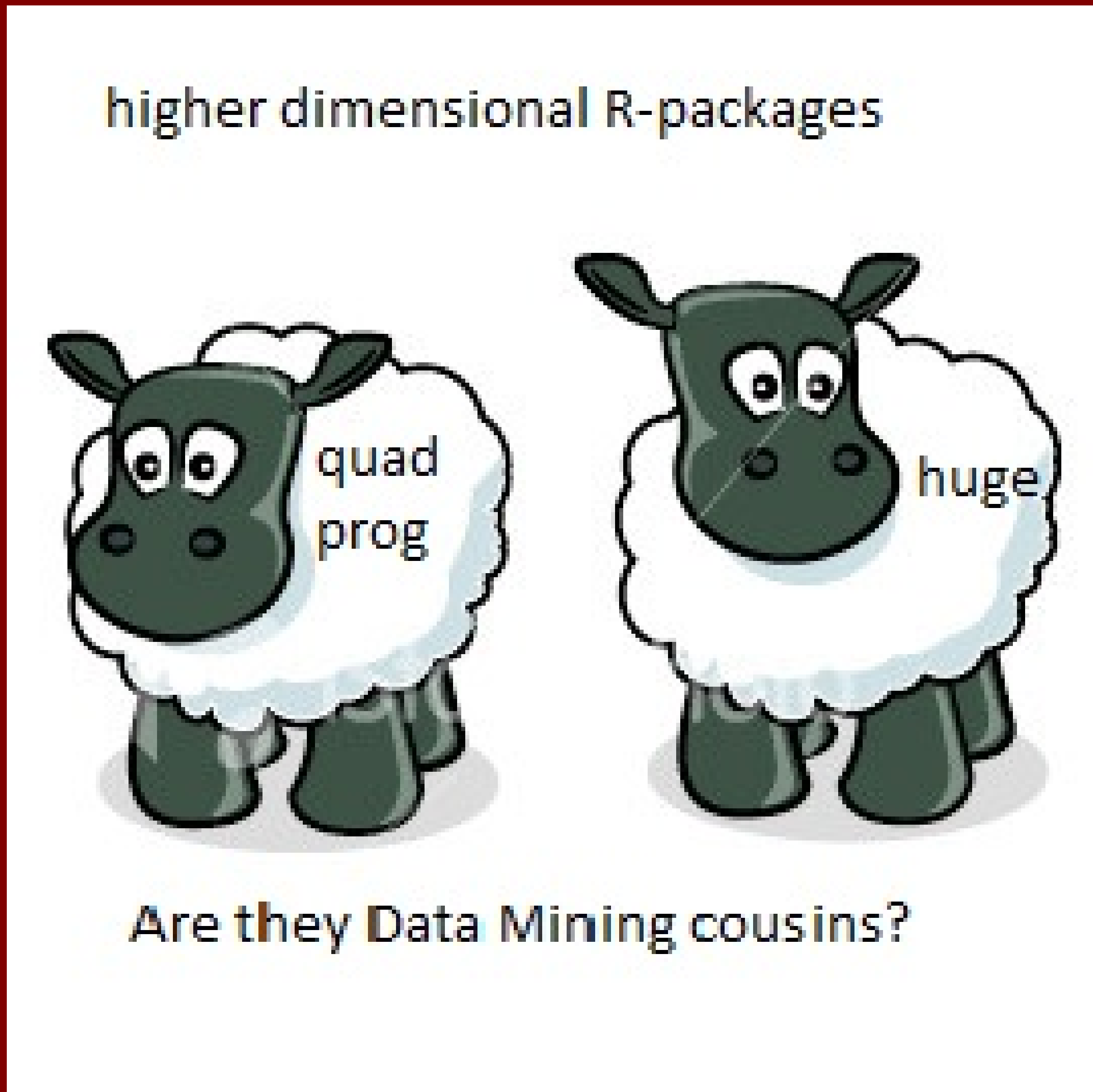
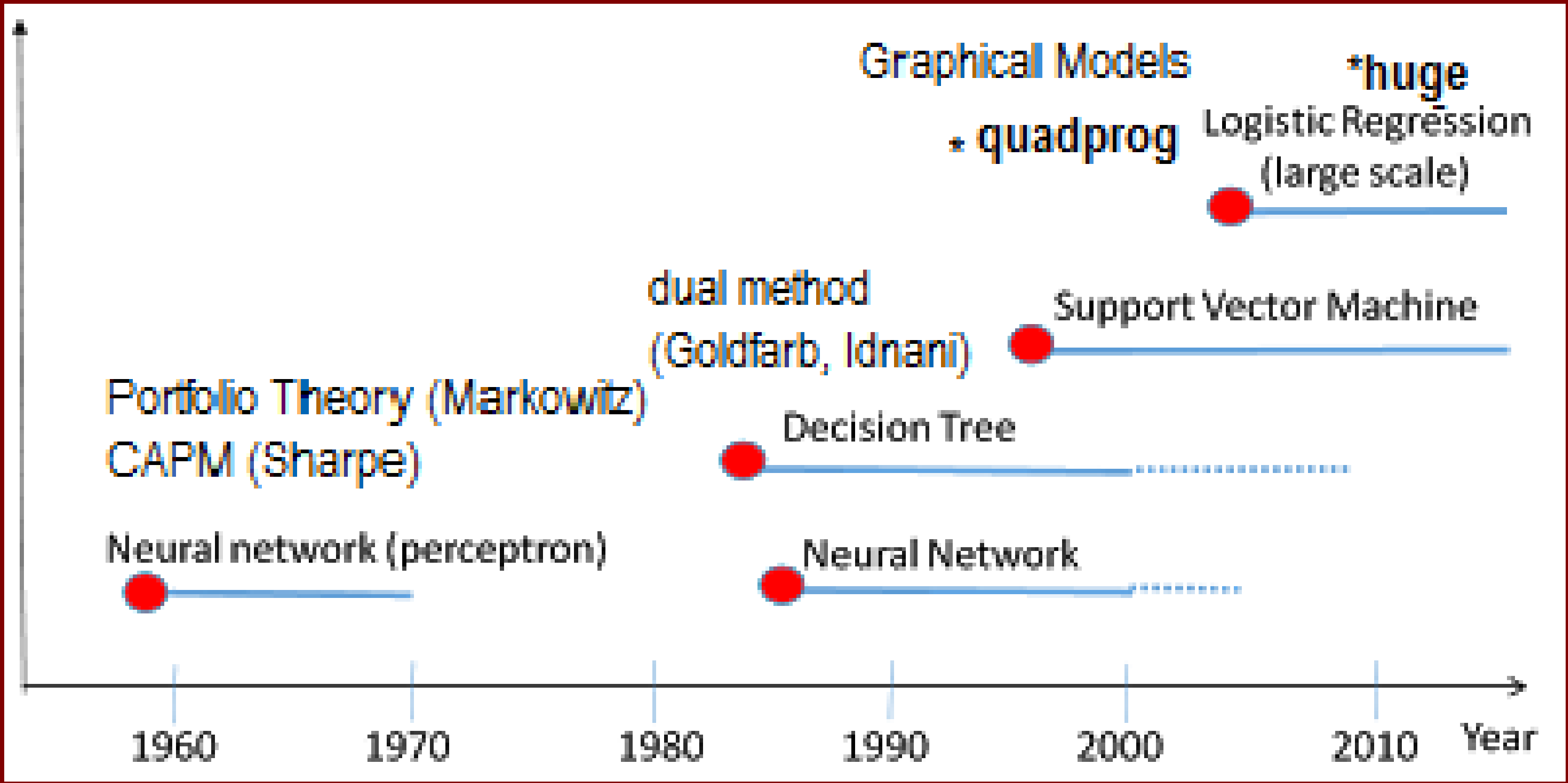
R in Finance, May, 2014


higher dimensional R-packages
quad
prog
huge
Are they Data Mining cousins?

Historical Timeline



Why Are These Techniques Important?

- Portfolio Theory and Graphical Models are Natural Tools for Market Uncertainty, Portfolio Complexity

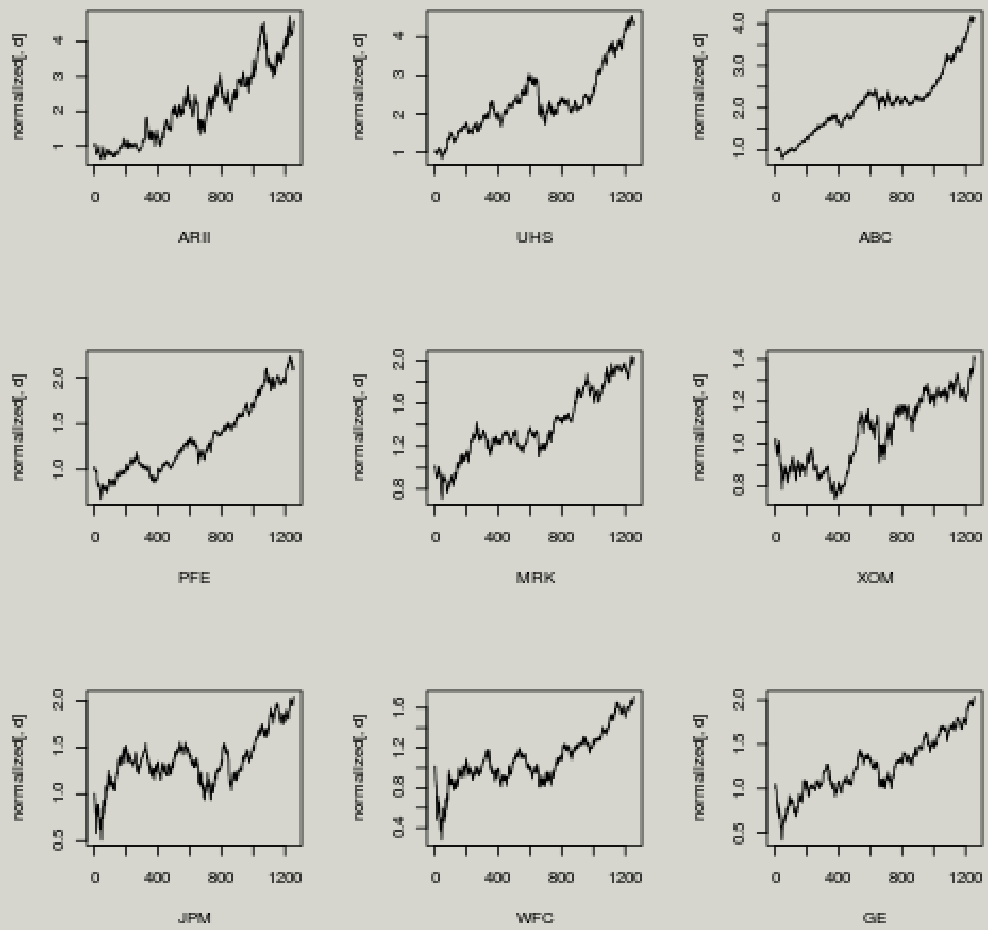
- Tie together Machine Learning and Financial Analytics: looking for Comovement in Multivariate Gaussian distributed returns

- Given dataset: N Training vectors X p Dimensions

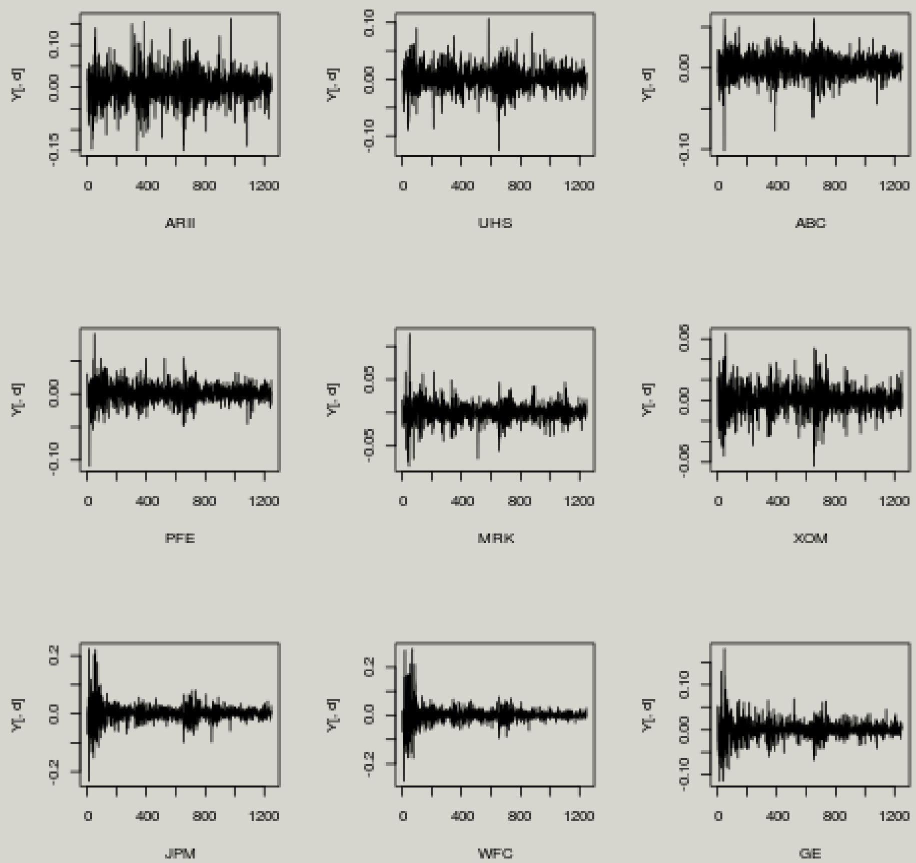
- Perform data mining where $N \approx p$

- Inspect and clean dataset: split-adjust

Historical $N + 1 = 1255$ prices for $p = 9$ securities



Historical $N = 1254$ log returns for $p = 9$ securities



Environment:

When we have p time series of stock prices: S_{ij} at time i for security j, log returns are $R_{ij} = ln(S_{ij}/S_{i-1j})$.

The full matrix of log returns is: $\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1p} \\ R_{21} & R_{22} & \dots & R_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{Np} \end{bmatrix}$

We can find: $E\{\mathbf{R}\} = \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_p \end{bmatrix}$ and $cov(\mathbf{R}) = \Sigma_{p \times p}$

Portfolio Optimization:

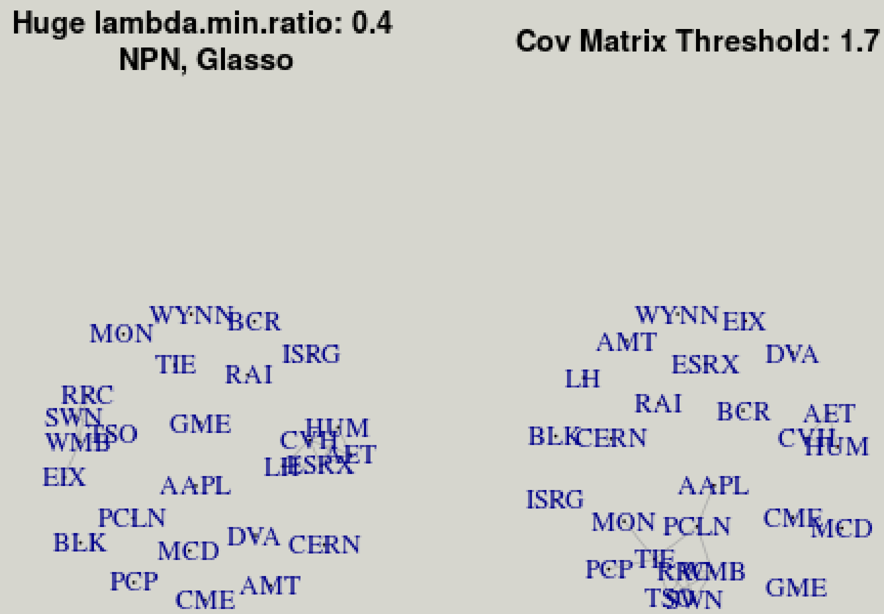
Find $\boldsymbol{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_p \end{bmatrix}$ the *optimal weights* for the p securities given historical series of $N \times p$ prices in order to forecast the best portfolio going forward:

$$\underset{w}{\text{argmin}}\ w^T \Sigma w, \qquad (1)$$

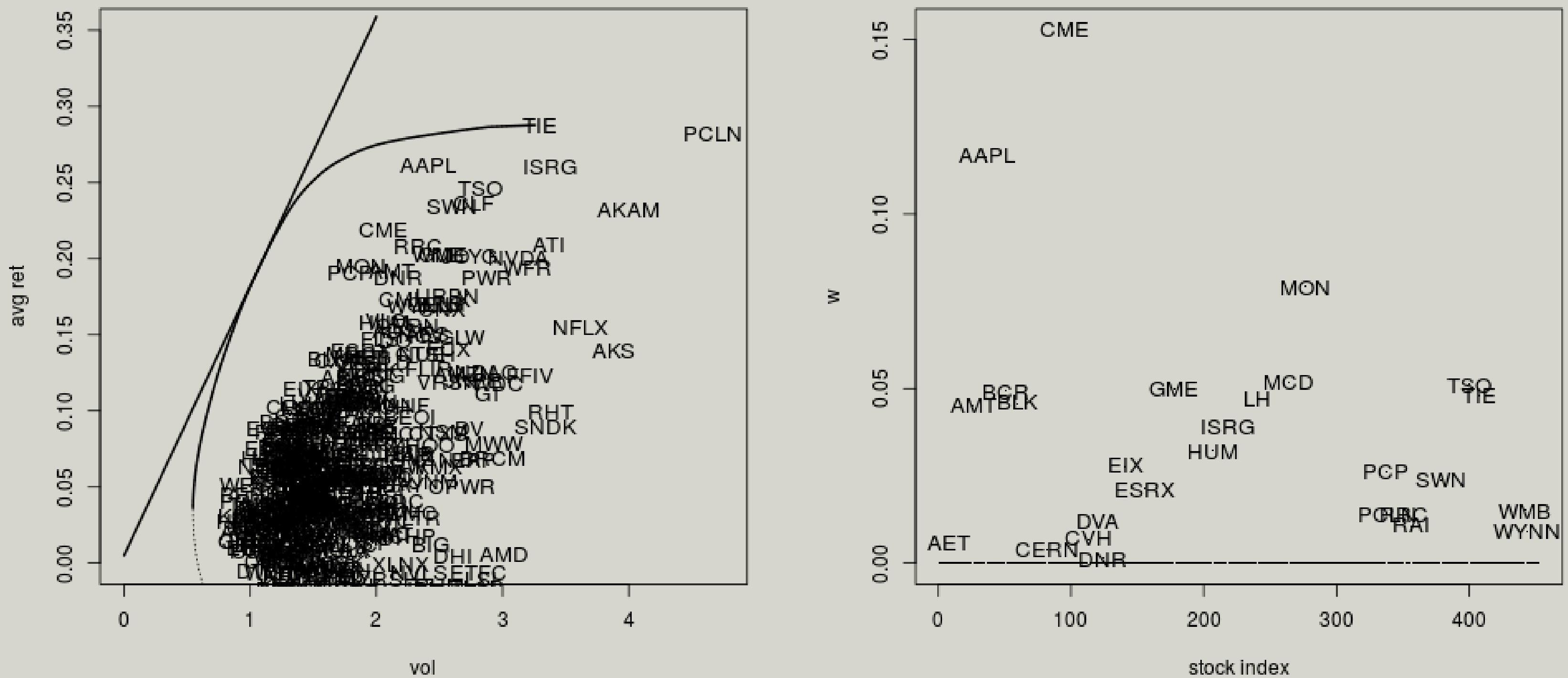
for each target level of return μ_P where $w^T \Sigma w$ is the return variance of the portfolio, weighted by the vector w where below are the weight sum, portfolio mean, and no short sales constraints:

$$\begin{pmatrix} 1^T w \\ \mu^T w \end{pmatrix} = \begin{pmatrix} 1 \\ \mu_P \end{pmatrix}; w \geq 0 \qquad (2)$$

Comovement Clusters for Weighted Stocks $p = 452$ securities



Result Portfolio for $p = 452$ securities: Favoring CME, AAPL



Thank you!

References:

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- Zhao, T., Liu, H., Roeder, K., Lafferty, J., Wasserman, L., The huge Package for High-dimensional Undirected Graph Estimation in R, Journal of Machine Learning Research 13 (2012) 1059-1062, April 2012.