The imprecision of volatility indexes

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Volatility indexes

- The volatility index (VIX) is an implied volatility estimate that measures the market's expectation of future volatility.
- VIX has gained significant importance in the recent past due to its applications. These include:
  1. Volatility forecasting: applications in option pricing, value at risk etc.
  2. Predicting the direction of the market due to the negative correlation with market returns.
  3. Hedging tool: using VIX futures and options.

VIX is imprecise!

Example: Vega VIX

In our sample, the size of the 95% confidence band for Vega VIX (VVIX) is 2.9 precentage points in the median case.

Concern about imprecision in a VIX estimator arises due to aggregation of imprecise implied volatilities (IVs). Latane and Rendleman, 1976; Hentschel, 2003; Jiang and Tian, 2007

Consequences of imprecision

1. Imprecise option prices.
   - For example, a 6100 OTM call option on the Nifty index at 5464.75 with 29 days to expiry is priced at Rs. 1.92 when employing a VVIX of 17.82 %.
   - The 95% CI for VVIX ranges from 16.03% to 19.91% $\implies$ the option's price may lie between Rs. 0.89 and Rs. 3.86.
2. Imprecise VaR and portfolios based on it.
3. Difficulty with pricing derivatives on a fuzzy underlying.

Summary

- Estimate the imprecision of model based volatility indexes.
- Bootstrapping to estimate the imprecision in a VIX estimator.
- The σ and confidence bands are computed to measure this imprecision.
- For a cross-section of SPX options with 29 and 64 days to expiry, the VVIX estimate is 21.53% with 95% confidence limits of 20.8% to 22.32%.
- Similarly, for Nifty options with 29 and 57 days to expiry, the VVIX estimate is 17.82% with confidence limits of 16.03% to 19.91%.
- Imprecision indicators are used for model selection: vega, liquidity, and elasticity weighted VIXs.
- VVIX has the lowest imprecision with a median confidence interval width of 2.9pp.

Outline

- Concerns about measurement
- Measuring the imprecision in a VIX
- Two empirical examples
- Using this measure of imprecision for model selection
- Conclusion

Concerns about measurement

Two approaches to measurement

- Model based approach - uses option pricing model - VXO, Vega VIX etc.
  - Measurement errors in prices - imprecise IVs (Hentschel, 2003)
  - Hentschel (2003) derives CI's from B-S formula.
  - For an ATM stock option with 20 days to expiry, the 95% CIs are of the order +/- 6 pp.
  - For VXO, the 95% CIs are of the order +/- 25 bps.
- Model free approach - pricing of variance swap - CBOE VIX
  - Methodological errors (Jiang and Tian, 2005)
  - Imprecise intra-day VIX due to varying strike range (Andersen et al., 2011)

Measuring the imprecision in a volatility index

Our approach to the problem

- Non-parametric methodology; contrast with Hentschel (2003).
- Model based; contrast with model free.
- Agnostic about the distribution of errors.
- Each option price is an imprecise transformation of the true implied volatility index.
- Bootstrapping to estimate the imprecision in the VIX estimator.

An example: Vega weighted VIX

The VVIX is computed from all option prices as follows:

1. Estimation of IVs using the Black-Scholes model for the two nearest maturities.
2. Computation of the average weighted IV for each maturity i:

$$IV_i = \frac{\sum_{j=1}^{n} w_{ij} IV_{ij}}{\sum_{j=1}^{n} w_{ij}}$$

   where, IV_{ij} refers to a vector of IVs for $j = \{1 \ldots n\}$ and two nearest maturities, $i = \{near, next\}$, w_{ij} refers to the vega weight for the corresponding IV_{ij}.
3. The vega weighted average IVs are interpolated to compute the 30 day expected volatility, VVIX.

Bootstrap inference: The case of LIBOR

- Calculation of VIX involves aggregation of several imprecise estimates.
- Estimation of LIBOR poses a similar challenge:
  - The true price on the OTC market is unobserved.
  - Multiple noisy estimates from polled dealers.
  - Aggregation into a bootstrapped, robust, and precise estimate (Cita and Lien, 1992; Berkowitz, 1999; Shah, 2000).
- The parallel with LIBOR suggests a bootstrap inference approach for VVIX.

Steps involved

1. At each maturity, we sample with replacement among all observed option prices to construct a bootstrap replicate.
2. Thousands of times, we obtain an estimate of VVIX at each maturity and thus an overall VVIX estimate.
3. The standard deviation (σ) and confidence bands are computed from the bootstrapped sampling distribution of VVIX using the adjusted bootstrap percentile method (Efron, 1987).

Data description

- S&P 500 index (SPX) options end-of-day data.
- The data is available for the months of Sep, Oct, and Nov 2010.
- Nifty options tick-by-tick data ($\sim$ 200K obs. per day):
- The data is available from Feb, 2009 to Sep, 2010.
- Each dataset includes:
  - Transaction date
  - Expiry date of the options contract
  - Strike price
  - Type of the option i.e. call or put
  - Price of the underlying index
  - Best buy price and ask price of option
- The one and three month MIBOR rates provided by NSE as the riskfree rates.
- The one and three month US Treasury bill rates provided by the US department of the Treasury as the riskfree rates.

Sampling procedure

- We follow Andersen et al. (2011) and sample options as follows:
  1. Construct fifteen seconds series for each individual option using the *previous tick method* from tick-by-tick data.
  2. Retain the last available quotes prior to the end of each fifteen second interval throughout the trading day.
  3. If no new quote arrives in a fifteen second interval, the last available quote prior to the interval is retained.
  4. If no quote is available in the previous interval, the last available quote from the last five minutes is retained.
  5. Filter out options with zero traded volume (optional).
- For robustness check, sampling frequencies of thirty and sixty seconds are also used.

Two empirical examples

Intuition

- We use a sample of near-the-money SPX options.

Strike	Type	Underlying	Mid-Quote	Maturity (Days)	Risk-free (%)	IVol (%)
5500	c	5464.75	72.57	29	6.29	12.43
5600	c	5464.75	30.55	29	6.29	11.57
5400	c	5464.75	133.95	29	6.29	13.11
5600	p	5464.75	160.65	29	6.29	15.75
5400	p	5464.75	68.75	29	6.29	17.80
5500	p	5464.75	105.30	29	6.29	16.48

Note: We define near-the-money-options as call and put options with strike-to-spot ratio between 0.97 and 1.03 (Pan Poteshman, 2006).

- 95% CI of sample mean: [12.53, 16.52]

A sample of SPX options

```
> library(ifrogs)
> set.seed(101)
> data(vix_spx)
> str(vix_spx)

List of 2
 $ opt_near:'data.frame': 239 obs. of  7 variables:
  ..$ maturity  : num [1:239] 0.0795 0.0795 0.0795 0.0795 0.0795 ...
  ..$ riskfree  : num [1:239] 0.0012 0.0012 0.0012 0.0012 0.0012 0.0012 0.0012 0
  ..$ type      : chr [1:239] "p" "p" "p" "p" ...
  ..$ strike    : num [1:239] 675 680 690 700 710 715 720 725 730 740 ...
  ..$ underlying: num [1:239] 1126 1126 1126 1126 1126 ...
  ..$ bid       : num [1:239] 0.05 0.05 0.05 0.1 0.1 0.1 0.1 0.1 0.1 0.1 ...
  ..$ ask       : num [1:239] 0.1 0.1 0.1 0.15 0.15 0.15 0.15 0.15 0.15 0.2 ...
 $ opt_next:'data.frame': 242 obs. of  7 variables:
  ..$ maturity  : num [1:242] 0.175 0.175 0.175 0.175 0.175 ...
  ..$ riskfree  : num [1:242] 0.0016 0.0016 0.0016 0.0016 0.0016 0.0016 0.0016 0
  ..$ type      : chr [1:242] "p" "p" "p" "p" ...
  ..$ strike    : num [1:242] 600 620 625 630 640 650 660 670 675 680 ...
  ..$ underlying: num [1:242] 1126 1126 1126 1126 1126 ...
  ..$ bid       : num [1:242] 0.05 0.1 0.1 0.1 0.15 0.2 0.1 0.1 0.2 0.05 ...
  ..$ ask       : num [1:242] 0.2 0.25 0.25 0.35 0.35 0.3 0.45 0.5 0.5 0.55 ...
```

Preparing the options dataset

```
> spx_near <- prep_maturity(maturity=vix_spx$opt_near$maturity[[1]],
+                            riskfree=vix_spx$opt_near$riskfree[[1]],
+                            carry=vix_spx$opt_near$riskfree[[1]],
+                            type=vix_spx$opt_near$type,
+                            strike=vix_spx$opt_near$strike,
+                            underlying=vix_spx$opt_near$underlying,
+                            schemes="vega",
+                            bid=vix_spx$opt_near$bid,
+                            ask=vix_spx$opt_near$ask,
+                            tv_filter=FALSE)
```

```
> str(spx_near)

List of 3
 $ maturity: num 0.0795
 $ schemes : chr "vega"
 $ out     :'data.frame': 186 obs. of  11 variables:
  ..$ maturity  : num [1:186] 0.0795 0.0795 0.0795 0.0795 0.0795 ...
  ..$ riskfree  : num [1:186] 0.0012 0.0012 0.0012 0.0012 0.0012 0.0012 0.0012 0
  ..$ carry     : num [1:186] 0.0012 0.0012 0.0012 0.0012 0.0012 0.0012 0.0012 0
  ..$ type      : Factor w/ 2 levels "c","p": 1 1 1 1 1 1 1 1 1 1 ...
  ..$ strike    : num [1:186] 965 970 975 980 985 ...
  ..$ underlying: num [1:186] 1126 1126 1126 1126 1126 ...
  ..$ bid       : num [1:186] 158 153 148 143 138 ...
  ..$ ask       : num [1:186] 163 158 153 149 144 ...
  ..$ value     : num [1:186] 161 156 151 146 141 ...
  ..$ iv        : num [1:186] 0.18 0.219 0.212 0.223 0.216 ...
  ..$ vega      : num [1:186] 1.18 6.36 6.51 10.35 10.61 ...
```

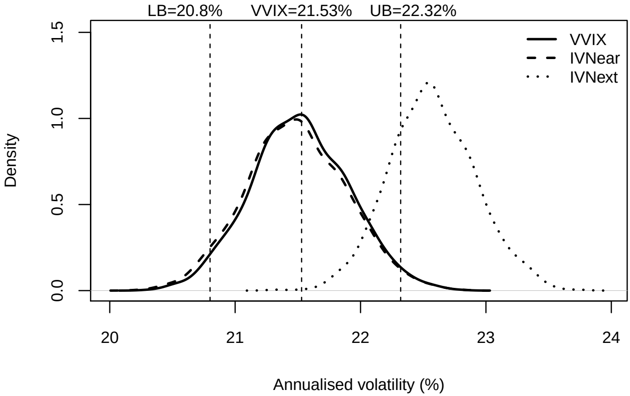
```
> spx_next <- prep_maturity(maturity=vix_spx$opt_next$maturity[[1]],
+                           riskfree=vix_spx$opt_next$riskfree[[1]],
+                           carry=vix_spx$opt_next$riskfree[[1]],
+                           type=vix_spx$opt_next$type,
+                           strike=vix_spx$opt_next$strike,
+                           underlying=vix_spx$opt_next$underlying,
+                           schemes="vega",
+                           bid=vix_spx$opt_next$bid,
+                           ask=vix_spx$opt_next$ask,
+                           tv_filter=FALSE)
```

Estimation of confidence band

```
> spx_ci <- vix_ci(prep_near=spx_near,
+                  prep_next=spx_next,
+                  n_samples=1e3, conf=0.95,
+                  verbose=TRUE)
> str(spx_ci)
```

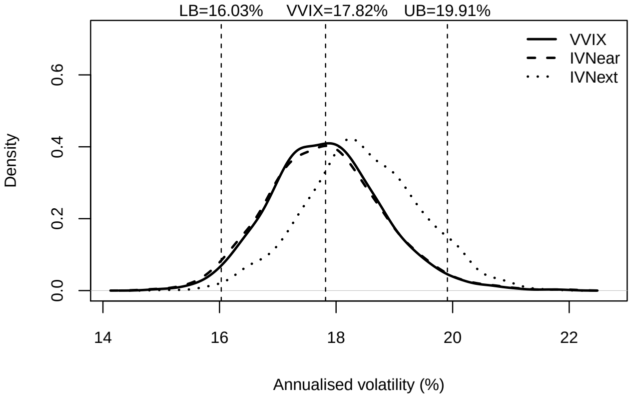
```
List of 3
 $ point  : Named num 21.5
  ..- attr(*, "names")= chr "vega"
 $ ci     : num [1:2, 1] 20.8 22.3
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : chr [1:2] "lower" "upper"
  .. ..$ : chr "vega"
 $ samples: num [1:1000, 1] 21.1 21.5 21.6 22.1 21.8 ...
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : NULL
  .. ..$ : chr "vega"
```

The distribution of VVIX on 2010-09-17: SPX



The one-day change in VVIX is smaller than 1.5pp on 62% of the days.

The distribution of VVIX on 2010-09-01: Nifty



The one-day change in VVIX is smaller than 4pp on 92% of the days.

Imprecision of VVIX over a large sample of Nifty options

- The imprecision indicators are computed from Feb 2009 to Sep 2010.
- The median CI for VVIX is 2.9pp which is an economically significant one.
- This is larger than the one-day change in VVIX of 1.18pp.

Using this measure of imprecision for model selection

Benchmarking performance of VIXs

- Alternatives to Vega: elasticity, liquidity etc. (Grover & Thomas, 2012).
- Precision is desirable in an estimator.
- Smaller σ and confidence interval $\implies$ higher precision.

Methodology

- Competitors:
  - Vega weighted VIX: VVIX
  - Liquidity weighted VIX: SVIX, TVVIX
  - Elasticity weighted VIX: EVIX
- Period of analysis: February 2009 - September 2010. Four snapshots a day.
- Sampling frequency: 15, 30, and 60 seconds.
- Performance indicators: σ and width of CI.
- Significant test: Pair wise Wilcoxon signed rank test.
- Results: VVIX has the highest precision with median CI width of 2.902 pp and σ of 0.733 pp.
- Presented results are for 15 seconds. The results are robust to the sampling frequency.

Summary statistics

	SVIX	TVVIX	VVIX	EVIX
Size of confidence band (pp)				
Min	0.929	1.362	1.033	2.177
1st Qu	2.713	2.743	2.271	6.201
Median	3.546	3.418	2.923	7.368
Mean	4.542	4.024	3.907	8.245
3rd Qu	4.845	4.440	4.064	9.262
Max	52.940	23.790	50.490	51.080
Std Dev	3.803	2.109	3.636	3.926
σ of the bootstrap estimates (pp)				
Min	0.239	0.344	0.255	0.571
1st Qu	0.706	0.706	0.581	1.576
Median	0.913	0.877	0.739	1.868
Mean	1.139	1.028	0.945	2.053
3rd Qu	1.252	1.141	1.025	2.324
Max	13.390	4.772	11.780	10.580
Std Dev	0.822	0.530	0.754	0.875

Pairwise comparisons: Wilcoxon sign rank test

	Size of confidence band		σ of the bootstrap estimates	
	Median Diff	Pval	Median Diff	Pval
EVIX - SVIX	3.745	0.000	0.923	0.000
EVIX - TVVIX	3.846	0.000	0.962	0.000
EVIX - VVIX	4.326	0.000	1.097	0.000
SVIX - TVVIX	-0.004	1.000	0.013	0.341
SVIX - VVIX	0.641	0.000	0.183	0.000
TVVIX - VVIX	0.618	0.000	0.165	0.000

Ranking: VVIX, SVIX & TVVIX, EVIX

Reproducible research

R package ifrogs has been released into the public domain, with an open source implementation of the methods of this paper.

Conclusion

- Hentschel (2003) argues that imprecision in VIX is small.
- Our result disagrees substantially with his.
- These differences result from our use of a non-parametric methodology, free from any distributional assumption about errors, and a large sample of options.
- Also, we compute VIX derived from a wide range of strikes in contrast to VXO.
- We find that the imprecision for VIX is significant when estimated from SPX or Nifty options.
- We use the imprecision indicators for model selection and find VVIX to be the most precise estimator.

Thank you