Portfolio Optimization: Price Predictability, Utility Functions, Computational Methods, and Applications

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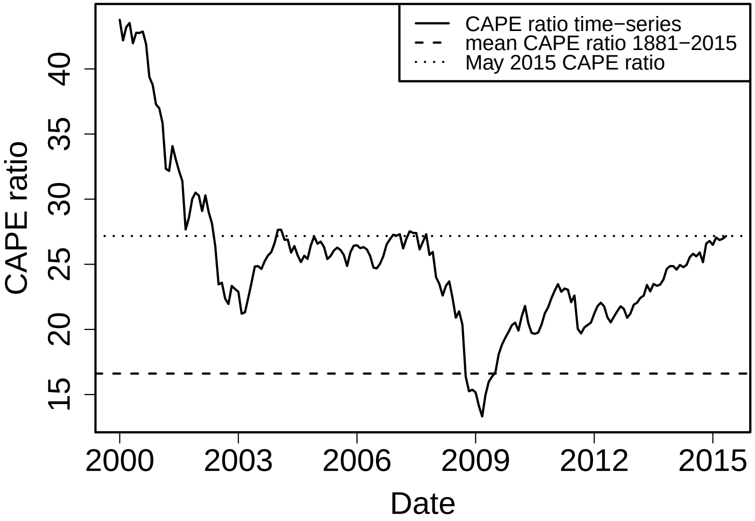
Wanted: a portfolio optimization method that

- exploits asset prices' predictable features, if they exist
- accommodates changes in the distribution of returns, if they occur
- makes normative use of expected utility theory
- utilizes global optimization methods
- is simple enough for practical application

Cyclically-adjusted price-earnings ratio (P/E10)



Sample selection

For the 25 assets of interest, consistent monthly data on real after-tax total returns start in January 2000. Should all these observations, or just a subset, be used to approximate the distribution of future returns?

The **Energy** package's eqdist.etest function rejects the equal distribution hypothesis for a pair of samples with *contrasting* initial CAPE ratios. It does *not* reject the hypothesis for a pair of samples with *similar* initial CAPE ratios.

A sample with an initial CAPE ratio similar to the current one starts in October 2001. It is used in the portfolio optimization process outlined below.

Desiderata for utility as a function of wealth, $U(w)$

Boundedness: Arrow (1965) and Samuelson (1977)
Derivatives alternating in sign: Eeckhoudt and Schlesinger (2006)

- $U^{(1)} > 0$, "non-satiation," prefer high mean
- $U^{(2)} < 0$, "risk aversion," prefer low variance
- $U^{(3)} > 0$, "prudence," prefer positive skewness
- $U^{(4)} < 0$, "temperance," prefer low kurtosis
- $U^{(5)} > 0$, "edginess," prefer high fifth central moment

Simple utility functions with the desired properties and useful implications for preferences on gross returns R

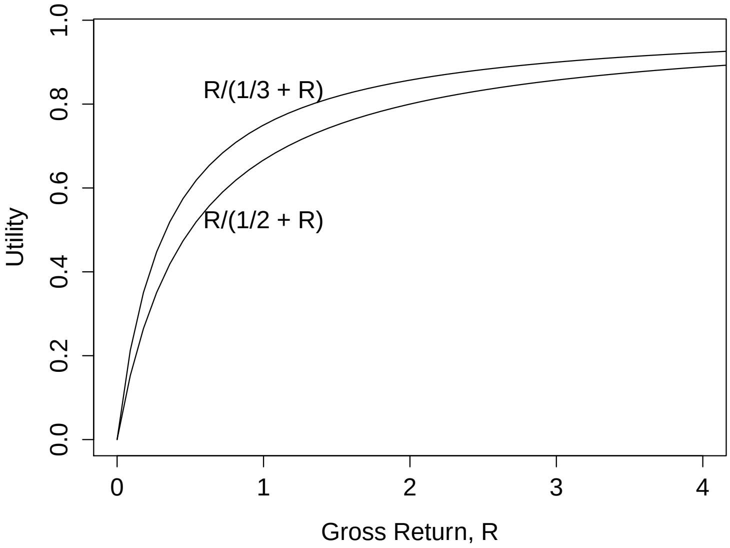
$U(w) = \frac{w}{c+w}$, where $c > 0$, $w \geq 0$.

It's easy to show that $0 \leq U \leq 1$ and that $U^{(1)} > 0$, $U^{(2)} < 0$, $U^{(3)} > 0$, $U^{(4)} < 0$, $U^{(5)} > 0$, etc.

Let $w = w_0 R$. Then $U(R) = \frac{w_o R}{c+w_0 R}$

If $w_0 > 0$, then $U(R) = \frac{R}{(c/w_0)+R}$

Utility functions for two values of c/w_0



Maximizing expected utility

- Gross return R on portfolio is a function of asset weights
- Utility is a function of R and thus asset weights
- Expected utility depends on the distribution of future returns, approximated by the distribution of past returns
- Expected utility is maximized by choosing asset weights, using a differential evolution algorithm (Hagströmer and Binner 2009) implemented in **DEoptim**.
- Result when, for example, $c/w_0 = 1/2$:

stock	ctb	acgl	wnc	merc
weight	.1047	.6593	.0876	.1484

References

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Louis Eeckhoudt and Harris Schlesinger. Putting risk in its proper place. *American Economic Review*, 96(1):289–89, March 2006.

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