Characteristic-based equity portfolios

Economic value and dynamic style-allocation

Kris Boudt[1] & Marjan Wauters[2] & David Ardia[3]

[1]Vrije Universiteit Brussel & VU Amsterdam

[2]KU Leuven

[3]Université Laval

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Motivation

- Increasing popularity of exchange-traded funds (ETFs) which track simple characteristic-based portfolios
- Characteristic-based portfolios
  - Market capitalization
    - World's largest ETF is SPDR S&P 500 with a value of $169bn (Nasdaq, August 1, 2014)
  - Alternative characteristic-based portfolios
    - Accelerating intrest: global value of funds increased from $58bn (2010) to $350bn (2014)

Motivation

- Characteristic-based portfolios as **mean–variance efficient portfolio proxies**
  - Market capitalization portfolio
    - Capital Asset Pricing Model
    - In practice: assumptions often violated
  - Alternative portfolios
    - Long run risk-adjusted outperformance
    - Life cycle specificity: diversification opportunity

Research questions

1. Is the mean–variance criterion a useful criterion to allocate across characteristic-based portfolios?
   - Timing gains: unlikely that one characteristic is the mean–variance efficient proxy
   - Model the mean–variance efficient weights as a time-varying linear combination of the characteristic-based portfolios
   - Implied loadings provide information on the expected individual performance of the proxies for each period
2. Is there a gain in switching from mean–variance allocation at the stock–level to mean–variance allocation across the characteristic-based portfolios?
   - Estimation error (out-of-sample)
   - Imposing constraints reduces the estimation risk and can improve the portfolio performance (e.g., Jagannathan and Ma, 2003)

Modelling framework

- Investment universe
  - Directly invest in the *N stocks* belonging to investment universe
    - $\mathbf{w}$ the $(N \times 1)$ vector of portfolio weights
  - Allocate budget across *K characteristic-based portfolios*
    - $\boldsymbol{\theta}$ the $(K \times 1)$ vector of portfolio weights
  - $K \ll N$

Single characteristic-based portfolios

In a portfolio based on characteristic k for a universe of N stocks, we set the portfolio weights at time t as:

$$x_{k,i,t} = \frac{\max\{0, z_{k,i,t}\}}{\sum_{i=1}^{N} \max\{0, z_{k,i,t}\}}$$

in which $z_{k,i,t}$ is the generic characteristic k of stock i at time t

- The considered characteristics are:
  - Market capitalization
  - Fundamental metrics of company size
  - Risk-based weighting: low volatility
  - S&P 500 inclusion dummy, i.e. equally-weighted portfolio

Portfolio construction

- Style allocation to characteristic-based portfolios
  - Mean–variance efficiency criterion

$$h_\gamma(\boldsymbol{\theta}_t; \tilde{\boldsymbol{\mu}}_{t|t-1}, \tilde{\boldsymbol{\Sigma}}_{t|t-1}) \doteq \boldsymbol{\theta}_t' \tilde{\boldsymbol{\mu}}_{t|t-1} - \frac{\gamma}{2} \boldsymbol{\theta}_t' \tilde{\boldsymbol{\Sigma}}_{t|t-1} \boldsymbol{\theta}_t$$

Efficiency of dynamic style portfolios

- Suppose
  - $\mathbf{w}_t^*$ is the true mean–variance optimal portfolio weight vector over $[t-1, t]$
  - $\mathbf{X}_{t-1}$ is the $N \times K$ matrix of characteristic-based portfolio weights
- In sample, by allocating over K characteristics instead of N stocks, we expect to be inefficient, except when there exists a $\boldsymbol{\theta}_t^*$ such that:
  - $\mathbf{w}_t^* = \mathbf{X}_{t-1}\boldsymbol{\theta}_t^*$
  - The true mean–variance portfolio is a linear combination of the characteristic-based portfolio weights

Efficiency of dynamic style portfolios

- Special case when the efficiency result arises
  - Mean–variance utility investor

$$\underset{\mathbf{w}}{\operatorname{argmax}}\, \mathbf{w}'\boldsymbol{\mu}_{t|t-1} - \frac{\gamma}{2}\mathbf{w}'\boldsymbol{\Sigma}_{t|t-1}\mathbf{w} \tag{1}$$

  - Stocks' return–generating process: multifactor model of Haugen and Baker (1996)

$$r_{i,t} = \sum_{k=1}^{K} \lambda_{k,t} x_{i,k,t-1} + \epsilon_{i,t}. \tag{2}$$

  - From FOC, it follows that:

$$\mathbf{w}_t^* \doteq \gamma^{-1}\boldsymbol{\Sigma}_{t|t-1}^{-1}\boldsymbol{\mu}_{t|t-1} \tag{3}$$

  - Under (2):

$$\boldsymbol{\mu}_{t|t-1} = E[\mathbf{r}_t|\mathbf{x}_{t-1}] = \mathbf{X}_{t-1}\boldsymbol{\lambda}_t \tag{4}$$

  - Thus

$$\mathbf{w}_t^* = \gamma^{-1}\boldsymbol{\Sigma}_{t|t-1}^{-1}\mathbf{X}_{t-1}\boldsymbol{\lambda}_t \tag{5}$$

Efficiency of dynamic style portfolios

- Assume homoskedasticity and no cross correlation across asset returns (Hjalmarsson and Manchev, 2012)
- Then:

$$\mathbf{w}_t^* = \mathbf{X}_{t-1}\boldsymbol{\theta}_t^*, \tag{6}$$

with $\boldsymbol{\theta}_t^* = \gamma^{-1}\boldsymbol{\sigma}_{t|t-1}^{-2}\boldsymbol{\lambda}_t$

Data and estimation

- Investment universe is restricted to S&P 500 stocks
- Evaluation period 1990–2012
- γ is set at 5 representing medium risk-aversion (Martellini & Ziemann, 2010)
- Monthly rebalancing frequency
- $\Sigma_{t|t-1}$ and $\tilde{\Sigma}_{t|t-1}$ are estimated over a 156 week rolling window
  - Exponentially weighted moving average (EWMA)

Data and estimation

- Market capitalization
- Fundamental value (cfr. Arnott et al.,2005)
  - Book value of common equity, dividends, net operating cash flow and revenues
- Risk-based weighting
  - $1/\sigma$
- Equally-weighted

Characteristic-based portfolios

- Long run outperformance of the alternative characteristic-based portfolios

	Mean (%)	Vol (%)	Sharpe	MDD (%)	Alpha	Turnover (%)
Market capitalization	8.70	16.84	0.29*	53.87	0.042	**0.68**
Fundamental value	9.94	17.73	0.34**	63.07	**0.119****	6.18
Volatility ($1/\sigma$)	9.91	**12.50**	**0.49****	**39.69**	0.174	15.67
Equally–weighted (1/N)	**10.58**	18.46	0.36**	60.34	**0.162***	7.00

* p-value < 0.10, ** p-value < 0.05, *** p-value < 0.01

Out-of-sample performance characteristics

1. Is the mean–variance criterion a useful criterion to allocate across characteristic-based portfolios?

	Mean (%)	Vol (%)	Sharpe	MDD (%)	Alpha	Turnover (%)
Panel A: Single-Characteristic Portfolios						
Market Capitalization	8.70	16.84	0.29*	53.87	0.042	0.68
Fundamental Value	9.94	17.73	0.34**	63.07	0.119**	6.18
Volatility ($1/\sigma$)	9.91	12.50	0.49**	39.69	0.174	15.67
Equally Weighted (1/N)	10.58	18.46	0.36**	60.34	0.162*	7.00
Panel B: Mean–Variance Dynamic Style Portfolio						
$MVDS_{\gamma=5}$	**10.06**	**13.99**	**0.44****	**41.09**	**0.195****	**17.90**

* p-value $<$ 0.10, ** p-value $<$ 0.05, *** p-value $<$ 0.01

Out-of-sample performance characteristics

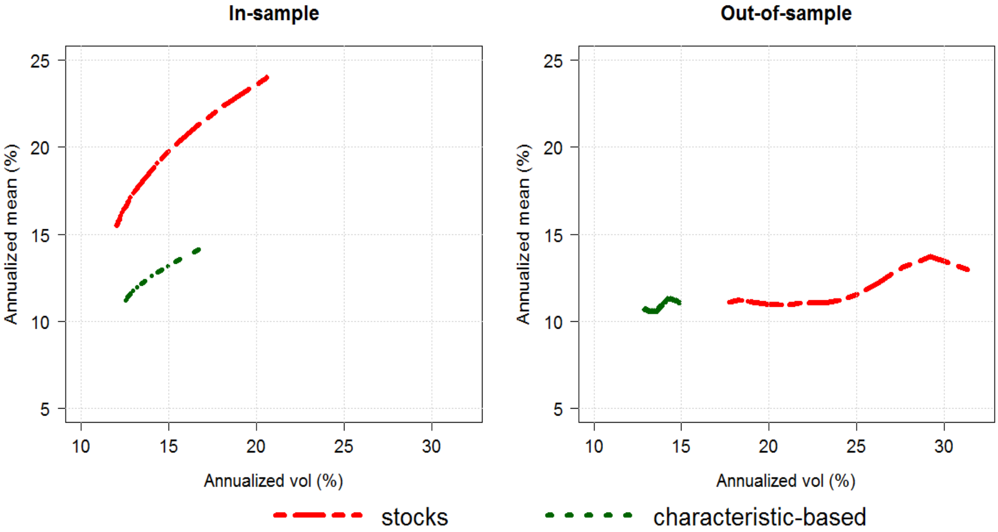
- Long-run outperformance of the single-characteristic portfolios compared to market capitalization portfolio
- Absolute outperformance of dynamic style portfolio is similar to best performing single-characteristic portfolios
- **Advantages dynamic portfolio**
  - Addresses selection problem
  - Dynamic allocation approach is purely data-driven
  - Few periods of underperformance: investors tend to allocate funds based on short-term relative performance

Risk–return trade-off

2. Is there a gain in switching from mean–variance allocation at the stock-level to mean–variance allocation across the characteristic-based portfolios?

- Plot the risk–return trade-off
  - Optimization over set of γ's ranging from 1.47 to 10.79
  - Mean–variance allocation at stock-level and across characteristic-based portfolios
  - Out-of-sample: rebalance portfolios every month over period ranging from 1990 to 2012
  - Calculate the out-of-sample realized annualized mean returns and volatility

Risk–return trade-off



Efficient frontier

- In-sample
  - Characteristic-based investor is restricted
  - In-sample loss compared to mean–variance optimization across all stocks
- Out-of-sample
  - Impact of estimation error
  - Imposing restrictions increases portfolio performance

Out-of-sample performance characteristics

	Mean (%)	Vol (%)	Sharpe	MDD (%)	Alpha	Turnover (%)
Panel A: Mean–Variance Dynamic Style Portfolio						
$MVDS_{\gamma=5}$	**10.06**	**13.99**	**0.44****	**41.09**	**0.195****	**17.90**
Panel B: Mean–Variance Optimized Portfolio Over All Stocks using EWMA						
$MV_{\gamma=5}$	11.20	24.56	0.30**	68.77	0.197	70.35

* p-value < 0.10, ** p-value < 0.05, *** p-value < 0.01

Robustness

- Diversification-based criteria
- No–trade zone and/or turnover constraint
- Inclusion of risk-free asset in investment universe
- Alternative levels of risk-aversion
- Alternative estimators for $\Sigma_{t|t-1}$ and $\widetilde{\Sigma}_{t|t-1}$

Conclusion

- Goal: exploit life-cycle specificity of characteristic-based portfolios

1. Is the mean–variance criterion a useful criterion to allocate across characteristic-based portfolios?
    - Absolute outperformance of dynamic style portfolio is similar to best performing single-characteristic portfolios
    - **Advantages dynamic portfolio**
        - Addresses selection problem
        - Less periods of underperformance
2. Is there a gain in switching from mean–variance allocation at the stock-level to mean–variance allocation across the characteristic-based portfolios?
    - Substantial improvement in out-of-sample performance

Thank you!

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