Characteristic-based equity portfolios
Economic value and dynamic style-allocation

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Motivation

- Increasing popularity of exchange-traded funds (ETFs) which track simple characteristic-based portfolios

- Characteristic-based portfolios
  - Market capitalization
    - World’s largest ETF is SPDR S&P 500 with a value of $169bn (Nasdaq, August 1, 2014)
  - Alternative characteristic-based portfolios
    - Accelerating interest: global value of funds increased from $58bn (2010) to $350bn (2014)
Motivation

- Characteristic-based portfolios as \textit{mean–variance efficient portfolio proxies}
  - Market capitalization portfolio
    - Capital Asset Pricing Model
    - In practice: assumptions often violated
  - Alternative portfolios
    - Long run risk-adjusted outperformance
    - Life cycle specificity: diversification opportunity
Research questions

1. Is the mean–variance criterion a useful criterion to allocate across characteristic-based portfolios?
   - Timing gains: unlikely that one characteristic is the mean–variance efficient proxy
   - Model the mean–variance efficient weights as a time-varying linear combination of the characteristic-based portfolios
   - Implied loadings provide information on the expected individual performance of the proxies for each period

2. Is there a gain in switching from mean–variance allocation at the stock–level to mean–variance allocation across the characteristic-based portfolios?
   - Estimation error (out-of-sample)
   - Imposing constraints reduces the estimation risk and can improve the portfolio performance (e.g., Jagannathan and Ma, 2003)
Modelling framework

- Investment universe
  - Directly invest in the $N$ 	extit{stocks} belonging to investment universe
    - $w$ the ($N \times 1$) vector of portfolio weights
  - Allocate budget across $K$ 	extit{characteristic-based portfolios}
    - $\theta$ the ($K \times 1$) vector of portfolio weights
  - $K \ll N$
Single characteristic-based portfolios

In a portfolio based on characteristic $k$ for a universe of $N$ stocks, we set the portfolio weights at time $t$ as:

$$x_{k,i,t} = \frac{\max\{0, z_{k,i,t}\}}{\sum_{i=1}^{N} \max\{0, z_{k,i,t}\}}$$

in which $z_{k,i,t}$ is the generic characteristic $k$ of stock $i$ at time $t$

- The considered characteristics are:
  - Market capitalization
  - Fundamental metrics of company size
  - Risk-based weighting: low volatility
  - S&P 500 inclusion dummy, i.e. equally-weighted portfolio
Portfolio construction

- Style allocation to characteristic-based portfolios
  - Mean–variance efficiency criterion

\[
h_\gamma(\theta_t; \tilde{\mu}_{t|t-1}, \tilde{\Sigma}_{t|t-1}) = \theta_t' \tilde{\mu}_{t|t-1} - \frac{\gamma}{2} \theta_t' \tilde{\Sigma}_{t|t-1} \theta_t
\]
Suppose

- $w_t^*$ is the true mean–variance optimal portfolio weight vector over $[t-1, t]$
- $X_{t-1}$ is the $N \times K$ matrix of characteristic-based portfolio weights

In sample, by allocating over $K$ characteristics instead of $N$ stocks, we expect to be inefficient, except when there exists a $\theta_t^*$ such that:

- $w_t^* = X_{t-1} \theta_t^*$

The true mean–variance portfolio is a linear combination of the characteristic-based portfolio weights
Efficiency of dynamic style portfolios

- Special case when the efficiency result arises
  - Mean–variance utility investor
    \[
    \text{argmax} \ w' \mu_{t|t-1} - \frac{\gamma}{2} w' \Sigma_{t|t-1} w
    \]  
    (1)
    \[
    r_{i,t} = \sum_{k=1}^{K} \lambda_{k,t} x_{i,k,t-1} + \epsilon_{i,t}.
    \]  
    (2)
  - From FOC, it follows that:
    \[
    w_t^* = \gamma^{-1} \Sigma_{t|t-1}^{-1} \mu_{t|t-1}
    \]  
    (3)
  - Under (2):
    \[
    \mu_{t|t-1} = E[r_t|x_{t-1}] = X_{t-1} \lambda_t
    \]  
    (4)
  - Thus
    \[
    w_t^* = \gamma^{-1} \Sigma_{t|t-1}^{-1} X_{t-1} \lambda_t
    \]  
    (5)
Efficiency of dynamic style portfolios

- Assume homoskedasticity and no cross correlation across asset returns (Hjalmarsson and Manchev, 2012)

Then:

$$w_t^* = X_{t-1} \theta_t^*,$$

(6)

with $$\theta_t^* = \gamma^{-1} \sigma_{t|t-1}^{-2} \lambda_t$$
Investment universe is restricted to S&P 500 stocks

Evaluation period 1990–2012

\( \gamma \) is set at 5 representing medium risk-aversion (Martellini & Ziemann, 2010)

Monthly rebalancing frequency

\[ \Sigma_{t|t-1} \] and \[ \tilde{\Sigma}_{t|t-1} \] are estimated over a 156 week rolling window
  - Exponentially weighted moving average (EWMA)
Data and estimation

- Market capitalization

- Fundamental value (cfr. Arnott et al., 2005)
  - Book value of common equity, dividends, net operating cash flow and revenues

- Risk-based weighting
  - $1/\sigma$

- Equally-weighted
Characteristic-based portfolios

- Long run outperformance of the alternative characteristic-based portfolios

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Is the mean–variance criterion a useful criterion to allocate across characteristic-based portfolios?

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**Panel B: Mean–Variance Dynamic Style Portfolio**

| MVDSγ=5                                  | 10.06    | 13.99   | 0.44** | 41.09   | 0.195**| 17.90        |

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Out-of-sample performance characteristics

- Long-run outperformance of the single-characteristic portfolios compared to market capitalization portfolio

- Absolute outperformance of dynamic style portfolio is similar to best performing single-characteristic portfolios

- **Advantages dynamic portfolio**
  - Addresses selection problem
  - Dynamic allocation approach is purely data-driven
  - Few periods of underperformance: investors tend to allocate funds based on short-term relative performance
Is there a gain in switching from mean–variance allocation at the stock-level to mean–variance allocation across the characteristic-based portfolios?

Plot the risk–return trade-off

- Optimization over set of $\gamma$'s ranging from 1.47 to 10.79
- Mean–variance allocation at stock-level and across characteristic-based portfolios
- Out-of-sample: rebalance portfolios every month over period ranging from 1990 to 2012
- Calculate the out-of-sample realized annualized mean returns and volatility
Risk–return trade-off

In-sample

Out-of-sample

- Annualized mean (%)
- Annualized vol (%)

stocks
characteristic-based
Efficient frontier

- **In-sample**
  - Characteristic-based investor is restricted
  - In-sample loss compared to mean–variance optimization across all stocks

- **Out-of-sample**
  - Impact of estimation error
  - Imposing restrictions increases portfolio performance
## Out-of-sample performance characteristics

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Robustness

- Diversification-based criteria
- No-trade zone and/or turnover constraint
- Inclusion of risk-free asset in investment universe
- Alternative levels of risk-aversion
- Alternative estimators for $\Sigma_{t|t-1}$ and $\tilde{\Sigma}_{t|t-1}$
Conclusion

- Goal: exploit life-cycle specificity of characteristic-based portfolios

1. Is the mean–variance criterion a useful criterion to allocate across characteristic-based portfolios?
   - Absolute outperformance of dynamic style portfolio is similar to best performing single-characteristic portfolios
   - **Advantages dynamic portfolio**
     - Addresses selection problem
     - Less periods of underperformance

2. Is there a gain in switching from mean–variance allocation at the stock-level to mean–variance allocation across the characteristic-based portfolios?
   - Substantial improvement in out-of-sample performance
Thank you!
References


