The `partialAR` package for modeling time series with both permanent and transient components

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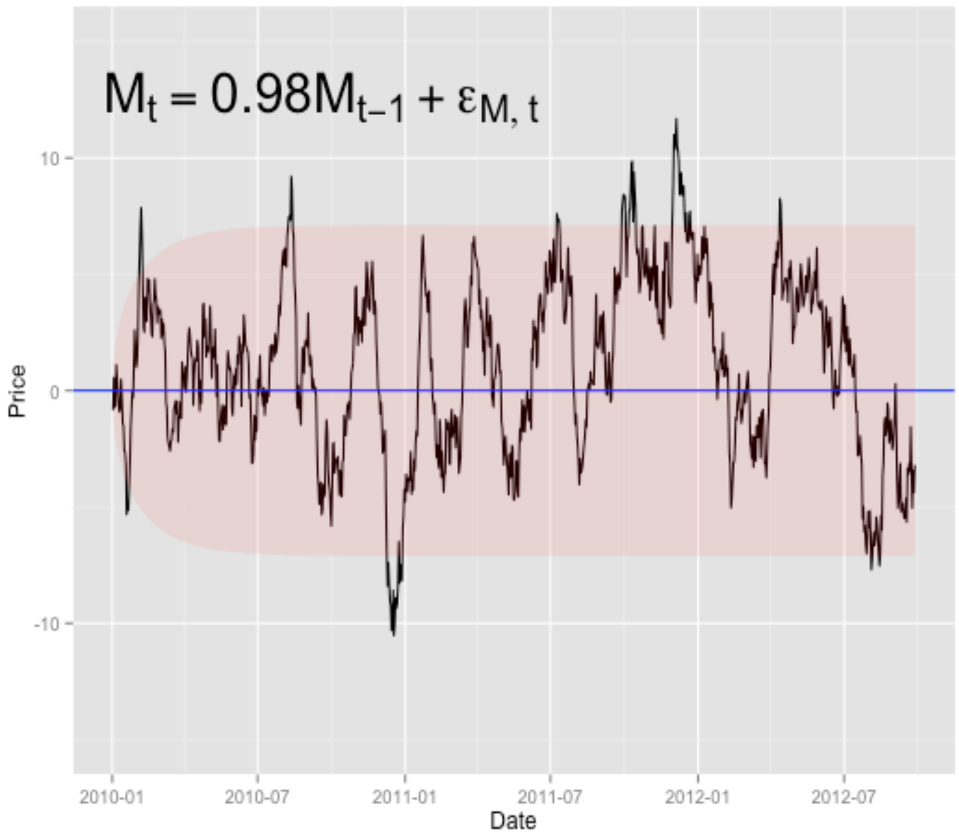
Stationary Autoregressive Processes



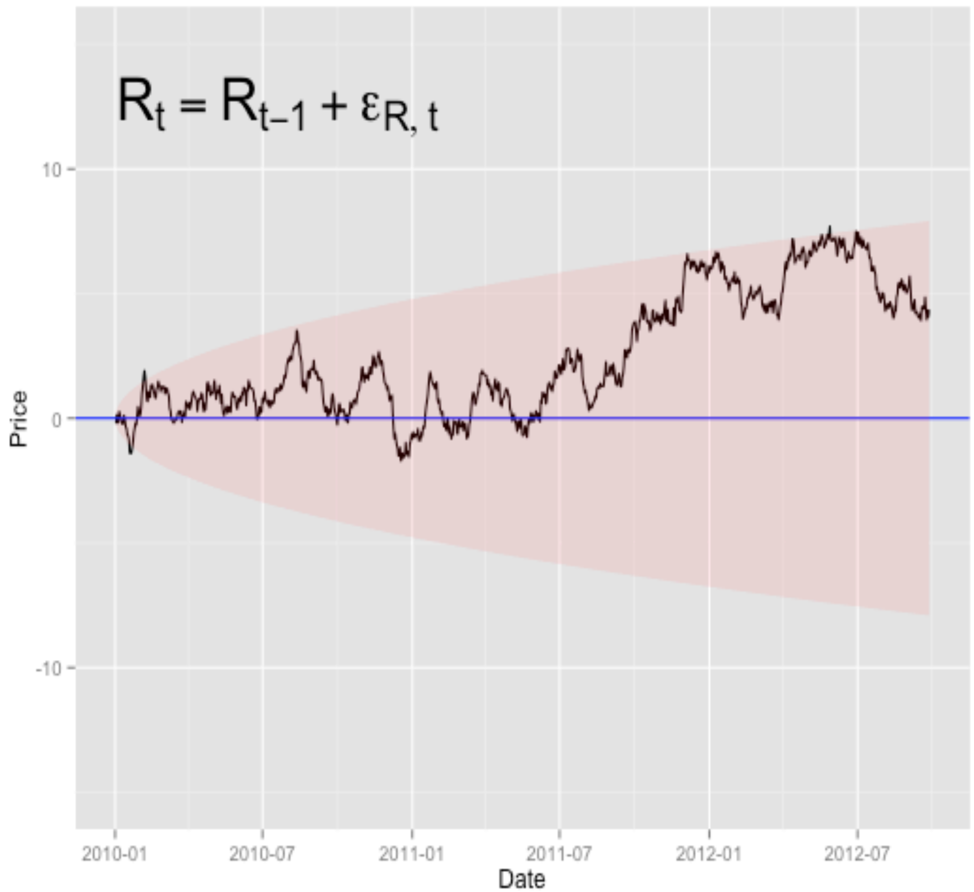
Key Features:

- Well-defined mean
- Finite variance
- Mean-reverting

When they can be found in real life, profitable trading opportunities exist.

But: It assumes all shocks are transient. This is unrealistic except in special cases.

Unit Root Processes



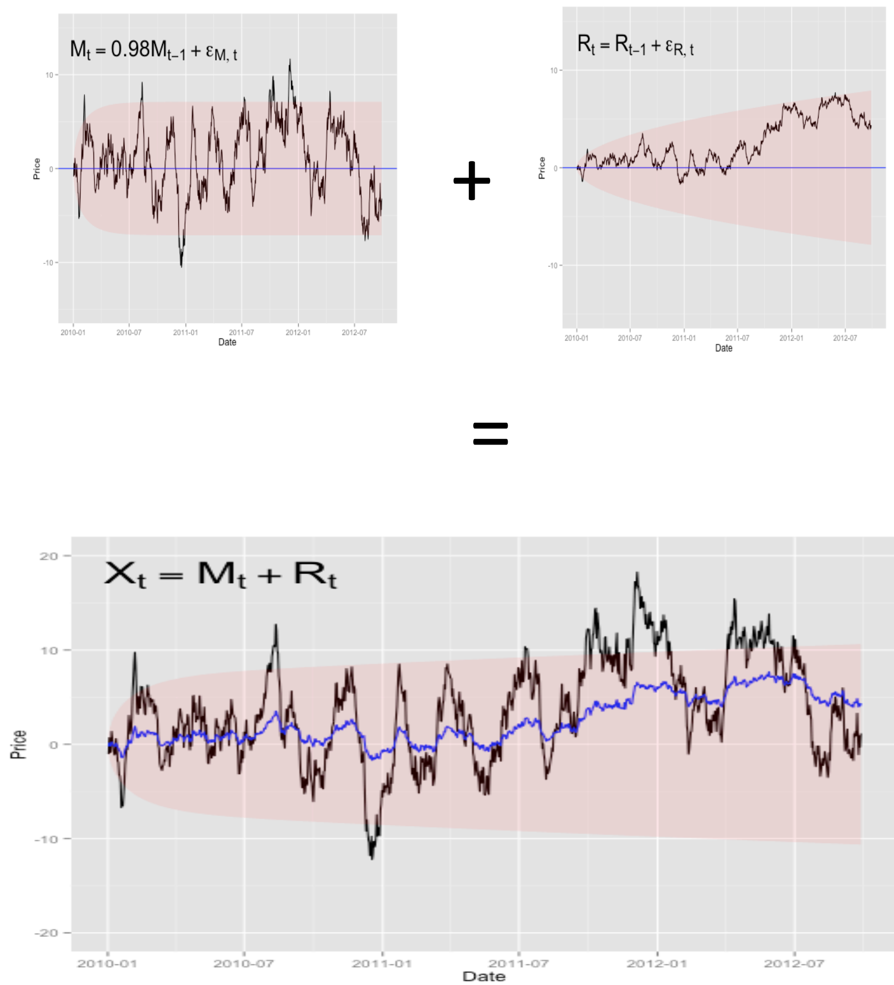
Key Features:

- Undefined variance
- Random walk

Often this seems to be a more realistic model of price movements, but it is not useful for trading.

In this model, all shocks are permanent.

Partially Autoregressive Processes



Key Features:

- Globally it looks like a random walk
- Locally, it is mean reverting

Shocks contain both permanent and transient components.

Possibly useful when a mean-reverting financial time series is contaminated with a small random walk.

Previously considered by Summers [1986] and Poterba and Summers [1988].

Model Parameters

ρ	Coefficient of mean reversion
σ_M^2	Variance of innovations in mean-reverting component
σ_R^2	Variance of innovations in random walk component

Estimation can be performed using a Kalman filter.

The `partialAR` package

`fit.par(X)`

Finds the maximum likelihood fit of a partially autoregressive model to the `zoo` series `X`.

`test.par(X)`

Tests the alternative hypothesis of partial autoregression against the null hypothesis that X is either a random walk or stationary AR(1).

Proportion of Variance Attributable to Mean Reversion

$$R_{MR}^2 = \frac{2\sigma_M^2}{2\sigma_M^2 + (1+\rho)\sigma_R^2}$$

Measures whether a particular series is closer to a random walk or a mean-reverting series:

- For a pure random walk, it will be 0
- For a partially autoregressive series, it will be between 0 and 1
- For a pure autoregressive series, it will be 1

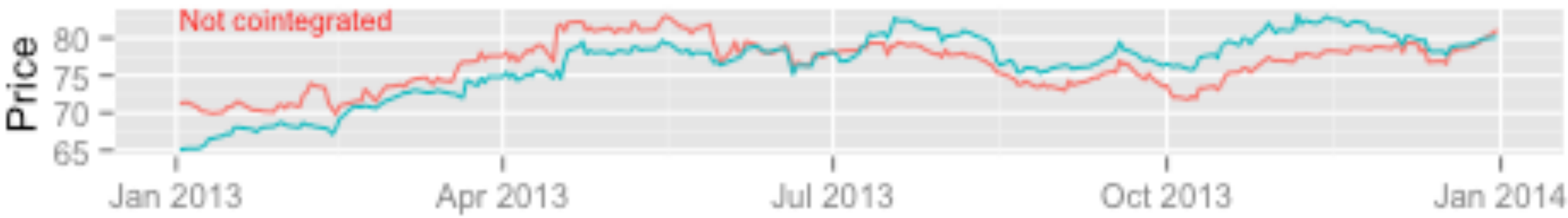
When two series are cointegrated, this value will be one when computed for the residual series.

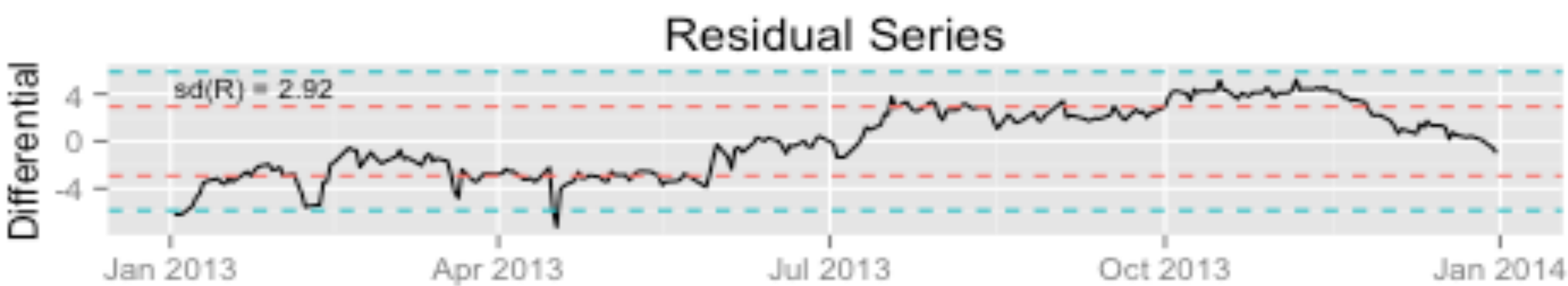
Example: Testing Cointegration of Price of Coca-Cola vs. Pepsi, 2013

Sample R Code:

```
> library(quantmod)
> library(tseries)
> library(egcm)
> library(partialAR)
> getSymbols("KO")
> getSymbols("PEP")
> KO.PEP <- merge(KO,PEP)
> KO.PEP2013 <- window(KO.PEP, start=as.Date("2013-01-01"),
end=as.Date("2013-12-31"))
> KO.PEP2013 <- KO.PEP2013[,c("KO.Adjusted","PEP.Adjusted")]
> colnames(KO.PEP2013) <- c("KO","PEP")
> egcm(KO.PEP2013)
> adf.test(lm(KO~PEP,KO.PEP2013)$residuals)
> pp.test(lm(KO~PEP,KO.PEP2013)$residuals)
> fit.par(egcm(KO.PEP2013)$residuals)
> test.par(egcm(KO.PEP2013)$residuals)
```

Example: Testing Cointegration of price of Coca-Cola vs. Pepsi, 2013





Coca Cola vs. Pepsi 2013: Finding a partially autoregressive fit

```
> fit.par(egcm(KO.PEP2013)$residuals)
Fitted model:
  X[t] = M[t] + R[t]
  M[t] = 0.6396 M[t-1] + eps_M,t,  eps_M,t ~ N(0, 0.4623^2)
        (0.2201)                                (0.1076)
  R[t] = R[t-1] + eps_R,t,         eps_R,t ~ N(0, 0.4569^2)
                                                (0.1088)
  M_0 = 0.0000, R_0 = -6.0535
           (NA)       (0.6536)
Proportion of variance attributable to mean reversion (pvmr) = 0.5553
Negative log likelihood = 249.01
```

Coca Cola vs. Pepsi 2013: Testing the Goodness of Fit

```
> test.par(egcm(KO.PEP2013)$residuals)

   Test of [Random Walk or AR(1)] vs Almost AR(1) [LR test
for AR1]

data:  egcm(KO.PEP2013)$residuals

Hypothesis                  Statistic    p-value
Random Walk                     -2.83      0.058
AR(1)                           -2.67      0.010
Combined                                   0.040
```

Limitations

- It works best when the half-life of mean reversion is short.
- Test does not have great power, especially when ρ is close to 1.
- If used on a price series with significant bid-ask bounce, the model may focus on the bid-ask spread.
- Results of fit can be biased by GARCH effects.
- See paper for further details.

Conclusion

Partial autoregression is a possibly useful model when confronted with a mean-reverting time series that has been contaminated with a random walk component.

If you think your data may fit this model, the `partialAR` package can be used to find the best fit.

Feel free to contact me for further details and assistance:

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References

Summers, Lawrence H. "Does the stock market rationally reflect fundamental values?" *The Journal of Finance* 41.3 (1986): 591-601.

Poterba, James M. and Lawrence H. Summers, "Mean reversion in stock prices: Evidence and implications." *Journal of Financial Economics* 22.1 (1988): 27-59.

Clegg, Matthew. "Modeling Time Series with Both Permanent and Transient Components Using the Partially Autoregressive Model." (January 28, 2015). Available at SSRN: http://ssrn.com/abstract=2556957