Risk Decomposition for Fund Managers

Matthew Dixon

HedgeFacts
www.hedgefacts.com

R in Finance 2015
May 29th, 2015

Problem statement

Investment management firms seek to measure the contribution of a sub-manager's positions to the overall Value-at-Risk.
These components should

- be additive across the portfolio
- fully capture the correlations between instrument returns in the portfolio
- account for nonlinear instruments in the portfolio
- be ranked by their net impact on the VaR

Examples of multi-manager funds

Examples of such funds include:

- large multi-strategy funds that employ multiple traders
- large asset management firms such as pension funds
- family offices and endowments
- multi-manager 40-act investment funds
- proprietary trading firms
- fund of funds who receive position transparency

Overview of presentation

- Revisit non-linear parametric methodologies for estimating Value-at-Risk
- Extend instrument Component VaR to non-linear loss functions
- Present a manager Component VaR approach which enables sub-portfolio managers to concentrate on the most significant risk factors
- Evaluate and compare the decomposition approach with other approaches using a representative CTA portfolio

Reference materials

- Dixon, M., *Risk Decomposition for Fund Managers*, R/Finance 2015, `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2610188`.
- Boudt K., Peterson B.G. and Croux C., *Estimation and Decomposition of Downside Risk for Portfolios with Non-normal Returns*, Journal of Risk, Volume 11 (2), pp. 79–103, 2008.
- Britten-Jones M. and Schaefer S.M., *Non-Linear Value-at-Risk*, European Finance Review, 2004.
- Castellacci F. and Siclari M.J., *The Practice of Delta-Gamma VaR: Implementing the Quadratic Portfolio Model*, European Journal of Operational Research, 2003, pp. 529–545.
- Jorion P., *Value-at-Risk*, Second Edition.

Delta-Gamma approximation

Definition (Delta-Gamma Approximation)

$$dP = \sum_{i,j} \underbrace{\Delta_i dR_i}_{delta} + \underbrace{\frac{1}{2} dR_i \Gamma_{ij} dR_j}_{gamma},$$

where

- $\Delta_i = \frac{\partial P}{\partial R_i}$ is the sensitivity of the portfolio price to the i^{th} risk factor
- $\Gamma_{ij} = \frac{\partial^2 P}{\partial R_i \partial R_j}$ is the second order sensitivity of the portfolio price to the i^{th} and j^{th} risk factors.

Cornish-Fisher expansion

Definition (Cornish-Fisher expansion)

$$VaR_{c,dt}[dP_t] = -\left(\mu_1 + (z + \frac{1}{6}(z^2 - 1)s + \frac{1}{24}(z^3 - 3z)(\kappa - 3) - \frac{1}{36}(2z^3 - 5z)s^2\right)\sqrt{\mu_2}$$

- $z = \Phi^{-1}(1 - c)$ is the inverse standard normal cumulative distribution function $\Phi(z)$ evaluated at $1 - c$.
- $s = \frac{\mu_3}{\mu_2^{3/2}}$ is the skewness.
- κ denotes kurtosis and is given by $\frac{\mu_4}{\mu_2^2}$

Moments of the distribution

$$\mu_1 := \mathbb{E}[dP_t] = \frac{1}{2} tr(\Gamma\Sigma) \quad (1)$$

$$\mu_2 := \mathbb{E}[dP_t - \mu_1]^2 = \Delta^T \Sigma \Delta + \frac{1}{2} tr(\Gamma\Sigma)^2 \quad (2)$$

$$\mu_3 := \mathbb{E}[dP_t - \mu_1]^3 = 3\Delta^T \Sigma \Gamma \Sigma \Delta + tr(\Gamma\Sigma)^3 \quad (3)$$

$$\mu_4 := \mathbb{E}[dP_t - \mu_1]^3 = 12\Delta^T \Sigma (\Gamma\Sigma)^2 \Delta + 3tr(\Gamma\Sigma)^4 + 3\mu_2^2 \quad (4)$$

Linear component of variance

Definition (Linear Instrument Component VaR)

$$(\sigma_P^{[i]})^2 = \frac{1}{2} \sum_{k \in K_i} w_i(\Delta_k) \left(\nabla_\Delta \sigma_P^2\right)_k$$

where

- $\nabla_\Delta \sigma_P = 2\Delta^T \Sigma$ is the sensitivity of σ_P^2 to Δ
- $w_i(\Delta_k)$ is the exposure of instrument i to risk factor k (or equivalently the contribution of instrument i to Δ_k)
- K_i is the set of k indices corresponding to the non-zero terms of $\mathbf{w}_i(\Delta)$.

Convexity adjusted component of variance

The convexity adjusted contribution of the i^{th} instrument to the standard deviation of the portfolio loss σ_P is

$$(\sigma_P^{[i]})^2 = \frac{1}{2}\sum_{k \in K_i} w_i(\Delta_k)\left(\nabla_\Delta \sigma_P^2\right)_k + \left(w_i(\Gamma)\nabla_\Gamma \sigma_P^2\right)_{kk}$$

where

- $w_i(\Gamma)$ is a matrix whose $(l,m)^{th}$ elements stores the contribution of instrument i to $\Gamma_{l,m}$
- $\nabla_\Gamma \sigma_P^2 = tr(\Sigma\Gamma)\Sigma$ is the matrix of sensitivities to Γ, whose $(l,m)^{th}$ element is just the sensitivity of σ_P^2 to Γ_{lm}.

Non-linear component VaR

Definition (Non-linear Instrument Component VaR)

$$VaR^{[i]}_{c,dt}[dP_t] = -\left[\mu_1^{[i]} + \left(z + \frac{1}{6}(z^2-1)s + \frac{1}{24}(z^3-3z)(\kappa-3) - \frac{1}{36}(2z^3-5z)s^2\right)(\sigma_P^{[i]})^2/\sigma_P\right],$$

where the i^{th} instrument's contribution to the first moment of the portfolio loss distribution is

$$\mu_1^{[i]} = \frac{1}{2}\sum_{k\in K_i}(w_i(\Gamma)\Sigma)_{kk}$$

Manager component VaR

Definition (Manager Component VaR)

$$\widehat{VaR}^{[j]}_{c,dt}[dP_t] = -\left[\hat{\mu}^{[j]}_1 + \left(z + \frac{1}{6}(z^2-1)s + \frac{1}{24}(z^3-3z)(\kappa-3) - \frac{1}{36}(2z^3-5z)s^2\right)(\hat{\sigma}^{[j]}_P)^2/\sigma_P\right],$$

$$\hat{\mu}^{[j]}_i = \sum_{i \in I_j} \mu^{[i]}_1,$$

and

$$(\hat{\sigma}^{[j]}_P)^2 = \sum_{i \in I_j} (\sigma^{[i]}_P)^2$$

Composition of the CTA portfolio

Symbol	Expiry	Description	Sector	Holding	Currency
EC	Sep 2014	EURO FX CURR	Currencies	-27	USD
BP	Sep 2014	BRITISH POUND	Currencies	-52	USD
HO	Sep 2014	HEATING OIL	Energies	-10	USD
CL	Sep 2014	CRUDE OIL	Energies	-10	USD
NG	Sep 2014	HENRY HUB NATURAL GAS	Energies	-10	USD
G	Sep 2014	LONG GILT	Interest Rates	15	GBP
FGBL	Sep 2014	Euro-Bund Futures	Interest Rates	14	EUR
TY	Sep 2014	10 Year U.S. Treasury Notes	Interest Rates	-26	USD
LH	Oct 2014	LEAN HOGS	Livestock	10	USD
GC	Aug 2014	GOLD	Metals	-7	USD
VG	Sep 2014	DJ EURO STOXX 50	Stock Indices	-30	EUR
ES	Sep 2014	S&P500 EMINI	Stock Indices	50	USD
L	Mar 2015	90DAY STERLING	Interest Rates	45	GBP
ED	Mar 2015	EURODOLLAR	Interest Rates	-16	USD
PUT NG 2.5	Oct 2014	HENRY HUB NATURAL GAS	Energies	215	USD
PUT NG 2.25	Sep 2014	HENRY HUB NATURAL GAS	Energies	1600	USD
PUT ES 1270	Sep 2014	S&P500 EMINI	Stock Indices	-170	USD
PUT CL 85	Sep 2014	CRUDE OIL	Energies	82	USD
CALL GC 1740	Aug 2014	GOLD	Metals	2000	USD
PUT NQ 2450	Aug 2014	NASDAQ 100 EMINI	Stock Indices	-196	USD

Table: Composition of each sub-portfolio in the CTA portfolio.

Verification of component VaR properties

- *Observe the effect of the convexity adjustment by instrument*: Compare the Delta Component VaR of each instrument with the Delta-Gamma Component VaR
- *Observe the overall effect of the convexity adjustment*: Compare the Delta VaR of the portfolio with the Delta-Gamma VaR.
- *Ensure that each sub-manager tracks the most significant risk factors*: For each sub-portfolio, we rank the instruments that are associated with the most significant risk factors.
- *Ensure that the instrument component delta-gamma VaR is additive*: Check that the instrument component VaRs sum to the overall VaR

Instrument component VaR

Symbol	Expiry	$\Delta - \Gamma$ Component VaR	Δ Component VaR	Risk Factor Rank
NG	Sep 2014	$13,519.09	$15,849.46	1
CL	Sep 2014	$15,762.75	$18,883.80	2
HO	Sep 2014	$14,737.84	$17,662.77	4
BP	Sep 2014	$12,617.50	$15,002.18	5
EC	Sep 2014	$11,766.81	$14,455.78	6
G	Sep 2014	$3,764.95	$4,659.97	8
FGBL	Sep 2014	$3,661.06	$4,549.75	9
VG	Sep 2014	$5,627.52	$6,753.13	7
ED	Mar 2015	$107.96	$106.10	11
L	Mar 2015	$10.34	$73.86	12
LH	Oct 2014	-$476.19	-$608.78	13
TY	Sep 2014	-$2,055.80	-$2,665.35	14
ES	Sep 2014	-$3,545.11	-$4,181.75	15
GC	Aug 2014	$4,691.23	$5,621.43	16
PUT NG 2.25	Sep 2014	$88,568.48	$108,096.15	1
PUT CL 85	Sep 2014	$35,052.87	$43,922.64	2
PUT NG 2.5	Oct 2014	$38,985.18	$46,780.38	3
PUT NQ 2450	Aug 2014	$1,281.51	-$66.31	10
PUT ES 1270	Sep 2014	-$704.78	-$1,780.56	15
CALL GC 1740	Aug 2014	-$8,979.55	-$6,897.13	16
Sum		$234,393.65	$286,217.53	
99% Portfolio VaR		$234,393.65	$286,217.53	

Table: VaR is estimated at the 99% percentile.

Component VaR

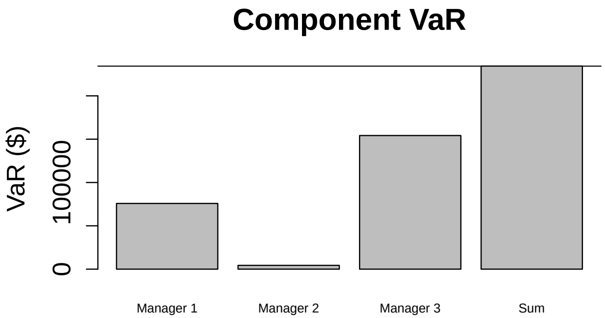

Figure: We observe that the sum equals the overall VaR, as shown by the horizontal line.

Independent VaR

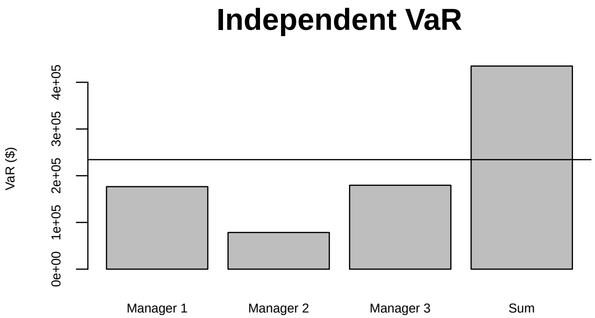

Figure: Independent VaR measures the VaR on the sub-portfolios separately and ignores correlations between instruments across sub-portfolios. We observe that the sum does not match the overall VaR, as shown by the horizontal line.

Incremental VaR

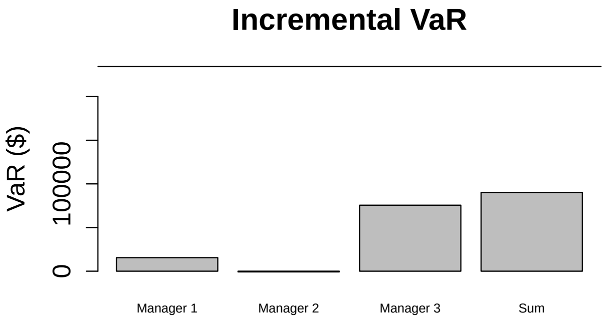

Figure: Incremental VaR is a two step procedure. First the target sub-portfolio is removed from the portfolio and the VaR measured on the residual sub-portfolios. Next this VaR amount is subtracted from the overall VaR. This approach also ignores correlations between instruments across sub-portfolios and we observe that the sum does not match the overall VaR, as shown by the horizontal line.

Conclusion

We've presented a manager Component VaR methodology that

- is additive across the portfolio
- fully captures the correlations between instrument returns in the portfolio
- accounts for nonlinear instruments in the portfolio
- ranks instruments in each sub-portfolio by the net impact of the associated risk factors on the overall portfolio