Risk Decomposition for Fund Managers

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Investment management firms seek to measure the contribution of a sub-manager’s positions to the overall Value-at-Risk. These components should:

- be additive across the portfolio
- fully capture the correlations between instrument returns in the portfolio
- account for nonlinear instruments in the portfolio
- be ranked by their net impact on the VaR
Examples of multi-manager funds

Examples of such funds include:

- large multi-strategy funds that employ multiple traders
- large asset management firms such as pension funds
- family offices and endowments
- multi-manager 40-act investment funds
- proprietary trading firms
- fund of funds who receive position transparency
Overview of presentation

- Revisit non-linear parametric methodologies for estimating Value-at-Risk
- Extend instrument Component VaR to non-linear loss functions
- Present a manager Component VaR approach which enables sub-portfolio managers to concentrate on the most significant risk factors
- Evaluate and compare the decomposition approach with other approaches using a representative CTA portfolio
Reference materials


Delta-Gamma approximation

Definition (Delta-Gamma Approximation)

\[ dP = \sum_{i,j} \Delta_i dR_i + \frac{1}{2} dR_i \Gamma_{ij} dR_j, \]

where

- \( \Delta_i = \frac{\partial P}{\partial R_i} \) is the sensitivity of the portfolio price to the \( i^{th} \) risk factor
- \( \Gamma_{ij} = \frac{\partial^2 P}{\partial R_i \partial R_j} \) is the second order sensitivity of the portfolio price to the \( i^{th} \) and \( j^{th} \) risk factors.
Cornish-Fisher expansion

Definition (Cornish-Fisher expansion)

\[ \text{VaR}_{c, dt}[dP_t] = -\left( \mu_1 + (z + \frac{1}{6}(z^2 - 1)s + \frac{1}{24}(z^3 - 3z)(\kappa - 3) - \frac{1}{36}(2z^3 - 5z)s^2) \right) \sqrt{\mu_2} \]

- \( z = \Phi^{-1}(1 - c) \) is the inverse standard normal cumulative distribution function \( \Phi(z) \) evaluated at \( 1 - c \).
- \( s = \frac{\mu^3}{\mu_2^{3/2}} \) is the skewness.
- \( \kappa \) denotes kurtosis and is given by \( \frac{\mu^4}{\mu_2^2} \).
Moments of the distribution

\[ \mu_1 := \mathbb{E}[dP_t] = \frac{1}{2} \text{tr}(\Gamma \Sigma) \]  
(1)

\[ \mu_2 := \mathbb{E}[dP_t - \mu_1]^2 = \Delta^T \Sigma \Delta + \frac{1}{2} \text{tr}(\Gamma \Sigma)^2 \]  
(2)

\[ \mu_3 := \mathbb{E}[dP_t - \mu_1]^3 = 3 \Delta^T \Sigma \Gamma \Sigma \Delta + \text{tr}(\Gamma \Sigma)^3 \]  
(3)

\[ \mu_4 := \mathbb{E}[dP_t - \mu_1]^4 = 12 \Delta^T \Sigma (\Gamma \Sigma)^2 \Delta + 3 \text{tr}(\Gamma \Sigma)^4 + 3 \mu_2^2 \]  
(4)
Linear component of variance

Definition (Linear Instrument Component VaR)

\[
(\sigma_P^i)^2 = \frac{1}{2} \sum_{k \in K_i} w_i(\Delta_k) (\nabla \Delta \sigma_P^2)_k
\]

where

- \( \nabla \Delta \sigma_P = 2\Delta^T \Sigma \) is the sensitivity of \( \sigma_P^2 \) to \( \Delta \)
- \( w_i(\Delta_k) \) is the exposure of instrument \( i \) to risk factor \( k \) (or equivalently the contribution of instrument \( i \) to \( \Delta_k \))
- \( K_i \) is the set of \( k \) indices corresponding to the non-zero terms of \( w_i(\Delta) \).
Convexity adjusted component of variance

The convexity adjusted contribution of the $i^{th}$ instrument to the standard deviation of the portfolio loss $\sigma_P$ is

$$
(\sigma_P[i])^2 = \frac{1}{2} \sum_{k \in K_i} w_i(\Delta_k) (\nabla_{\Delta} \sigma_P^2)_k + (w_i(\Gamma) \nabla_{\Gamma} \sigma_P^2)_{kk}
$$

where

- $w_i(\Gamma)$ is a matrix whose $(l, m)^{th}$ elements stores the contribution of instrument $i$ to $\Gamma_{l,m}$
- $\nabla_{\Gamma} \sigma_P^2 = tr(\Sigma \Gamma) \Sigma$ is the matrix of sensitivities to $\Gamma$, whose $(l, m)^{th}$ element is just the sensitivity of $\sigma_P^2$ to $\Gamma_{lm}$. 
Non-linear component VaR

Definition (Non-linear Instrument Component VaR)

\[
\text{VaR}_{c,dt}^{[i]}[dP_t] = - \left[ \mu_1^{[i]} + \left( z + \frac{1}{6}(z^2 - 1)s + \frac{1}{24}(z^3 - 3z)(\kappa - 3) - \frac{1}{36}(2z^3 - 5z)s^2 \right) \left( \sigma_P^{[i]} \right)^2 / \sigma_P \right],
\]

where the \( i^{th} \) instrument’s contribution to the first moment of the portfolio loss distribution is

\[
\mu_1^{[i]} = \frac{1}{2} \sum_{k \in K_i} (w_i(\Gamma)\Sigma)_{kk}
\]
Definition (Manager Component VaR)

\[
\text{VaR}_{c, dt}[dP_t] = - \left[ \hat{\mu}_j + \left( z + \frac{1}{6} (z^2 - 1)s + \frac{1}{24} (z^3 - 3z)(\kappa - 3) - \frac{1}{36} (2z^3 - 5z)s^2 \right) (\hat{\sigma}_j^2 / \sigma_P) \right],
\]

\[
\hat{\mu}_j = \sum_{i \in l_j} \mu_{1i},
\]

and

\[
(\hat{\sigma}_j^2) = \sum_{i \in l_j} (\sigma_{1i}^2)
\]
## Composition of the CTA portfolio

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Expiry</th>
<th>Description</th>
<th>Sector</th>
<th>Holding</th>
<th>Currency</th>
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**Table:** Composition of each sub-portfolio in the CTA portfolio.
Verification of component VaR properties

- *Observe the effect of the convexity adjustment by instrument*: Compare the Delta Component VaR of each instrument with the Delta-Gamma Component VaR.

- *Observe the overall effect of the convexity adjustment*: Compare the Delta VaR of the portfolio with the Delta-Gamma VaR.

- *Ensure that each sub-manager tracks the most significant risk factors*: For each sub-portfolio, we rank the instruments that are associated with the most significant risk factors.

- *Ensure that the instrument component delta-gamma VaR is additive*: Check that the instrument component VaRs sum to the overall VaR.
## Instrument component VaR

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Expiry</th>
<th>$\Delta - \Gamma$ Component VaR</th>
<th>$\Delta$ Component VaR</th>
<th>Risk Factor Rank</th>
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<tr>
<td>NG</td>
<td>Sep 2014</td>
<td>$13,519.09$</td>
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<td>$15,762.75$</td>
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<td>$14,737.84$</td>
<td>$17,662.77$</td>
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<td>$12,617.50$</td>
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<td>$4,659.97$</td>
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<tr>
<td>FGBL</td>
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<td>$3,661.06$</td>
<td>$4,549.75$</td>
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<td>$6,753.13$</td>
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<td>$106.10$</td>
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<td>L</td>
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<td>Sum</td>
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<tr>
<td>99% Portfolio VaR</td>
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<td>$234,393.65$</td>
<td>$286,217.53$</td>
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**Table:** VaR is estimated at the 99% percentile.
Component VaR

Manager 1
Manager 2
Manager 3
Sum

Figure: We observe that the sum equals the overall VaR, as shown by the horizontal line.
Independent VaR

Figure: Independent VaR measures the VaR on the sub-portfolios separately and ignores correlations between instruments across sub-portfolios. We observe that the sum does not match the overall VaR, as shown by the horizontal line.
Figure: Incremental VaR is a two step procedure. First the target sub-portfolio is removed from the portfolio and the VaR measured on the residual sub-portfolios. Next this VaR amount is subtracted from the overall VaR. This approach also ignores correlations between instruments across sub-portfolios and we observe that the sum does not match the overall VaR, as shown by the horizontal line.
Conclusion

We’ve presented a manager Component VaR methodology that

- is additive across the portfolio
- fully captures the correlations between instrument returns in the portfolio
- accounts for nonlinear instruments in the portfolio
- ranks instruments in each sub-portfolio by the net impact of the associated risk factors on the overall portfolio