Intraday Trading Invariance in the E-Mini S&P 500 Futures Market

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Joint with

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Intraday Pattern for Market Activity Variables

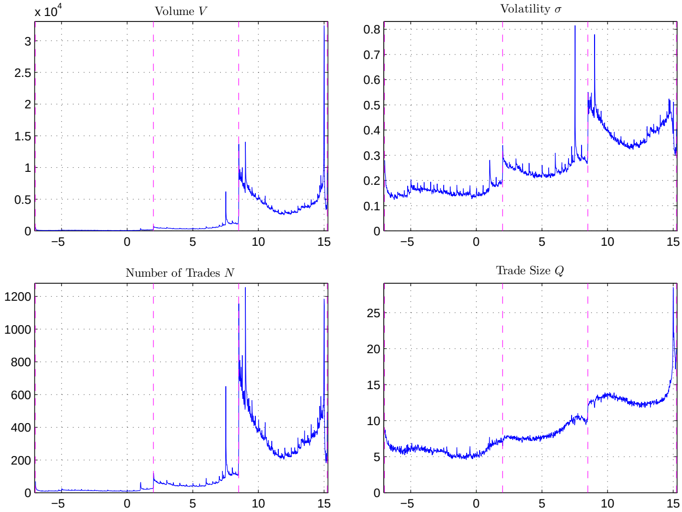

Figure: S&P 500 E-mini. Sample averages per 1 min. Dashed lines separate trading hours in Asia, Europe, and America.

Hypotheses on How Trading Drives Volatility

Three Theories:

1. Volume drives volatility: Clark (1973)
2. Transactions drive volatility: Mandelbroit and Taylor (1967), Jones, Kaul & Lipson (1994), and Ané & Geman (2000
3. Market Microstructure Invariance: Kyle and Obizhaeva (2014)

Imply different **stochastic clocks**

Theories almost Invariably tested in **Time Series** (daily data)

Why High-Frequency Analysis

- Pronounced **intraday market activity patterns**
- News incorporated into prices quickly; Trading fast
- Huge systematic variation over 24-hour trading day
- Does any basic regularity apply in this setting?

- **Macroeconomic announcements** particular challenge
  - Large price jump on impact without (much) trading
  - Subsequent price discovery process

- **Sudden market turmoil: Crisis, Flash Crash**
  - Do same or different regularities apply in this context?

S&P 500 E-Mini Futures Market

- BBO files from CME; Jan 4, 2008 – Nov 2, 2011
  - Extraordinary active market – Price discovery for equities
  - Time-stamped to second, Sequenced in actual order
  - Use front month contract (most liquid)
- Three daily **Regimes** (CT):
  - Asia, 17:00 – 2:00
  - Europe, 2:00 – 8:30
  - America, 8:30 – 15:15
- $\mathbf{D} = 969$ trading days; $\mathbf{T} = 1{,}335$ 1-minute intervals per day

  $\mathbf{N_{dt}} =$ Number of transactions per min;
  $\mathbf{V_{dt}} =$ Volume (Number of contracts per min);
  $\mathbf{Q_{dt}} =$ Average Trade Size;
  $\mathbf{P_{dt}} =$ Average Price;
  $\sigma_{\mathbf{dt}} =$ Volatility;
  $\mathbf{W_{dt}} =$ Trading Activity (Dollars at Risk per min) $= \mathbf{P_{dt}}\,\mathbf{V_{dt}}\,\sigma_{\mathbf{dt}}$

Descriptive Statistics for S&P 500 E-mini Futures

	Asia	Europe	America	Combined			
				Mean	Min	Max	Ratio
Volatility	0.16	0.25	0.40	0.26	0.12	0.81	6.5
Volume	95	601	4726	1647	51	32398	638
# Trades	14	67	360	134	9	1256	142
Notional Value, $Mln	5	34	266	93	3	1814	626
Trade Size	5.9	8.4	13.3	8.9	4.6	28.5	6.2
Market Depth	54	265	984	398	35	3519	101
Bid-Ask Spread	26.5	25.7	25.1	25.9	25.1	28.0	1.1
Business Time	24.6	5.8	1.0	12.0	0.2	37.7	172

Notes: Sample averages per 1 min. Volatility is annualized (in decimal form). Business Time is proportional to $W^{-2/3}$ (normalized to 1 in America)

Market Microstructure Invariance

Market Microstructure Invariance (MMI): Risk transfers, transactions costs, resilience, market depth, etc., are constant across assets and market environments when measured in units of business time

In particual, **Dollar-Risk Transfer per Bet in Business Time is i.i.d.**

$$\mathcal{I} \ = \ \mathbf{P} \cdot \mathbf{Q_B} \cdot \boldsymbol{\sigma} \cdot \mathbf{N_B^{-1/2}}$$

where $Q_B =$ Bet size and $N_B =$ expected number of Bets.

Define "Trading Activity" $\mathbf{W} \ = \ \mathbf{P} \cdot \mathbf{V} \cdot \boldsymbol{\sigma}$, then

$$\mathbf{N_B} \ \sim \ \mathbf{W^{2/3}} \qquad \text{and} \qquad \mathbf{Q_B} \ \sim \ \mathbf{W^{1/3}}$$

Interpretation: Since $V = Q_B \cdot N_B$,

Variation in Volume: $2/3$ from N_B, $1/3$ from Q_B.

Intraday Trading Invariance

Bets are not observable; Q_B, N_B, σ and latent variables.

Invoke auxiliary hypotheses to develop testable variant of MMI:

$$\mathcal{I}_{\mathbf{dt}} \;=\; \mathbf{P_{dt}} \cdot \mathbf{Q_{dt}} \cdot \boldsymbol{\sigma}_{\mathbf{dt}} \cdot \mathbf{N_{dt}^{-1/2}}$$

Replace unobserved Q_B and arrival rate N_B with observed parallels, Trade Size Q and expected Number of trades N

Proxy N by observed transactions, estimate σ by RV using HF returns

Note: Intraday variation in expected price change trivial. Henceforth, for intraday tests, we ignore variation in P

Intraday Trading Invariance

Invariance Inspired Hypothesis: $\mathbf{\log\left(N_{dt}\right) = c + \boldsymbol{\beta} \cdot \log\left(W_{dt}\right)}$

where Invariance predicts slope of $\boldsymbol{\beta} = \mathbf{2/3}$.

Relies on expectations approximated by realizations or noisy estimators

Aggregate relationship across days to diversify measurement errors

$$\mathbf{n_t = \frac{1}{D} \sum_{d=1}^{D} \log\left(N_{dt}\right) = c + \boldsymbol{\beta} \cdot \left[\frac{1}{D} \sum_{d=1}^{D} \log\left(W_{dt}\right)\right] + \boldsymbol{\nu}_t}$$

for $t = 1, \ldots, T = 1{,}335$.

Suggestive Intraday Trading Invariance Check

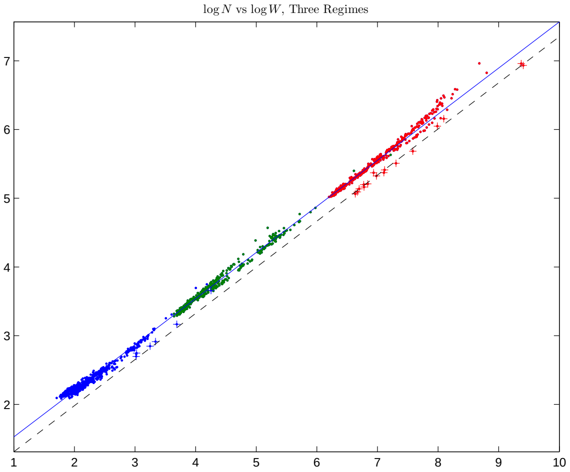

Figure: Asia (blue), Europe (green), America (red). Crosses: First 6 min of trading (blue) and last 16 min (red). Solid line: $n_t = c + 0.671 \cdot w_t$; Dashed line: Same slope, Fit to red crosses.

Suggestive Test for Alternative Theories

Ignoring P, the three theories may be restated as

$$\log \mathbf{N} = \mathbf{c} + \boldsymbol{\beta} \cdot \log\left(\mathbf{W} / \mathbf{Q}^{\frac{3}{2}}\right) \qquad \text{[Clark]}$$

$$\log \mathbf{N} = \mathbf{c} + \boldsymbol{\beta} \cdot \log\left(\mathbf{W} / \mathbf{Q}\right) \qquad \text{[Ané \& Geman]}$$

$$\log \mathbf{N} = \mathbf{c} + \boldsymbol{\beta} \cdot \log\left(\mathbf{W}\right) \qquad \text{[Invariance]}$$

with $\boldsymbol{\beta} = \mathbf{2/3}$ for each theory.

Table: Intraday OLS Regression of $\log N$

	Nobs	c	β	se(c)	se(β)	$\bar{R}^2$
Clark	1335	2.41	0.976	0.0031	0.0016	0.997
Ané & Geman	1335	1.75	0.849	0.0018	0.0006	0.999
Invariance	1335	0.85	0.671	0.0034	0.0007	0.998

Suggestive Test for Alternative Theories

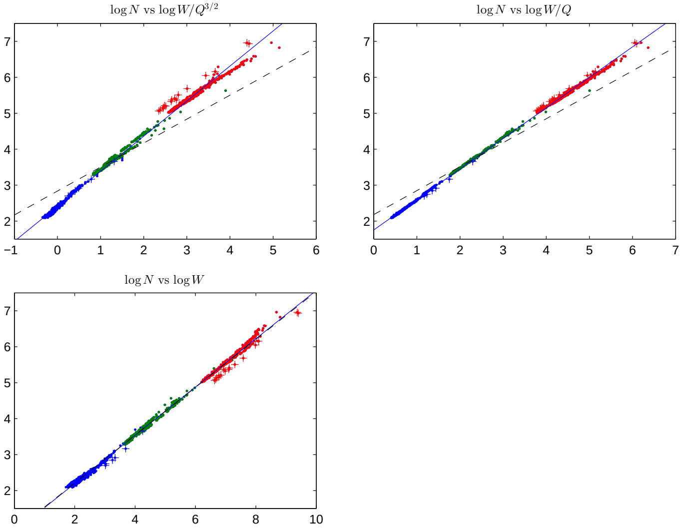

Figure: OLS Regression Line (solid) and Model Predicted (dashed).

Formal Tests of Alternative Theories

Regressions above (at best) informal; N on both sides $\rightarrow R^2$ inflated

Alternative representations: For **Clark**, **Ané & Geman** and **Invariance**

$$\boldsymbol{\sigma^2 \sim \mathbf{NQ}, \qquad \sigma^2 \sim \mathbf{N}, \qquad \sigma^2 \sim \mathbf{N/Q^2}.}$$

These imply, respectively, $\boldsymbol{\beta = 1}$, $\boldsymbol{\beta = 0}$, or $\boldsymbol{\beta = -2}$ below

$$\mathbf{s_t - n_t \;=\; \frac{1}{D} \sum_{d=1}^{D} \log\left(\sigma_{dt}^2 \,/\, N_{dt}\right) \;=\; c + \boldsymbol{\beta} \cdot q_t + \boldsymbol{\nu}_t}$$

Critical role of Trade Size

Nested Test of Alternative Theories

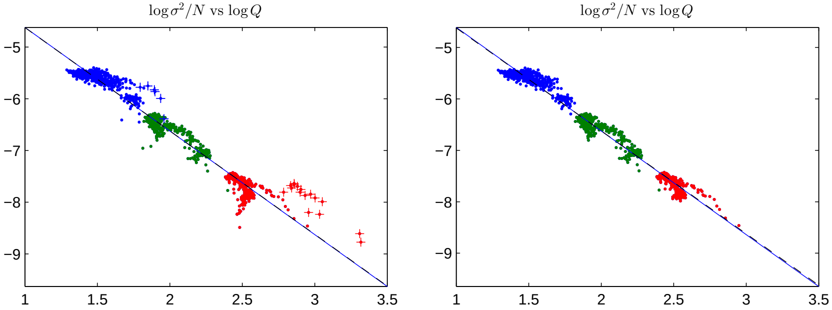

Figure: Scatter plots of $(s_t - n_t)$ versus q_t. Regression line (solid), Invariance predicted line (dashed). Crosses: first 6 min (blue) and last 16 min (red).

Right Panel Removes:

6 Mins at the Beginning of trading (Asia);
3 Mins at 1:00 and 2:00 (Europe);
3+30 Mins at 8:30 (America, 9:00 News);
16 Mins at the End of Trading.

Nested Test of Alternative Theories

Table: Indraday OLS Regression of $\log \frac{\sigma^2}{N}$ onto $\log Q$

	Nobs	c	β	se(c)	se(β)	$\bar{R}^2$
Unfiltered	1335	-2.61	-2.005	0.0205	0.0102	0.966
Filtered	1273	-2.59	-2.015	0.0161	0.0081	0.980

Invariance yields vastly superior fit to intraday activity patterns

Trading Invariance and Macro Announcements

Extremes?

Macro Announcements involve dramatic spikes

7:30 CT: Employment, CPI, PPI, Retail Sales, Housing Starts, . . .

9:00 CT: Home Sales, Confidence Survey, Factory Orders . . .

Trading Invariance for 7:30 Macro Announcements

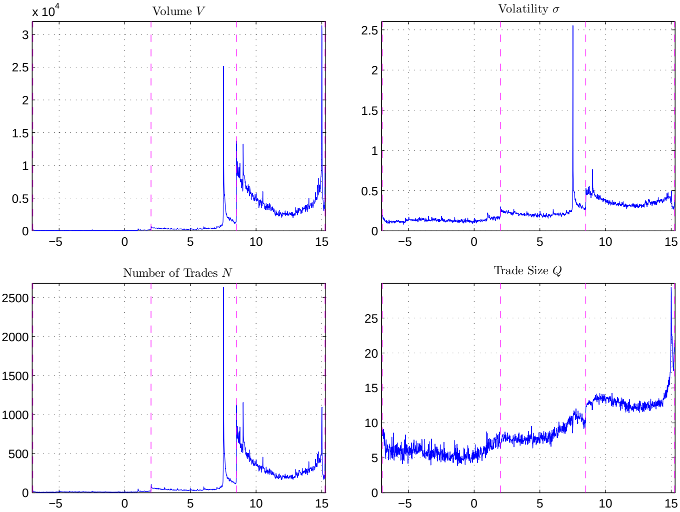

Figure: One-minute averages for 7:30 Announcement days.

Trading Invariance for 7:30 Macro Announcements

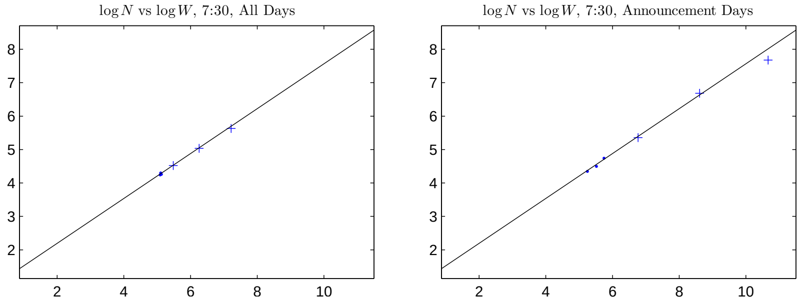

Figure: Left panel: All days, Right panel: 7:30 Announcement days. Solid line is prior OLS fit.

Trading Invariance during the Flash Crash

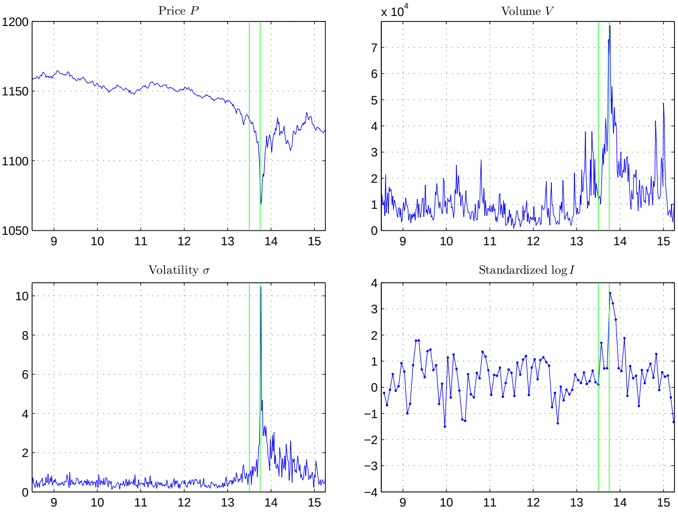

Figure: Market activity, May 6, 2010. 1-min observations, except $\log I$ 4-Min.

Trading Invariance during the Flash Crash

Invariance operative until last minute of Crash period – Fails during first 8-12 minutes following Crash.

Viewed across all trading days, these May 6 intervals fall in 78%, 100%, 99.9%, and 99.4% of $\log I$ distribution

This fits with **Menkveld and Yueshen (2013):** Co-integration of E-mini and SPDR fails in same period

Suggests **Intraday Trading Invariance** tied to standard operation and functioning of liquid financial markets

Implied Trade Size

Invariance has **Implied Trade Size**:

$$\mathbf{q}_t^* \;=\; c \;+\; \frac{1}{3}\left[\, v_t \;-\; s_t \,\right]$$

Can compare Actual and Implied log average trade size over days.

Evident pattern:

- Close of one Regime and Open of another creates deviation.
- **Regime Opening**: Trade Size lower than predicted
- **Regime Closing**: Trade Size higher than predicted

Asymmetric information concerns?

Failure: Market Regime Transitions

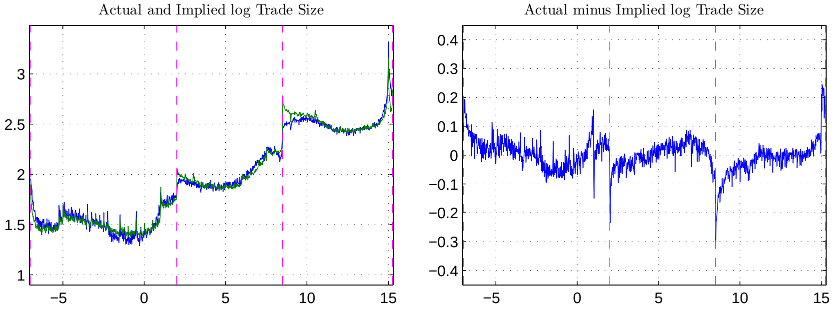

Figure: Intraday q (actual) and q^* (implied), and their difference (Prediction error).

Formal Tests – Aggregation within Days

Existing tests employ daily data. Can we mimic this?

Now aggregation within days, but only Regime-wise

Predictions: $\boldsymbol{\beta} = \mathbf{1}$, $\boldsymbol{\beta} = \mathbf{0}$, or $\boldsymbol{\beta} = -\mathbf{2}$

$$\mathbf{s_{di}} - \mathbf{n_{di}} = \frac{\mathbf{1}}{\mathbf{T_i}} \sum_{\mathbf{t=1}}^{\mathbf{T_i}} \mathbf{log}\left(\boldsymbol{\sigma}^{\mathbf{2}}_{\mathbf{dt}} / \mathbf{N_{dt}}\right) = \mathbf{c} + \boldsymbol{\beta} \cdot \mathbf{q_{di}} + \boldsymbol{\nu}_{\mathbf{di}}$$

where di indicates Regime i on Day d.

Suggestive Intertemporal Invariance Check

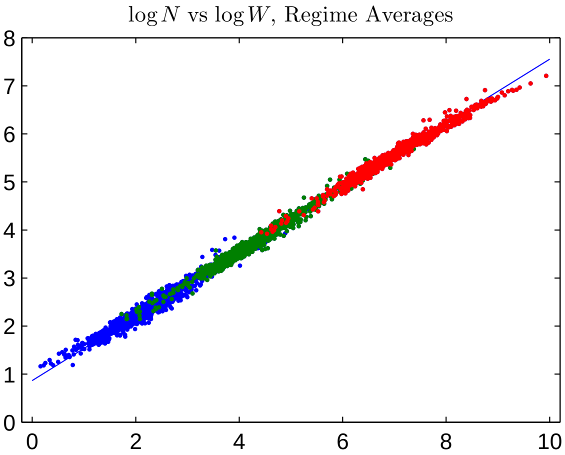

Figure: Scatter plot of n_{di} onto w_{di}. One observation per Regime. Slope is 0.668, $\bar{R}^2$ is 0.996

Formal Intertemporal Test

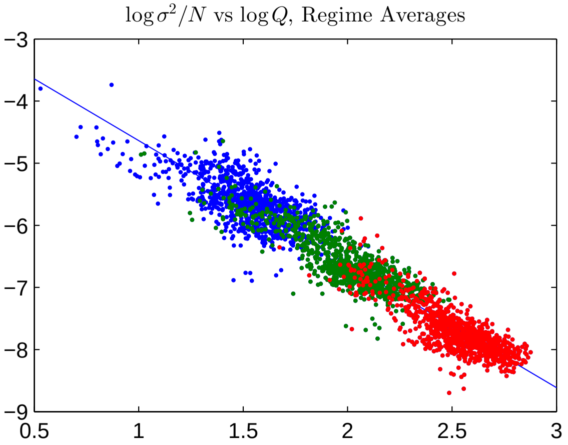

Figure: Time-series scatter plot of $s_{di} - n_{di}$ versus q_{di}. One observation per Regime. Slope is -1.98, $\bar{R}^2$ is 0.918.

Robustness Checks

Subsample Analysis:

Test Intraday Invariance for each Year

Test Intraday Invariance for each Regime

Test Intraday Invariance at High(er) Frequency

Binning with **105, 26, 5 Mins** in respective Regimes

- For all HF Observations
- For HF Observations each Year
- For HF Observations in each Regime

Robustness – High Frequency Observations

Table: OLS Regression of $\log N$: Binned Data, per Year

	Nobs	c	β	se(c)	se(β)	$\bar{R}^2$
2008	25108	0.882	0.657	0.0039	0.0006	0.982
2009	25357	0.737	0.674	0.0045	0.0007	0.976
2010	25416	0.724	0.685	0.0051	0.0008	0.968
2011	21706	0.798	0.671	0.0056	0.0008	0.968
All	97587	0.798	0.670	0.0024	0.0003	0.974

Table: OLS Regression of $\log N$: Binned Data, 3 Regimes

	Nobs	c	β	se(c)	se(β)	$\bar{R}^2$
Asia	4825	0.753	0.649	0.0055	0.0020	0.955
Europe	14497	0.882	0.650	0.0055	0.0012	0.956
America	78265	0.950	0.649	0.0045	0.0006	0.934

Robustness – High Frequency Observations

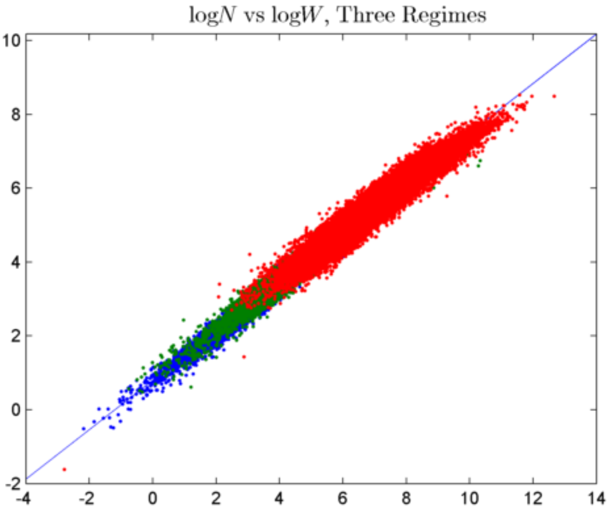

Figure: Scatter plot of n_{db} vs w_{db} with Binned data.

Conclusions

- **Intraday trading activity patterns** intimately related
- Traditional theories: Volume or Transactions govern Volatility
- Invariance (Kyle & Obizhaeva) motivates alternative relation
- Critically, Trade Size drops in specific proportion with Volatility
- For E-mini, tendency observed by Andersen & Bondarenko (RF, VPIN)
- Qualitative prediction verified for diurnal pattern
- Qualitative prediction verified for daily regimes (time series)
- Theoretical justification for Invariance in this context loom large
- How will findings generalize across market structures?