Intraday Trading Invariance in the E-Mini S&P 500 Futures Market

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Joint with
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Figure: S&P 500 E-mini. Sample averages per 1 min. Dashed lines separate trading hours in Asia, Europe, and America.
Hypotheses on How Trading Drives Volatility

Three Theories:

1. Volume drives volatility: Clark (1973)

Imply different *stochastic clocks*

Theories almost Invariably tested in *Time Series* (daily data)
Why High-Frequency Analysis

- Pronounced *intraday market activity patterns*
- News incorporated into prices quickly; Trading fast
- Huge systematic variation over 24-hour trading day
- Does any basic regularity apply in this setting?

- **Macroeconomic announcements** particular challenge
  - Large price jump on impact without (much) trading
  - Subsequent price discovery process

- **Sudden market turmoil**: Crisis, Flash Crash
  - Do same or different regularities apply in this context?
S&P 500 E-Mini Futures Market

- BBO files from CME; Jan 4, 2008 – Nov 2, 2011
  - Extraordinary active market – Price discovery for equities
  - Time-stamped to second, Sequenced in actual order
  - Use front month contract (most liquid)

- Three daily Regimes (CT):
  - Asia, 17:00 – 2:00
  - Europe, 2:00 – 8:30
  - America, 8:30 – 15:15

- \( D = 969 \) trading days; \( T = 1,335 \) 1-minute intervals per day
  - \( N_{dt} \) = Number of transactions per min;
  - \( V_{dt} \) = Volume (Number of contracts per min);
  - \( Q_{dt} \) = Average Trade Size;
  - \( P_{dt} \) = Average Price;
  - \( \sigma_{dt} \) = Volatility;
  - \( W_{dt} \) = Trading Activity (Dollars at Risk per min) = \( P_{dt} V_{dt} \sigma_{dt} \)
### Descriptive Statistics for S&P 500 E-mini Futures

|                  | Asia  | Europe | America | Combined
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Volatility</td>
<td>0.16</td>
<td>0.25</td>
<td>0.40</td>
<td>0.26</td>
</tr>
<tr>
<td>Volume</td>
<td>95</td>
<td>601</td>
<td>4726</td>
<td>1647</td>
</tr>
<tr>
<td># Trades</td>
<td>14</td>
<td>67</td>
<td>360</td>
<td>134</td>
</tr>
<tr>
<td>Notional Value, $Mln</td>
<td>5</td>
<td>34</td>
<td>266</td>
<td>93</td>
</tr>
<tr>
<td>Trade Size</td>
<td>5.9</td>
<td>8.4</td>
<td>13.3</td>
<td>8.9</td>
</tr>
<tr>
<td>Market Depth</td>
<td>54</td>
<td>265</td>
<td>984</td>
<td>398</td>
</tr>
<tr>
<td>Bid-Ask Spread</td>
<td>26.5</td>
<td>25.7</td>
<td>25.1</td>
<td>25.9</td>
</tr>
<tr>
<td>Business Time</td>
<td>24.6</td>
<td>5.8</td>
<td>1.0</td>
<td>12.0</td>
</tr>
</tbody>
</table>

**Notes:** Sample averages per 1 min. Volatility is annualized (in decimal form). Business Time is proportional to $W^{-2/3}$ (normalized to 1 in America)
Market Microstructure Invariance

**Market Microstructure Invariance (MMI):** Risk transfers, transactions costs, resilience, market depth, etc., are constant across assets and market environments when measured in units of business time.

In particular, **Dollar-Risk Transfer per Bet in Business Time is i.i.d.**

\[ I = P \cdot Q_B \cdot \sigma \cdot N_B^{-1/2} \]

where \( Q_B \) = Bet size and \( N_B \) = expected number of Bets.

Define “Trading Activity” \( W = P \cdot V \cdot \sigma \), then

\[ N_B \sim W^{2/3} \quad \text{and} \quad Q_B \sim W^{1/3} \]

**Interpretation:** Since \( V = Q_B \cdot N_B \),

Variation in Volume: 2/3 from \( N_B \), 1/3 from \( Q_B \).
Bets are not observable; $Q_B$, $N_B$, $\sigma$ and latent variables.

Invoke auxiliary hypotheses to develop testable variant of MMI:

$$I_{dt} = P_{dt} \cdot Q_{dt} \cdot \sigma_{dt} \cdot N_{dt}^{-1/2}$$

Replace unobserved $Q_B$ and arrival rate $N_B$ with observed parallels, Trade Size $Q$ and expected Number of trades $N$

Proxy $N$ by observed transactions, estimate $\sigma$ by RV using HF returns

**Note:** Intraday variation in expected price change trivial. Henceforth, for intraday tests, we ignore variation in $P$
**Intraday Trading Invariance**

**Invariance Inspired Hypothesis:** \( \log(N_{dt}) = c + \beta \cdot \log(W_{dt}) \)

where Invariance predicts slope of \( \beta = \frac{2}{3} \).

Relies on expectations approximated by realizations or noisy estimators

Aggregate relationship across days to diversify measurement errors

\[
n_t = \frac{1}{D} \sum_{d=1}^{D} \log(N_{dt}) = c + \beta \cdot \left[ \frac{1}{D} \sum_{d=1}^{D} \log(W_{dt}) \right] + \nu_t
\]

for \( t = 1, \ldots, T = 1,335 \).
Figure: Asia (blue), Europe (green), America (red). Crosses: First 6 min of trading (blue) and last 16 min (red). Solid line: \( n_t = c + 0.671 \cdot w_t \); Dashed line: Same slope, Fit to red crosses.
Suggestive Test for Alternative Theories

Ignoring $P$, the three theories may be restated as

\[
\log N = c + \beta \cdot \log \left( W/Q^{\frac{3}{2}} \right) \quad \text{[Clark]}
\]

\[
\log N = c + \beta \cdot \log (W/Q) \quad \text{[Ané & Geman]}
\]

\[
\log N = c + \beta \cdot \log (W) \quad \text{[Invariance]}
\]

with $\beta = 2/3$ for each theory.

**Table: Intraday OLS Regression of $\log N$**

<table>
<thead>
<tr>
<th></th>
<th>Nobs</th>
<th>$c$</th>
<th>$\beta$</th>
<th>$se(c)$</th>
<th>$se(\beta)$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clark</td>
<td>1335</td>
<td>2.41</td>
<td>0.976</td>
<td>0.0031</td>
<td>0.0016</td>
<td>0.997</td>
</tr>
<tr>
<td>Ané &amp; Geman</td>
<td>1335</td>
<td>1.75</td>
<td>0.849</td>
<td>0.0018</td>
<td>0.0006</td>
<td>0.999</td>
</tr>
<tr>
<td>Invariance</td>
<td>1335</td>
<td>0.85</td>
<td>0.671</td>
<td>0.0034</td>
<td>0.0007</td>
<td>0.998</td>
</tr>
</tbody>
</table>
Suggestive Test for Alternative Theories

Figure: OLS Regression Line (solid) and Model Predicted (dashed).
Formal Tests of Alternative Theories

Regressions above (at best) informal; $N$ on both sides $\rightarrow R^2$ inflated

Alternative representations: For Clark, Ané & Geman and Invariance

$$\sigma^2 \sim NQ \quad \sigma^2 \sim N \quad \sigma^2 \sim N/Q^2.$$  

These imply, respectively, $\beta = 1$, $\beta = 0$, or $\beta = -2$ below

$$s_t - n_t = \frac{1}{D} \sum_{d=1}^{D} \log \left( \frac{\sigma^2_{dt}}{N_{dt}} \right) = c + \beta \cdot q_t + \nu_t$$

Critical role of Trade Size
Nested Test of Alternative Theories

Figure: Scatter plots of \((s_t - n_t)\) versus \(q_t\). Regression line (solid), Invariance predicted line (dashed). Crosses: first 6 min (blue) and last 16 min (red).

Right Panel Removes:
- 6 Mins at the Beginning of trading (Asia);
- 3 Mins at 1:00 and 2:00 (Europe);
- 3+30 Mins at 8:30 (America, 9:00 News);
- 16 Mins at the End of Trading.
**Table:** Indraday OLS Regression of $\log \frac{\sigma^2}{N}$ onto $\log Q$

<table>
<thead>
<tr>
<th></th>
<th>Nobs</th>
<th>$c$</th>
<th>$\beta$</th>
<th>se($c$)</th>
<th>se($\beta$)</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfiltered</td>
<td>1335</td>
<td>-2.61</td>
<td>-2.005</td>
<td>0.0205</td>
<td>0.0102</td>
<td>0.966</td>
</tr>
<tr>
<td>Filtered</td>
<td>1273</td>
<td>-2.59</td>
<td>-2.015</td>
<td>0.0161</td>
<td>0.0081</td>
<td>0.980</td>
</tr>
</tbody>
</table>

Invariance yields vastly superior fit to intraday activity patterns
Extremes?

Macro Announcements involve dramatic spikes

7:30 CT: Employment, CPI, PPI, Retail Sales, Housing Starts, ...

9:00 CT: Home Sales, Confidence Survey, Factory Orders ...
Figure: One-minute averages for 7:30 Announcement days.
Figure: Left panel: All days, Right panel: 7:30 Announcement days. Solid line is prior OLS fit.
Figure: Market activity, May 6, 2010. 1-min observations, except log $I$ 4-Min.
Invariance operative until last minute of Crash period – Fails during first 8-12 minutes following Crash.

Viewed across all trading days, these May 6 intervals fall in 78%, 100%, 99.9%, and 99.4% of log $I$ distribution

This fits with Menkveld and Yueshen (2013): Co-integration of E-mini and SPDR fails in same period

Suggests Intraday Trading Invariance tied to standard operation and functioning of liquid financial markets
Implied Trade Size

Invariance has **Implied Trade Size**: 

$$q_t^* = c + \frac{1}{3} [v_t - s_t]$$

Can compare Actual and Implied log average trade size over days.

**Evident pattern:**

- Close of one Regime and Open of another creates deviation.
- **Regime Opening**: Trade Size lower than predicted
- **Regime Closing**: Trade Size higher than predicted

**Asymmetric information concerns?**
Figure: Intraday $q$ (actual) and $q^*$ (implied), and their difference (Prediction error).
Existing tests employ daily data. Can we mimic this?

Now aggregation within days, but only Regime-wise

Predictions: \( \beta = 1, \beta = 0, \) or \( \beta = -2 \)

\[
s_{di} - n_{di} = \frac{1}{T_i} \sum_{t=1}^{T_i} \log \left( \frac{\sigma_{dt}^2}{N_{dt}} \right) = c + \beta \cdot q_{di} + \nu_{di}
\]

where \( di \) indicates Regime \( i \) on Day \( d \).
Figure: Scatter plot of $n_{di}$ onto $w_{di}$. One observation per Regime. Slope is 0.668, $\bar{R}^2$ is 0.996
Intraday Trading Invariance

Figure: Time-series scatter plot of $s_{di} - n_{di}$ versus $q_{di}$. One observation per Regime. Slope is -1.98, $\bar{R}^2$ is 0.918.
Robustness Checks

Subsample Analysis:

Test Intraday Invariance for each Year

Test Intraday Invariance for each Regime

Test Intraday Invariance at High(er) Frequency

**Binning** with **105, 26, 5 Mins** in respective Regimes

- For all HF Observations
- For HF Observations each Year
- For HF Observations in each Regime
### Table: OLS Regression of log $N$: Binned Data, per Year

<table>
<thead>
<tr>
<th></th>
<th>Nobs</th>
<th>$c$</th>
<th>$\beta$</th>
<th>se($c$)</th>
<th>se($\beta$)</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>25108</td>
<td>0.882</td>
<td>0.657</td>
<td>0.0039</td>
<td>0.0006</td>
<td>0.982</td>
</tr>
<tr>
<td>2009</td>
<td>25357</td>
<td>0.737</td>
<td>0.674</td>
<td>0.0045</td>
<td>0.0007</td>
<td>0.976</td>
</tr>
<tr>
<td>2010</td>
<td>25416</td>
<td>0.724</td>
<td>0.685</td>
<td>0.0051</td>
<td>0.0008</td>
<td>0.968</td>
</tr>
<tr>
<td>2011</td>
<td>21706</td>
<td>0.798</td>
<td>0.671</td>
<td>0.0056</td>
<td>0.0008</td>
<td>0.968</td>
</tr>
<tr>
<td>All</td>
<td>97587</td>
<td>0.798</td>
<td>0.670</td>
<td>0.0024</td>
<td>0.0003</td>
<td>0.974</td>
</tr>
</tbody>
</table>

### Table: OLS Regression of log $N$: Binned Data, 3 Regimes

<table>
<thead>
<tr>
<th></th>
<th>Nobs</th>
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<th>$\beta$</th>
<th>se($c$)</th>
<th>se($\beta$)</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>4825</td>
<td>0.753</td>
<td>0.649</td>
<td>0.0055</td>
<td>0.0020</td>
<td>0.955</td>
</tr>
<tr>
<td>Europe</td>
<td>14497</td>
<td>0.882</td>
<td>0.650</td>
<td>0.0055</td>
<td>0.0012</td>
<td>0.956</td>
</tr>
<tr>
<td>America</td>
<td>78265</td>
<td>0.950</td>
<td>0.649</td>
<td>0.0045</td>
<td>0.0006</td>
<td>0.934</td>
</tr>
</tbody>
</table>
Figure: Scatter plot of $n_{db}$ vs $\omega_{db}$ with Binned data.
Conclusions

- **Intraday trading activity patterns** intimately related
- Traditional theories: Volume or Transactions govern Volatility
- Invariance (Kyle & Obizhaeva) motivates alternative relation
- Critically, Trade Size drops in specific proportion with Volatility
- For E-mini, tendency observed by Andersen & Bondarenko (RF, VPIN)
- Qualitative prediction verified for diurnal pattern
- Qualitative prediction verified for daily regimes (time series)
- Theoretical justification for Invariance in this context loom large
- How will findings generalize across market structures?