Equity Derivatives And Linked Default Intensity

rag·top
ˈragtäp/
noun
noun: ragtop; plural noun: ragtops; noun: rag-top; plural noun: rag-tops
1 a car with a convertible roof.

devtools::install_github('brianboonstra/ragtop')
Motivation

• Existing open-source derivatives pricing libraries lack important features

  • They handle only basic SDEs and ignore term structures of parameters

  • Calibration is at best an afterthought

• A great test case is convertible bonds

  • Proper pricing requires must treat subtle features of the bond and underlying

  • To a Q quant, convertible bonds are the most interesting asset class
I DON'T ALWAYS TRADE DERIVATIVES
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BUT WHEN I DO I HEDGE WITH MEXICAN BEER
Goals

- Reasonable stochastic process suitable for addressing convertible bonds, provides fancy pricing abilities to equity options as a special case
- Efficient pricing of multiple instruments at once
- Calibration included
- Pricing consistent with implied volatility skew
- Hasn’t somebody done this already? What about QuantLib?
Problems With QuantLib

- No equity-linked default intensity
- Does not handle discrete dividends that mix proportional and fixed
- Discrete pricing dates only: unsuitable for use close to option maturity
- Only available convertible bond model is 1980s era mixed discounting
- A lot of mental/coding overhead to handle simple cases
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Valeant Skew, 28 Day Options
Somewhat Explainable By Default Risk

Def. Intensity (%)

VRX=33.38
Problems With QuantLib

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Convertible Bond Value, Equity–Linked Default Model

- CB Unlinked
- Straight Bond Unlinked

Underlying

Convertible

Conversion Value
Out-of-the-Money Put, Linking Is Important

- Linked
- Unlinked

Put vs. Underlying
Problems With QuantLib

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Effect of Fixed Versus Proportional Discrete Dividends

Out-Of-Money Put Price vs. Dividend Proportion Assumed To Be Fixed (%)
Choices

- Use a 2D stochastic process
  - First dimension is Black-Scholes with term structures of deterministic coefficients
  - Second dimension is jump to bankruptcy
  - Link equity level to default jump intensity

\[
\frac{dS_t}{S_t} = (r(t) + h(S_t, t) - q(t))dt + \sigma(t)dZ - dJ(h(S_t, t))
\]
• Convert to a PDE using usual Feynman-Kac

\[
\frac{\partial V}{\partial t} - rV + h(\delta - V) + (r - q + h) S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = 0
\]
• Convert to a PDE using usual Feynman-Kac

• Solve using implicit finite difference scheme with Neumann boundary conditions
  • Include reasonable default grid parameters
  • Increase stability by working in log(S) space

• Calibration
  • Stage for efficiency: ATM volatilities, then the rest
  • Penalties computed on implied volatilities, to regularize

devtools::install_github('brianboonstra/ragtop')
Example

• You may think Tesla does not have convertibles

• In fact, they have millions

• March 1, 2019 Bond 88160RAB7
  • 0.25% Coupon
  • Each $1000 bond convertible for 2.7788 shares
  • “Green shoe” compound option
Analysis Process

- Choose a simple 3-parameter functional form for default intensity
  \[
  h(S) = h_0 \left( s + (1 - s) \left( \frac{S_0}{S} \right)^p \right)
  \]
- Calibrate model parameters to more liquid market instruments
  - Rates from treasury curve
  - Overall default intensity from lookup tables
  - Volatility and default intensity shape from equity options
Some Equity Option Data

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<th>time</th>
<th>mid</th>
<th>bid</th>
<th>ask</th>
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<td>205.225</td>
<td>203.45</td>
<td>207.00</td>
<td>3.55</td>
</tr>
</tbody>
</table>

...and 675 more.
Fit Volatilities And Default Intensity

disct_fcn = \texttt{ragtop::spot\_to\_df\_fcn(ragtop::TSLAMarket}\$\texttt{risk\_free\_rates)}

\texttt{S0 = ragtop::TSLAMarket}\$\texttt{S0}

cb = \texttt{ragtop::ConvertibleBond(}
\texttt{  maturity=2.87, conversion\_ratio=2.7788, notional=1000,}
\texttt{  coupons=\texttt{data.frame(payment\_time=seq(2.8, 0, by=-0.25)},}
\texttt{    payment\_size=1000*0.0025/4),}
\texttt{  discount\_factor\_fcn = disct\_fcn,}
\texttt{  name='TSLA\_CB'}}
Fit Volatilities And Default Intensity

```r
fit = ragtop::fit_to_option_market_df(  
  S0 = TSLAMarket$S0,  
  discount_factor_fcn = disct_fcn,  
  options_df = ragtop::TSLAMarket$options,  
  base_default_intensity = 0.05  
)
```
cb_by_S = `ragtop::form_present_value_grid`(S0=S0, grid_center=S0,
instruments=list(Convertible=cb),
um_time_steps=250,
default_intensity_fcn=fit$default_intensity_fcn,
discount_factor_fcn = disct_fcn,
variance_cumulation_fcn=fit$variance$cumulation_function,
std_devs_width=5)
Plot Our Results

cbgrid = na.omit(as.data.frame(cb_by_S))
present_value = spline(x=cbgrid[, "Underlying"],
  y=cbgrid[, "Convertible"],
  xout=S0)$y

cbplot = ( ggplot(cbgrid,
  aes(x=Underlying, y=Convertible)) +
  geom_line(size=1.2) +
  scale_x_continuous(limits=c(0, 2.5*S0)) +
  scale_y_continuous(limits=c(0, 2.5*cb$notional)) +
  geom_point(aes(x=S0, y=present_value), color="red") +
  labs(title="Convertible Bond Value")
)
What Can We Do With This?

- Price American and European options with *fixed* dividends
- Correct for skew with equity-linked default
- New (path-independent) payoffs are easy to add to the zoo
- Fit volatilities and default intensity functions simultaneously
- Price convertibles or other instruments consistently with economic intuition
- Local volatility model is a possible extension