Equity Derivatives And Linked Default Intensity

rag·top
ˈragtäp/
noun
noun: **ragtop**; plural noun: **ragtops**; noun: **rag-top**; plural noun: **rag-tops**
1 a car with a convertible roof.

```
devtools::install_github('brianboonstra/ragtop')
```

Motivation

- Existing open-source derivatives pricing libraries lack important features
  - They handle only basic SDEs and ignore term structures of parameters
  - Calibration is at best an afterthought
- A great test case is convertible bonds
  - Proper pricing requires must treat subtle features of the bond and underlying
  - To a Q quant, convertible bonds are the most interesting asset class


I DON'T ALWAYS TRADE DERIVATIVES


I DON'T ALWAYS TRADE DERIVATIVES
BUT WHEN I DO I HEDGE WITH MEXICAN BEER

Goals

- Reasonable stochastic process suitable for addressing convertible bonds, provides fancy pricing abilities to equity options as a special case
- Efficient pricing of multiple instruments at once
- Calibration included
- Pricing consistent with implied volatility skew
- Hasn't somebody done this already? What about QuantLib?

Problems With QuantLib

- No equity-linked default intensity
- Does not handle discrete dividends that mix proportional and fixed
- Discrete pricing dates only: unsuitable for use close to option maturity
- Only available convertible bond model is 1980s era mixed discounting
- A lot of mental/coding overhead to handle simple cases

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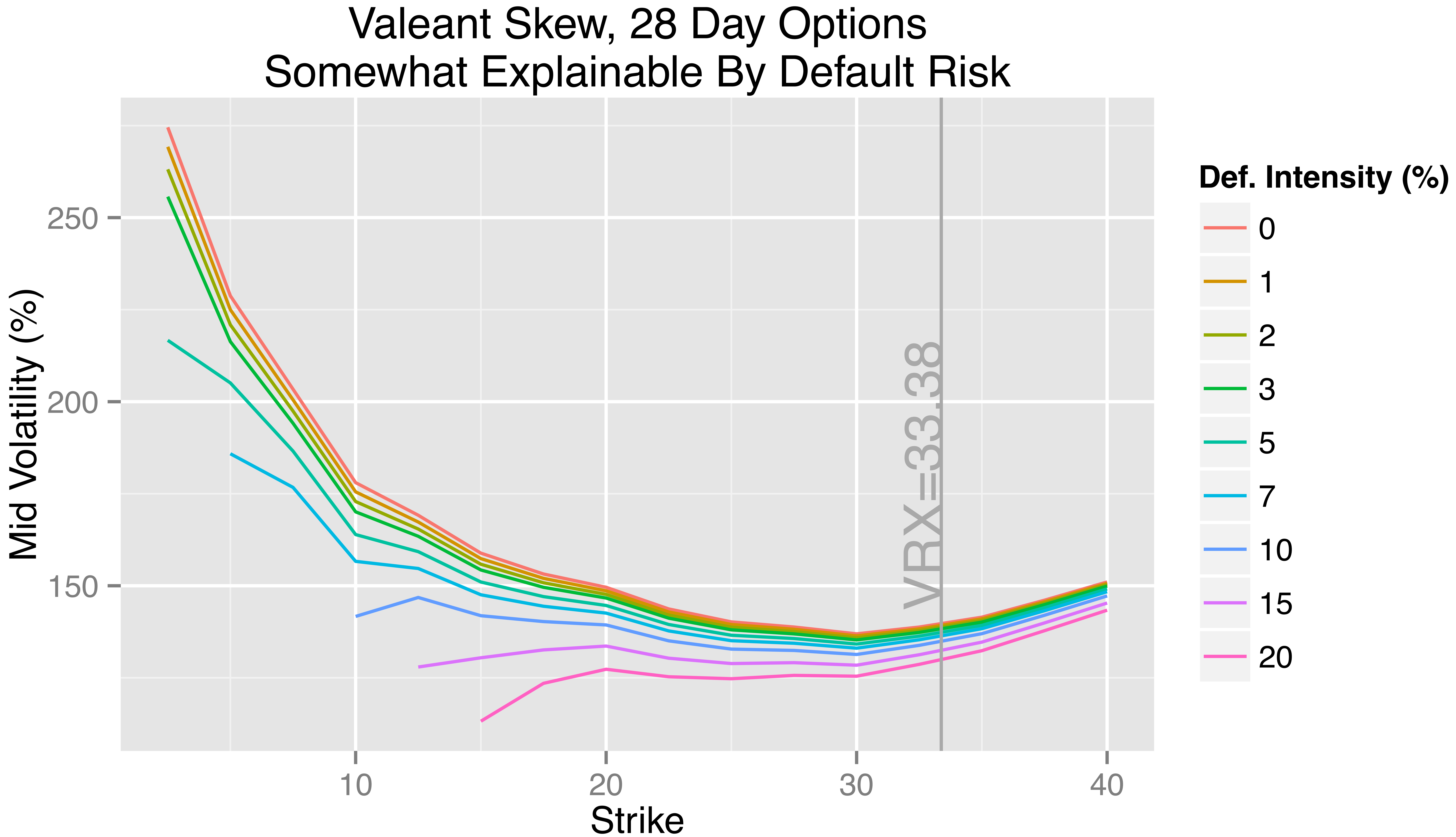
Valeant Skew, 28 Day Options
Somewhat Explainable By Default Risk
Mid Volatility (%)
250
200
150
10
20
30
40
Strike
VRX=33.38
Def. Intensity (%)
0
1
2
3
5
7
10
15
20

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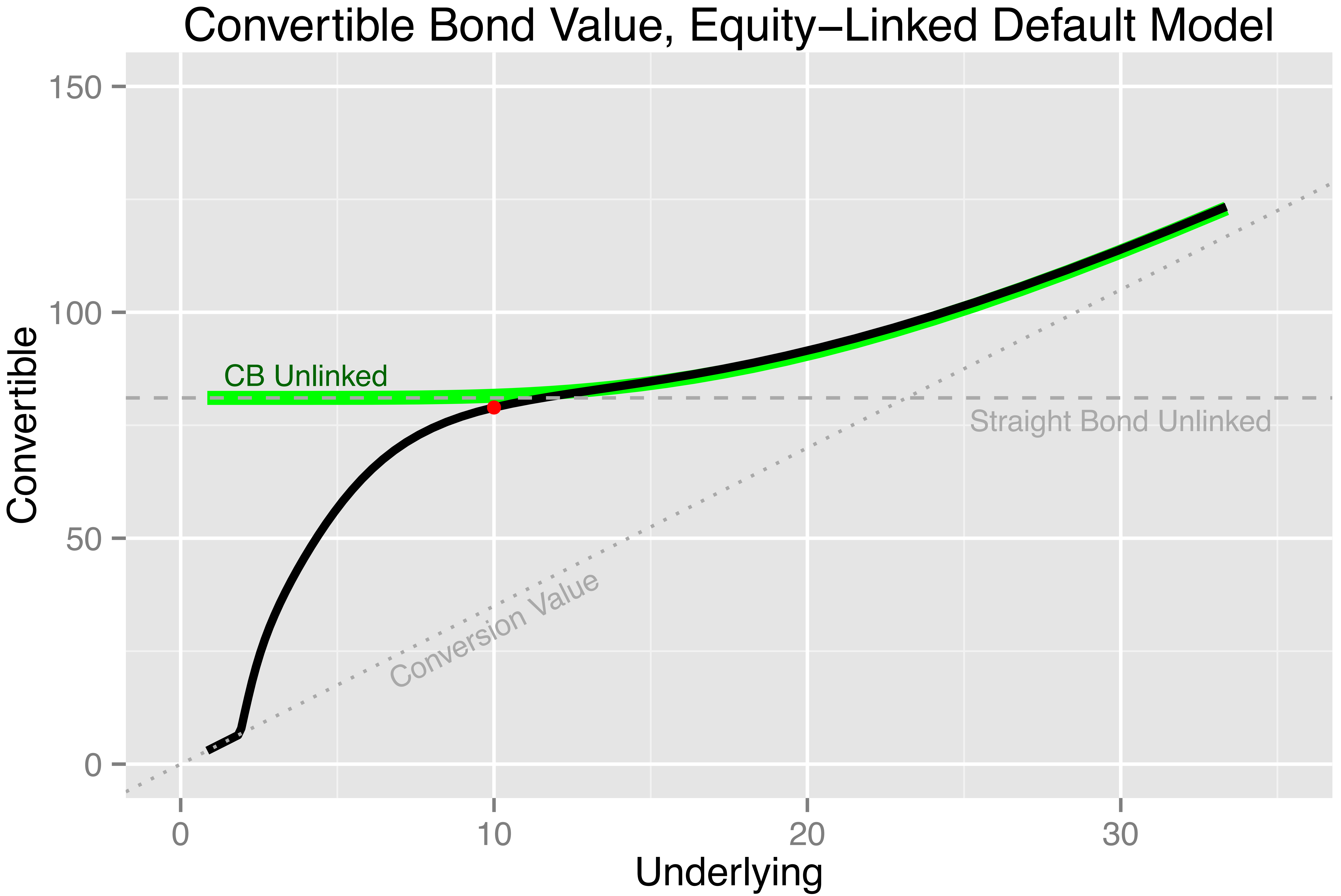
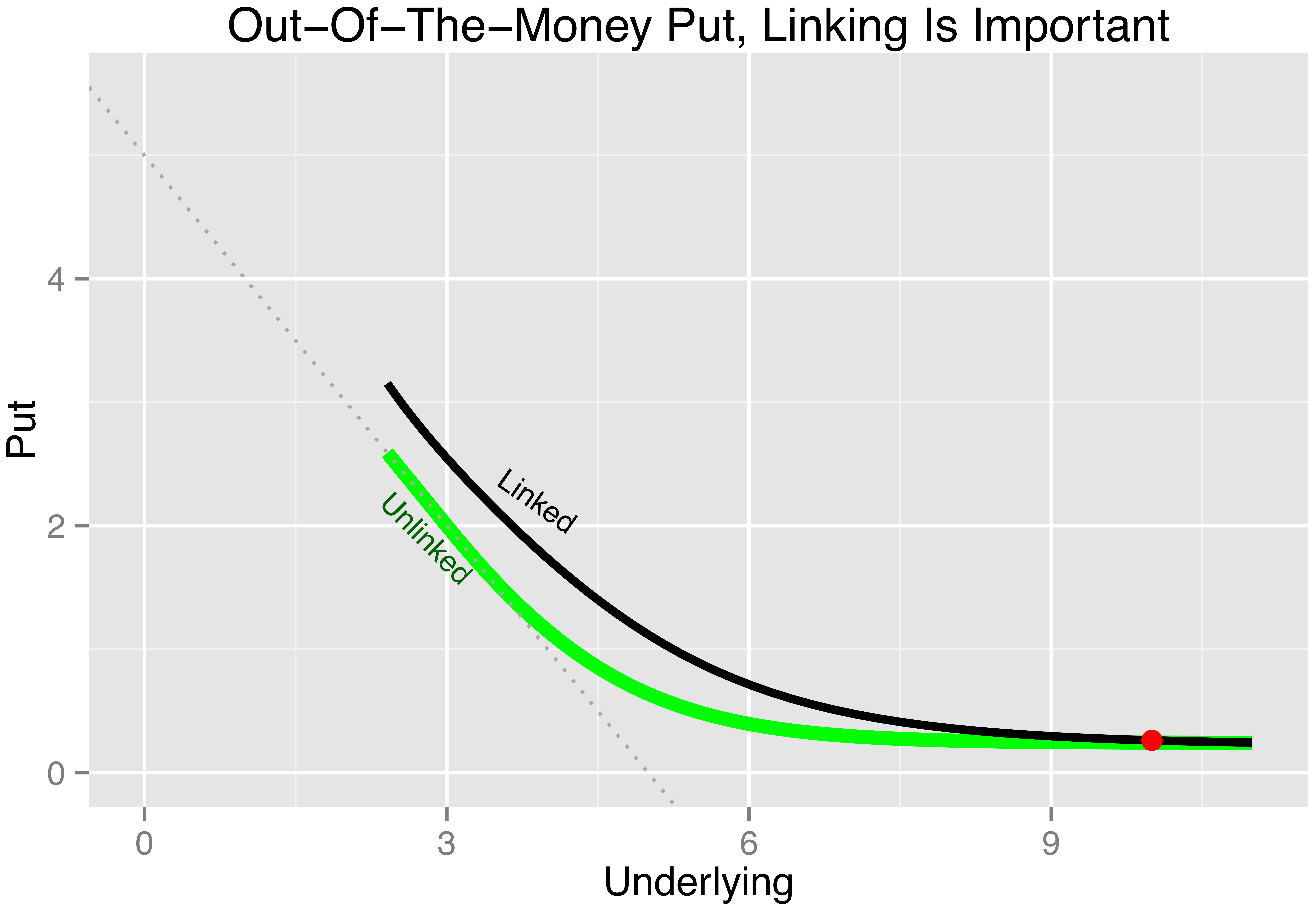

Convertible Bond Value, Equity−Linked Default Model
150
100
50
0
0
10
20
30
Underlying
Convertible
CB Unlinked
Straight Bond Unlinked
Conversion Value


Out–Of–The–Money Put, Linking Is Important
Put
Underlying
Linked
Unlinked
0
2
4
0
3
6
9

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Effect of Fixed Versus Proportional
Discrete Dividends
Out–Of–Money Put Price
7.4
7.2
7.0
6.8
0
25
50
75
100
Dividend Proportion Assumed To Be Fixed (%)

Choices

- Use a 2D stochastic process
  - First dimension is Black-Scholes with term structures of deterministic coefficients
  - Second dimension is jump to bankruptcy
  - Link equity level to default jump intensity

$$\frac{dS_t}{S_t} = (r(t) + h(S_t, t) - q(t))dt + \sigma(t)dZ - dJ(h(S_t, t))$$

- Convert to a PDE using usual Feynman-Kac

$$\frac{\partial V}{\partial t} - rV + h(\delta - V) + (r - q + h)\, S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = 0$$

```
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```

- Convert to a PDE using usual Feynman-Kac
- Solve using implicit finite difference scheme with Neumann boundary conditions
  - Include reasonable default grid parameters
  - Increase stability by working in log(S) space
- Calibration
  - Stage for efficiency: ATM volatilities, then the rest
  - Penalties computed on implied volatilities, to regularize

```
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```


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Example

- You may think Tesla does not have convertibles
- In fact, they have millions
- March 1, 2019 Bond 88160RAB7
  - 0.25% Coupon
  - Each $1000 bond convertible for 2.7788 shares
  - “Green shoe” compound option

Analysis Process

- Choose a simple 3-parameter functional form for default intensity

$$h(S) = h_0 \left(s + (1 - s) \left(\frac{S_0}{S} \right)^p \right)$$

- Calibrate model parameters to more liquid market instruments
    - Rates from treasury curve
    - Overall default intensity from lookup tables
    - Volatility and default intensity shape from equity options

Some Equity Option Data

callput	K	time	mid	bid	ask	spread
1	140	0.1265982	101.775	100.05	103.50	3.45
-1	300	0.1265982	61.200	60.00	62.40	2.40
1	360	0.3759132	1.745	1.57	1.92	0.35
1	185	0.6252283	64.750	63.80	65.70	1.90
-1	440	0.6252283	205.225	203.45	207.00	3.55

…and 675 more.

Fit Volatilities And Default Intensity

```
disct_fcn = ragtop::spot_to_df_fcn(ragtop::TSLAMarket$risk_free_rates)

S0 = ragtop::TSLAMarket$S0

cb = ragtop::ConvertibleBond(
  maturity=2.87, conversion_ratio=2.7788, notional=1000,
  coupons=data.frame(payment_time=seq(2.8,0, by=-0.25),
                     payment_size=1000*0.0025/4),
  discount_factor_fcn = disct_fcn,
  name='TSLA_CB'
)
```

Fit Volatilities And Default Intensity

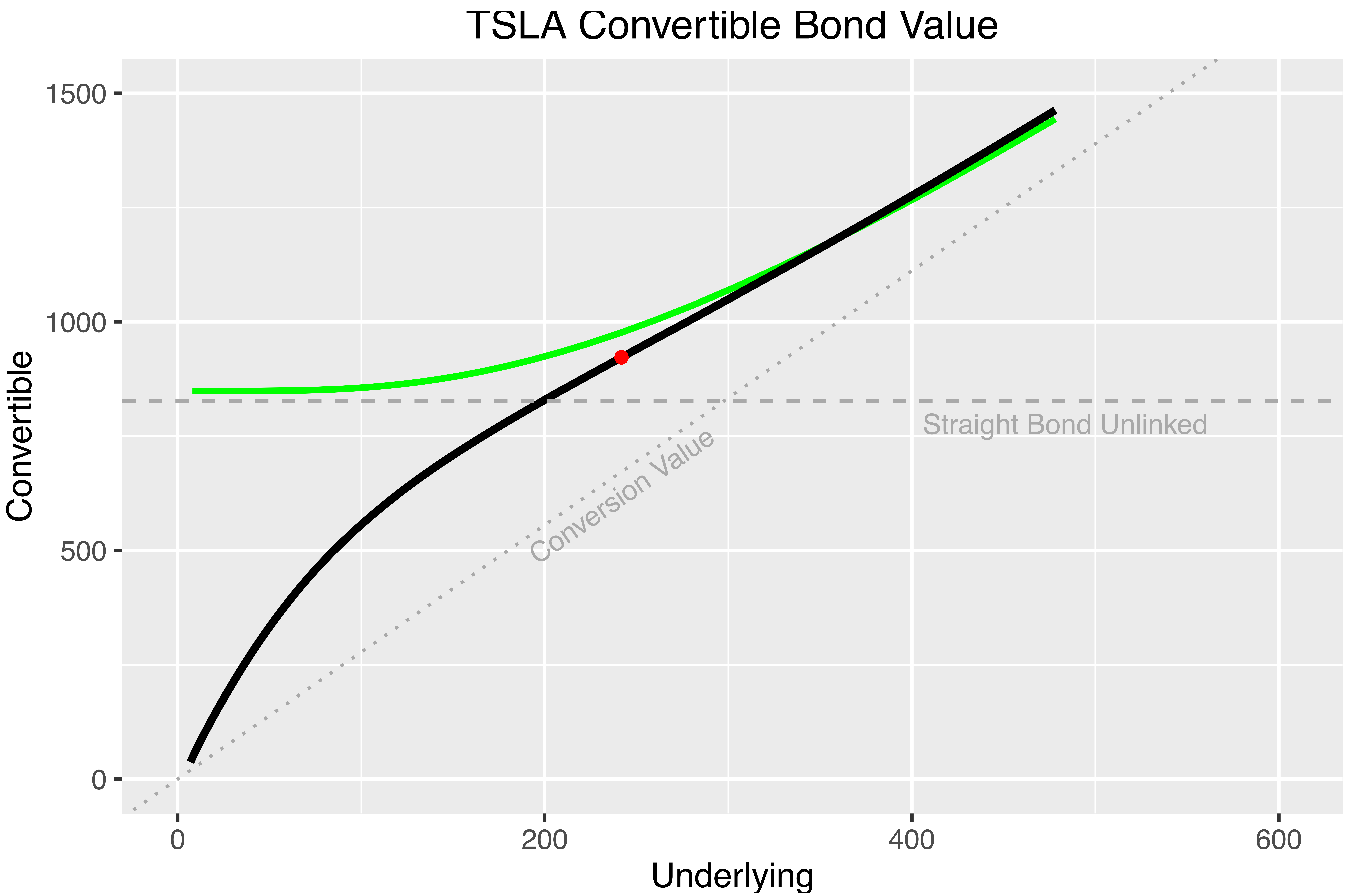
```
fit = ragtop::fit_to_option_market_df(
  S0 = TSLAMarket$S0,
  discount_factor_fcn = disct_fcn,
  options_df = ragtop::TSLAMarket$options,
  base_default_intensity=0.05
)
```

Price The Convertible Bond

```
cb_by_S = ragtop::form_present_value_grid(
  S0=S0, grid_center=S0,
  instruments=list(Convertible=cb),
  num_time_steps=250,
  default_intensity_fcn=fit$default_intensity_fcn,
  discount_factor_fcn = disct_fcn,
  variance_cumulation_fcn=fit$variance$cumulation_function,
  std_devs_width=5)
```

Plot Our Results

```
cbgrid = na.omit(as.data.frame(cb_by_S))
present_value = spline(x=cbgrid[,"Underlying"],
                       y=cbgrid[,"Convertible"],
                       xout=S0)$y
cbplot = ( ggplot(cbgrid,
                  aes(x=Underlying,y=Convertible)) +
             geom_line(size=1.2) +
             scale_x_continuous(limits=c(0,2.5*S0)) +
             scale_y_continuous(limits=c(0,2.5*cb$notional)) +
             geom_point(aes(x=S0,y=present_value), color="red") +
             labs(title="Convertible Bond Value")
)
```



TSLA Convertible Bond Value
Convertible
Underlying
1500
1000
500
0
0
200
400
600
Straight Bond Unlinked
Conversion Value

What Can We Do With This?

- Price American and European options with *fixed* dividends
- Correct for skew with equity-linked default
- New (path-independent) payoffs are easy to add to the zoo
- Fit volatilities and default intensity functions simultaneously
- Price convertibles or other instruments consistently with economic intuition
- Local volatility model is a possible extension