# Estimating Global Bank Network Connectedness

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# Financial Connectedness

- Market Risk
- Credit Risk
- Systemic Risk
- Counterparty Risk

# A Very General Environment

$$x_t = B(L)\,\varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

$$\boxed{C(x, B, \Sigma)}$$

# A Natural Financial/Economic Connectedness Question:

*What fraction of the H-step-ahead prediction-error variance of variable $i$ is due to shocks in variable $j$, $j \neq i$?*

# Variance Decompositions for Connectedness

$N$-Variable Connectedness Table

| | $x_1$ | $x_2$ | ... | $x_N$ | From Others to $i$ |
|---|---|---|---|---|---|
| $x_1$ | $d_{11}^H$ | $d_{12}^H$ | $\cdots$ | $d_{1N}^H$ | $\Sigma_{j=1}^N d_{1j}^H, j \neq 1$ |
| $x_2$ | $d_{21}^H$ | $d_{22}^H$ | $\cdots$ | $d_{2N}^H$ | $\Sigma_{j=1}^N d_{2j}^H, j \neq 2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ |
| $x_N$ | $d_{N1}^H$ | $d_{N2}^H$ | $\cdots$ | $d_{NN}^H$ | $\Sigma_{j=1}^N d_{Nj}^H, j \neq N$ |
| To Others<br>From $j$ | $\Sigma_{i=1}^N d_{i1}^H$<br>$i \neq 1$ | $\Sigma_{i=1}^N d_{i2}^H$<br>$i \neq 2$ | $\cdots$ | $\Sigma_{i=1}^N d_{iN}^H$<br>$i \neq N$ | $\Sigma_{i,j=1}^N d_{ij}^H$<br>$i \neq j$ |

Upper-left block is variance decomposition matrix, $D^H$

Connectedness involves the **non-diagonal** elements of $D^H$

*MES*, *CoVaR*

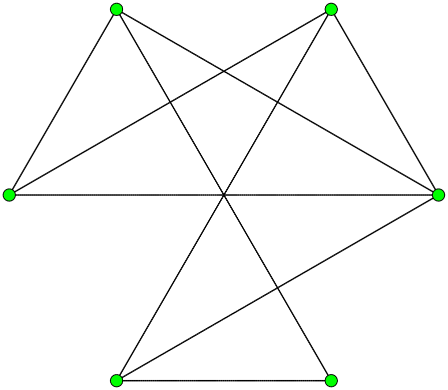
# Network Theory, Regularization, and Network Visualization

Network Theory:
The key to confidence in our connectedness measures

Regularization:
The key to network estimation in high dimensions

Visualization:
The key to network understanding in high dimensions

# Network Representation: Graph and Matrix



$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Symmetric adjacency matrix A

$A_{ij} = 1$ if nodes $i$, $j$ linked

$A_{ij} = 0$ otherwise

# Network Connectedness: The Degree Distribution

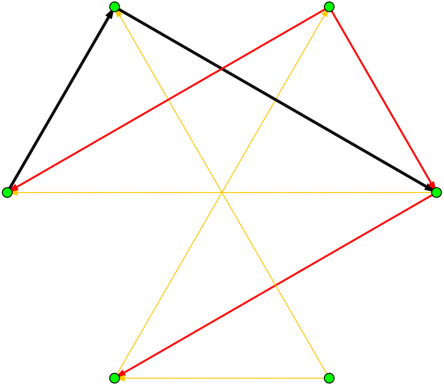
*Degree of node i*, $d_i$:

$$d_i = \sum_{j=1}^{N} A_{ij}$$

Discrete *degree distribution* on 0, ..., $N - 1$

*Mean degree*, $E(d)$, is the key connectedness measure

# Network Representation II (Weighted, Directed)



$$A = \begin{pmatrix} 0 & .5 & .7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .3 & 0 \\ 0 & 0 & 0 & .7 & 0 & .3 \\ .3 & .5 & 0 & 0 & 0 & 0 \\ .5 & 0 & 0 & 0 & 0 & .3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

"to $i$, from $j$"

# Network Connectedness II: The Degree Distribution(s)

$A_{ij} \in [0, 1]$ depending on connection strength

Two degrees:

$$d_i^{from} = \sum_{j=1}^{N} A_{ij}$$

$$d_j^{to} = \sum_{i=1}^{N} A_{ij}$$

"from-degree" and "to-degree" distributions

Mean degree remains the key connectedness measure

# $D$ as a Weighted, Directed Network

$N$-Variable Connectedness Table

| | $x_1$ | $x_2$ | ... | $x_N$ | From Others |
|---|---|---|---|---|---|
| $x_1$ | $d_{11}^H$ | $d_{12}^H$ | $\cdots$ | $d_{1N}^H$ | $\sum_{j\neq 1} d_{1j}^H$ |
| $x_2$ | $d_{21}^H$ | $d_{22}^H$ | $\cdots$ | $d_{2N}^H$ | $\sum_{j\neq 2} d_{2j}^H$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ |
| $x_N$ | $d_{N1}^H$ | $d_{N2}^H$ | $\cdots$ | $d_{NN}^H$ | $\sum_{j\neq N} d_{Nj}^H$ |
| To Others | $\sum_{i\neq 1} d_{i1}^H$ | $\sum_{i\neq 2} d_{i2}^H$ | $\cdots$ | $\sum_{i\neq N} d_{iN}^H$ | $\sum_{i\neq j} d_{ij}^H$ |

# Estimating Connectedness

Thus far we've worked under correct specification, in population:

$$C(x, B, \Sigma)$$

Now:

$$\widehat{C}\left(x, M(\hat{\theta})\right)$$

# Many Interesting Issues / Choices

- $x$ objects: Returns? **Return volatilities**?
- $x$ universe: How many and which ones? (**Major banks**)
- $x$ frequency: **Daily?** Monthly? Quarterly?

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- Approximating model $M$: **VAR**? DSGE?
- Perspective: Classical? Bayesian? **Hybrid**?
- Selection: Information criteria? Stepwise? **Lasso**?
- Shrinkage: BVAR? Ridge? **Lasso**?

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- Identification: Cholesky? **Generalized**? SVAR? DSGE?
- Display: Tables? Standard graphics? **Network graphics**?

# Selection and Shrinkage via Penalized Estimation of High-Dimensional Approximating Models

$$\hat{\beta} = argmin_{\beta} \sum_{t=1}^{T} \left( y_t - \sum_i \beta_i x_{it} \right)^2 \quad s.t. \quad \sum_{i=1}^{K} |\beta_i|^q \leq c$$

$$\hat{\beta} = argmin_{\beta} \left( \sum_{t=1}^{T} \left( y_t - \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i|^q \right)$$

Concave penalty functions non-differentiable at the origin produce selection. Smooth convex penalties produce shrinkage. $q \to 0$ produces selection, $q = 2$ produces ridge, $q = 1$ produces lasso.

# Lasso

$$\hat{\beta}_{Lasso} = argmin_{\beta}\left(\sum_{t=1}^{T}\left(y_t - \sum_i \beta_i x_{it}\right)^2 + \lambda \sum_{i=1}^{K} |\beta_i|\right)$$

$$\hat{\beta}_{ALasso} = argmin_{\beta}\left(\sum_{t=1}^{T}\left(y_t - \sum_i \beta_i x_{it}\right)^2 + \lambda \sum_{i=1}^{K} w_i|\beta_i|\right)$$

$$\hat{\beta}_{Enet} = argmin_{\beta}\left(\sum_{t=1}^{T}\left(y_t - \sum_i \beta_i x_{it}\right)^2 + \lambda \sum_{i=1}^{K} \left(\alpha|\beta_i| + (1-\alpha)\beta_i^2\right)\right)$$

$$\hat{\beta}_{AEnet} = argmin_{\beta}\left(\sum_{t=1}^{T}\left(y_t - \sum_i \beta_i x_{it}\right)^2 + \lambda \sum_{i=1}^{K} w_i \left(\alpha|\beta_i| + (1-\alpha)\beta_i^2\right)\right)$$

where $w_i = 1/|\hat{\beta}_i|^{\nu}$, $\hat{\beta}_i$ is OLS or ridge, and $\nu > 0$.

# Visualization via "Spring Graphs"

- Node size: Asset size
- Node color: Total directional connectedness "to others"
- Node location: Average pairwise directional connectedness (Equilibrium of repelling and attracting forces, where (1) nodes repel each other, but (2) edges attract the nodes they connect according to average pairwise directional connectedness "to" and "from.")
- Edge thickness: Average pairwise directional connectedness
- Edge arrow sizes: Pairwise directional connectedness "to" and "from"

# Estimating Global Bank Network Connectedness

- Daily range-based equity return volatilities
- Top 150 banks globally, by assets, 9/12/2003 - 2/7/2014
  - 96 banks publicly traded throughout the sample
  - 80 from 23 developed economies
  - 14 from 6 emerging economies
- Market-based approach:
  - Balance sheet data are hard to get and rarely timely
  - Balance sheet connections are just one part of the story
  - Hard to know more than the market

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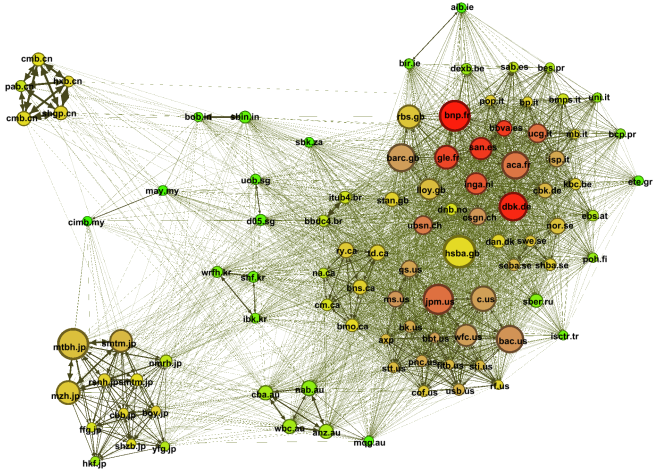
# Individual Bank Network, 2003-2014

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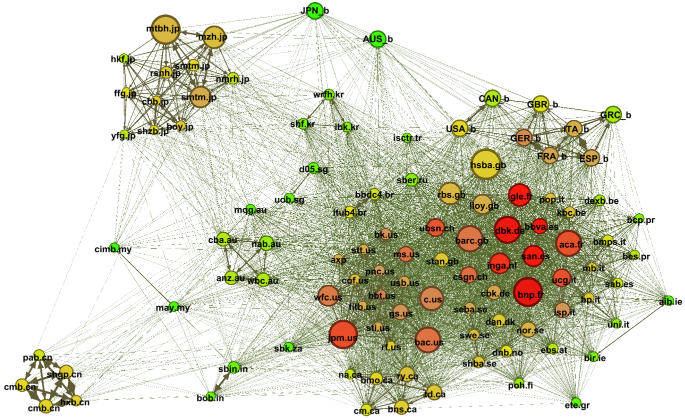
# Individual Bank / Sovereign Bond Network, 2003-2014



# Estimating Time-Varying Connectedness

Earlier:

$$C(x, B, \Sigma)$$

$$\widehat{C}\left(x, M(\hat{\theta})\right)$$

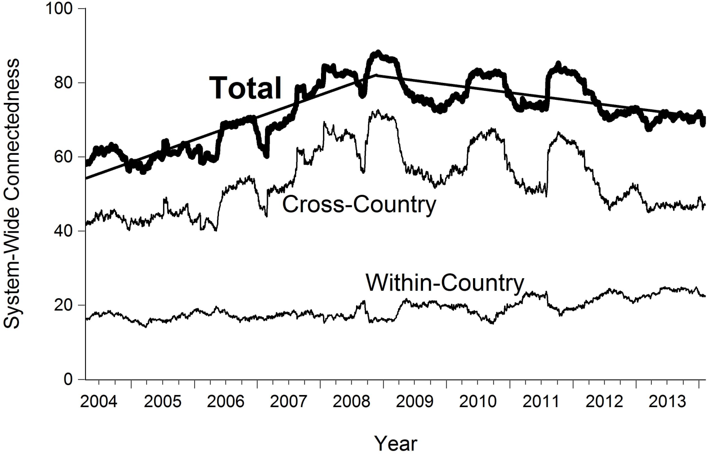
Now:

$$\widehat{C}_t\left(x, M(\hat{\theta}_t)\right)$$

Yet another interesting issue/choice:

- Time-variation: Explicit TVP model? Regime switching? **Rolling**?

# Dynamic System-Wide Connectedness
# 150-Day Rolling Estimation Window

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# Conclusions

- A fruitful approach to connectedness measurement can be based on variance decomposition networks
- High-dimensional success requires regularization and visualization
- Statically we see connectedness clustering first by asset type, and then by country/region
- Dynamically we see:
  - High-frequency changes in connectedness due to crises
  - Low-frequency changes in connectedness perhaps due to globalization
  - System-wide connectedness changes due mostly to changes in cross-country pairwise connectedness

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