The Implicit Value of Tracking the Market

Presented by Majeed Simaan
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Joint work with Dr. Brian Clark and Dr. Chanaka Edirisinghe at RPI
Motivation

- Under full information, tracking error portfolios (TEPs) are Mean-Variance (MV) suboptimal
- Nonetheless, estimation error is severe in portfolio analysis
- Richard Roll (1992) alludes to one possible benefit of TEP: reduction in estimation error

Roll was right: tracking the index does reduce estimation error
The benefits of tracking the index outweigh the cost of estimating additional parameters
Motivation

- Under full information, tracking error portfolios (TEPs) are Mean-Variance (MV) suboptimal.
- Nonetheless, estimation error is severe in portfolio analysis.

Findings

- **Roll** was right: tracking the index does reduce estimation error.
- The benefits of tracking the index outweigh the cost of estimating additional parameters.
The Model

- Two investors $A$ and $B$
- Both investors choose the same $d \in D$ set of assets
- Both investors have the same level of risk tolerance
- Investor $A$ solves for an MV optimal portfolio
- Investor $B$ tracks the benchmark while maximizing excess return

\[ x_B = x_A + \alpha_2 (1) \]

\[ \alpha_2 = \sigma_2 \beta \cdot B (2) \]

where $B$ is a non-linear function of $\Sigma$ and independent of $\mu$. 
The Model

- Two investors $A$ and $B$
- Both investors choose the same $d \in \mathcal{D}$ set of assets
- Both investors have the same level of risk tolerance
- Investor $A$ solves for an MV optimal portfolio
- Investor $B$ tracks the benchmark while maximizing excess return
- It follows that
  \[ x^B = x^A + \alpha_2 \]  
  (1)
- $\alpha_2$ is an arbitrage portfolio tilted toward the market direction, such that
  \[ \alpha_2 = \sigma_b^2 \cdot \mathbf{B} \beta \]  
  (2)
- where $\mathbf{B}$ is a non-linear function of $\Sigma$ and independent of $\mu$
From MV perspective, $x^B$ is sub-optimal to $x^A$
Estimation Error

- In practice, the model’s parameters are unknown and decisions are based on historical data.
- Let \( \hat{x} \) denote the estimate of \( x \), then we have
  \[
  \hat{x}^B = \hat{x}^A + \hat{\alpha}_2
  \] (3)

Main Findings

- The market component, \( \hat{\alpha}_2 \), is an unbiased estimate of \( \alpha_2 \).
- The bias in \( \hat{x}^B \) is mainly due to the MV portfolio, \( \hat{x}^A \).
- The difference in estimation error between portfolio \( \hat{x}^B \) and \( \hat{x}^A \) is determined by a scalar \( \lambda \), such that
  \[
  \Delta = \text{MSE}(\hat{x}^B) - \text{MSE}(\hat{x}^A) = \lambda^T - d - 1
  \] (4)

where,
  \[
  \text{MSE}(x) = \text{Var}(x) + \text{bias}(x)^2
  \]
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  \[
  \Delta = MSE(\hat{x}^B) - MSE(\hat{x}^A) = \frac{\lambda}{T - d - 1} \]
  (4)

  where, \( MSE(x) = Var(x) + bias(x)^2 \)
Looking at all stocks listed on S&P 500 starting from Jan 2000, we run the following test:

1. Set $r = 1$
2. Randomly draw a $d$ subset of assets
3. Pick the first $T$ days and estimate the model inputs
4. Compute all relevant metrics along with the MSE difference, $\Delta$, from (4)
5. Average the diagonal elements of $\Delta$, yielding $\bar{\Delta}(r)$
6. $r \rightarrow r + 1$ and repeat till $r > 1000$

Eventually, the experiment returns the following statistic:

$$Z_{\bar{\Delta}} = \frac{\bar{\Delta}}{\sigma(\bar{\Delta})} = \frac{\sum_{r=1}^{1000} \bar{\Delta}(r)/1000}{\sqrt{\sum_{r=1}^{1000} [\bar{\Delta}(r) - \bar{\Delta}]^2 /1000}}$$ (5)
### Table: Estimation error difference between $B$ and $A$

<table>
<thead>
<tr>
<th></th>
<th>$T$</th>
<th>$\Delta$</th>
<th>$Z_\Delta$</th>
<th>$\beta$</th>
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<td>$d = 10$</td>
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