The Implicit Value of Tracking the Market

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Long Story Short

Motivation

- Under full information, tracking error portfolios (TEPs) are Mean-Variance (MV) suboptimal
- Nonetheless, estimation error is severe in portfolio analysis
- **Richard Roll (1992)** alludes to one possible benefit of TEP: reduction in estimation error

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Findings

- **Roll** was right: tracking the index does reduce estimation error
- The benefits of tracking the index outweigh the cost of estimating additional parameters

The Model

- Two investors A and B
- Both investors choose the same $d \in \mathcal{D}$ set of assets
- Both investors have the same level of risk tolerance
- Investor A solves for an MV optimal portfolio
- Investor B tracks the benchmark while maximizing excess return

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- It follows that

$$x^B = x^A + \alpha_2 \tag{1}$$

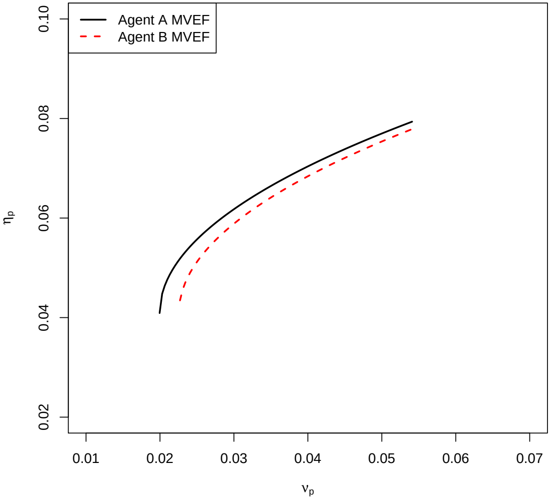
- α_2 is an arbitrage portfolio tilted toward the market direction, such that

$$\alpha_2 = \sigma_b^2 \cdot \mathbf{B}\beta \tag{2}$$

- where $\mathbf{B}$ is a non-linear function of Σ and independent of μ

MV Efficient Frontier

From MV perspective, x^B is sub-optimal to x^A



Estimation Error

- In practice, the model's parameters are unknown and decisions are based on historical data
- Let $\hat{x}$ denote the estimate of x, then we have

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Main Findings

- The market component, $\hat{\alpha}_2$, is unbiased estimate of α_2
- The bias in $\hat{x}^B$ is mainly due to the MV portfolio, $\hat{x}^A$
- The difference in estimation error between portfolio $\hat{x}^B$ and $\hat{x}^A$ is determined by a scalar λ, such that

$$\Delta = MSE(\hat{x}^B) - MSE(\hat{x}^A) = \frac{\lambda}{T - d - 1}\mathbf{B} \tag{4}$$

where, $MSE(x) = Var(x) + bias(x)^2$

Experiment

Looking at all stocks listed on S&P 500 starting from Jan 2000, we run the following test:

1. set $r = 1$
2. Randomly draw a d subset of assets
3. Pick the first T days and estimate the model inputs
4. Compute all relevant metrics along with the MSE difference, Δ, from (4)
5. Average the diagonal elements of Δ, yielding $\bar{\Delta}^{(r)}$
6. $r \rightarrow r + 1$ and repeat till $r > 1000$

Eventually, the experiment returns the following statistic:

$$Z_{\bar{\Delta}} = \frac{\bar{\Delta}}{\sigma(\bar{\Delta})} = \frac{\sum_{r=1}^{1000} \bar{\Delta}^{(r)} / 1000}{\sqrt{\sum_{r=1}^{1000} \left[\bar{\Delta}^{(r)} - \bar{\Delta}\right]^2 / 1000}} \tag{5}$$

Results

Table: Estimation error difference between B and A

T	$\bar{\Delta}$	$Z_{\bar{\Delta}}$	$\bar{\beta}$
$d = 10$			
500	-0.40	-0.79	0.77
1000	-0.54	-2.19	0.88
2000	-0.32	-2.79	0.89
$d = 20$			
500	-0.55	-1.65	0.94
1000	-0.58	-3.34	1.01
2000	-0.34	-4.26	1.00
$d = 40$			
500	-0.68	-2.81	1.25
1000	-0.59	-4.49	1.25
2000	-0.34	-5.67	1.20