Mixing Data of Different Sampling Frequencies in the Frequency Domain: a Daily System of Macro-Indicators

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May 17, 2016
R/Finance 2016, Chicago
## Table of contents

1. **Data and Estimation Problem**
   - Data Provider: Quandl
   - Estimation Problem: Mixed-Frequency

2. **Frequency-Domain Approach (MDFA)**
   - Formulation of the Estimation Problem
   - Optimization Criterion
   - Summary

3. **Empirical Examples**
   - Empirical Examples: Tracking Monthly IPI
   - Empirical Examples: Tracking Quarterly GDP
FRED/GDP, FRED/ICSA, YAHOO/INDEX_GSPC, FRED/UNRATE, FRED/PAYEMS; FRED/INDPRO

Target:
- FRED/GDP
- FRED/INDPRO

Explaining: FRED/ICSA, YAHOO/INDEX_GSPC, FRED/UNRATE, FRED/PAYEMS; FRED/INDPRO

Data transformation: log-returns
### Data and Estimation Problem

**Frequency-Domain Approach (MDFA)**

**Empirical Examples**

**Data Provider:** Quandl

**Estimation Problem:** Mixed-Frequency

<table>
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<tr>
<th>Date</th>
<th>GDP</th>
<th>ICSA</th>
<th>SP500</th>
<th>Unrate</th>
<th>NFP</th>
<th>IPI</th>
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</tbody>
</table>

**Table:** Data flow: quarterly, monthly, weekly and daily time series
Data flow (see above slide)

- On 2012-03-31: new (weekly) ICSA
- On 2012-04-01: new (quarterly) GDP
- On 2012-04-03: new (monthly) UNRATE
- On 2012-04-04: new (monthly) NFP
- ...

Filter coefficients depend on information flow at the ‘ragged end’ of the data set
A day-specific filter must be constructed for each day in the quarter (3*25=75 days)
Partition the data-set into many (75) non-overlapping blocks
Problem: each block would be too small to allow estimation of the coefficients of a (more or less complex) multivariate filter
We here propose a solution in the frequency-domain
Toy Example: **Monthly** $m_t$ and **Quarterly** $q_t$

- **Time axis**
  - Let $t^m$ indicate months and $t^q$ quarters
  - Then $t^m = 3 \cdot t^q + j$ where $j = 0, 1, 2$ depending on the month in the quarter
  - The index $j$ is the ‘month-in-the-quarter’ index

- **Filters**
  - Monthly: $\sum_{k=0}^{L^m} b_{k,j}^m m_{3 \cdot t^q + j - k}$
  - Quarterly: $\sum_{k=0}^{L^q} b_{k,j}^q q_{3 \cdot t^q - 3k}$
    - Lags of $q_t$ are multiples of three
    - $j$ appears in the lag-structure of $m_t$

- The coefficients $b_{k,j}^m$ and $b_{k,j}^q$ depend on $j = 0, 1, 2$
Frequency-Domain

- Discrete Fourier Transform: \( \Xi_{TX}(\omega) := \sum_{t=1}^{T} x_t \exp(-it\omega) \)
- Lead/lag mappings:
  - \( x_t \rightarrow \Xi_{TX}(\omega) \)
  - \( x_{t-1} \rightarrow \exp(-i\omega)\Xi_{TX}(\omega) \), unity lag
  - \( x_{t-k} \rightarrow \exp(-ik\omega)\Xi_{TX}(\omega) \), arbitrary integer lag
  - \( x_{t-\delta} \rightarrow \exp(-i\delta\omega)\Xi_{TX}(\omega) \), arbitrary fractional lag
  - \( \sum_{k=0}^{L} b_k x_{t-k} \rightarrow \left( \sum_{k=0}^{L} b_k \exp(-ik\omega) \right)\Xi_{TX}(\omega) \), lag-distribution
Filters in the Frequency-Domain (Toy-Example)

- **Bi-variate filter**

\[
\sum_{k=0}^{L_m} b_{k,j}^m m^3.t^q + j - k + \sum_{k=0}^{L_q} b_{k,j}^q q^3.t^q - 3k
\]

- **Frequency-domain formulation**

\[
\left( \sum_{k=0}^{L_m} b_{k,j}^m \exp(-ik\omega) \right) \Xi_{Tm}(\omega) + \exp(-ij\omega) \left( \sum_{k=0}^{L_q} b_{k,j}^q \exp(-i3k\omega) \right) \Xi_{Tq}(\omega)
\]

where

- \(\Xi_{Tm}(\omega)\) and \(\Xi_{Tq}(\omega)\) are the DFTs of the data
- \(\exp(-ij\omega)\) shifts the quarterly data by \(j\) months relative to the monthly series
Target Specification (GDP)

GDP: Log–Periodogram (Quarterly scale)

Target GDP: Folded (log–) Periodogram (Monthly Scale)
Optimization Criterion (Toy-Example)

- Target: $\Gamma(\omega_n) \Xi T_q(\omega_n)$
  - $\Gamma(\omega)$ is the target filter: typically symmetric low-pass/band-pass
- MSE-criterion for month $j = 0, 1, 2$ in the quarter

$$\sum_n \left| \Gamma(\omega_n) \Xi T_q(\omega_n) - \left( \sum_{k=0}^{L^m} b_{k,j}^m \exp(-ik\omega_n) \right) \Xi T_m(\omega_n) \right|^2 \rightarrow \min_{b_j, j=0,1,2}$$

- The negative lag $j$ in $\exp(-ij\omega_n)$ means a (fractional) shift of the quarterly data into the past by $j = 0, 1, 2$ months
Summary

- Optimal (MSE-) design
- No explicit data-interpolation: ‘fractional’ shift applied to low-frequency data
- No data-partitioning: each filter set \( b_j, j = 0, 1, 2 \) relies on the whole data
- Target is tracked on the high-frequency scale to obtain a high-frequency real-time estimate
- Concepts straightforwardly extend to mixing data of arbitrary sampling frequencies (in the examples below: quarterly, monthly, weekly and daily)
Empirical Example: Daily Indicator Design IPI

- Target: IPI (cutoff $= \pi/12$ or 24 months)
- Explaining series: monthly (IPI, NFP, UNEM), weekly (ICSA) and daily (SP500, VIX)
Data and Estimation Problem
Frequency-Domain Approach (MDFA)
Empirical Examples

IPI: Daily Indicator (Design 1)

Design: neutral (q−t−q log−diffs: 1990−01−01 to 2016−05−16)

FRED.INDPRO: log−returns, quarter−to−quarter
Symmetric target (ideal trend)
Indicator: growth q−t−q

1996−10−28 2003−05−05 2009−11−05 2016−05−13
−0.06 −0.04 −0.02 0.00 0.02

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MDFA
Data and Estimation Problem
Frequency-Domain Approach (MDFA)
Empirical Examples

Additional Information: which Sub-series Trigger Alarm

Design 1: Growth neutral (quarter-to-quarter)

Last 100 observations
IPI: System of Daily Indicators (Designs 1-4)

Indicator Comparison (1990-01-01 to 2016-05-16)

- Indicator: growth q−t−q (neutral)
- Indicator: emphasize cycle q−t−q
- Indicator: growth m−t−m
- Indicator: emphasize cycle m−t−m

last 100 observations

- Indicator: growth q−t−q (neutral)
- Indicator: emphasize cycle q−t−q
- Indicator: growth m−t−m
- Indicator: emphasize cycle m−t−m

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Typical ‘Patterns’ (Dynamics) at the Onset of Severe Contractions (Recessions)

- Single indicator: draw-downs observed across all sub-series
- System of indicators: draw-downs observed in all indicators
- Timing/Chronology: faster/leading series anticipate coincident/lagging series: *crossings* of the faster indicators below (above) the slower indicators
Trading Opportunities (S&P500)

Log-transformed S&P500 (green) vs. actively managed design (red)

Actively managed S&P500
Log-transformed S&P500
Outperformance

0.0 0.5 1.0 1.5 2.0 2.5

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MDFA
Indicator Design: Daily GDP Indicator

- Target: smoothed **quarterly** GDP (cutoff=$\pi/4$ or 8 Quarters)
- Explaining series: **monthly** (IPI,NFP,UNEM), **weekly** (ICSA) and **daily** (SP500, VIX)
  - GDP is **not** an explaining variable (publication lag, revisions)
- Estimation of filter coefficients accounts for publication lag and revisions
Estimation Lag 3*75 Days (3 Quarters)

Design: neutral (q−t−q log−diffs: 1990–01–01 to 2016–05–16)

FRED.GDP: log−returns, quarter−to−quarter
Symmetric target (ideal trend)
Indicator: growth q−t−q

1996−10−28 2003−05−05 2009−11−05 2016−05−13
−0.02 −0.01 0.00 0.01 0.02
Summary/Outlook

- **Daily real-time indicators**
  - Fast: Improved timing abilities due to the inclusion of high-frequency data
  - Smooth: strong noise suppression
  - Small revisions: real-time filters, GDP is not an explanatory variable

- **Recession tracking based on ‘typical patterns’: exploit**
  - Sub-series
  - Indicator system
  - Chronology of events (lead/lag patterns)

- **Macro-trading: indicator-system generates views for a (more sophisticated) multi-period optimization tool (current industry-project)**
Generic MDFA-framework

- AST-trilemma and customization
- Regularization
- Replicate and customize classic maximum likelihood approaches
Outlook

- Track performances and adjust order-sizing of a low-frequency trading algorithm (for example a daily system) based on high-frequency (intraday) data-flow.