Mixing Data of Different Sampling Frequencies in the Frequency Domain: a Daily System of Macro-Indicators

Marc Wildi
marc.wildi@zhaw.ch

Institute of data analysis and process design
Zurich University of Applied Sciences

May 17, 2016
R/Finance 2016, Chicago

Table of contents

https://www.quandl.com/

- FRED/GDP, FRED/ICSA, YAHOO/INDEX _GSPC, FRED/UNRATE, FRED/PAYEMS; FRED/INDPRO
- Target:
  - FRED/GDP
  - FRED/INDPRO
- Explaining: FRED/ICSA, YAHOO/INDEX_GSPC, FRED/UNRATE, FRED/PAYEMS; FRED/INDPRO
- Data transformation: log-returns

	GDP	ICSA	SP500	Unrate	NFP	IPI
2012-03-31		-0.0547	0.1165			
2012-04-01	0.0092		0.1208			
2012-04-02			0.1054			
2012-04-03			0.1014	-0.0359		
2012-04-04			0.0912		0.0063	

Table: Data flow: quarterly, monthly, weekly and daily time series

Day Within Month Effect

- Data flow (see above slide)
  - On 2012-03-31: new (weekly) ICSA
  - On 2012-04-01: new (quarterly) GDP
  - On 2012-04-03: new (monthly) UNRATE
  - On 2012-04-04: new (monthly) NFP
  - ...
- Filter coefficients depend on information flow at the 'ragged end' of the data set

Estimation Problem

- A day-specific filter must be constructed for each day in the quarter (3*25=75 days)
- Partition the data-set into many (75) non-overlapping blocks
- Problem: each block would be too small to allow estimation of the coefficients of a (more or less complex) multivariate filter
- We here propose a solution in the frequency-domain

Toy Example: **Monthly** m_t and **Quarterly** q_t

- Time axis
  - Let t^m indicate months and t^q quarters
  - Then $t^m = 3 \cdot t^q + j$ where $j = 0, 1, 2$ depending on the month in the quarter
  - The index j is the 'month-in-the-quarter' index
- Filters
  - Monthly: $\sum_{k=0}^{L^m} b_{k,j}^m m_{3 \cdot t^q + j - k}$
  - Quarterly: $\sum_{k=0}^{L^q} b_{k,j}^q q_{3 \cdot t^q - 3k}$
    - Lags of q_t are multiples of three
    - j appears in the lag-structure of m_t
- The coefficients $b_{k,j}^m$ and $b_{k,j}^q$ depend on $j = 0, 1, 2$

Frequency-Domain

- Discrete Fourier Transform: $\Xi_{TX}(\omega) := \sum_{t=1}^{T} x_t \exp(-it\omega)$
- Lead/lag mappings:

$$
\begin{aligned}
x_t &\rightarrow \Xi_{TX}(\omega) \\
x_{t-1} &\rightarrow \exp(-i\omega)\Xi_{TX}(\omega) \text{ , unity lag} \\
x_{t-k} &\rightarrow \exp(-ik\omega)\Xi_{TX}(\omega) \text{ , arbitrary integer lag} \\
x_{t-\delta} &\rightarrow \exp(-i\delta\omega)\Xi_{TX}(\omega) \text{ , arbitrary fractional lag} \\
\sum_{k=0}^{L} b_k x_{t-k} &\rightarrow \left(\sum_{k=0}^{L} b_k \exp(-ik\omega)\right) \Xi_{TX}(\omega) \text{ , lag-distribution}
\end{aligned}
$$

Filters in the Frequency-Domain (Toy-Example)

- Bi-variate filter

$$\sum_{k=0}^{L^m} b_{k,j}^m m_{3\cdot t^q+j-k} + \sum_{k=0}^{L^q} b_{k,j}^q q_{3\cdot t^q-3k}$$

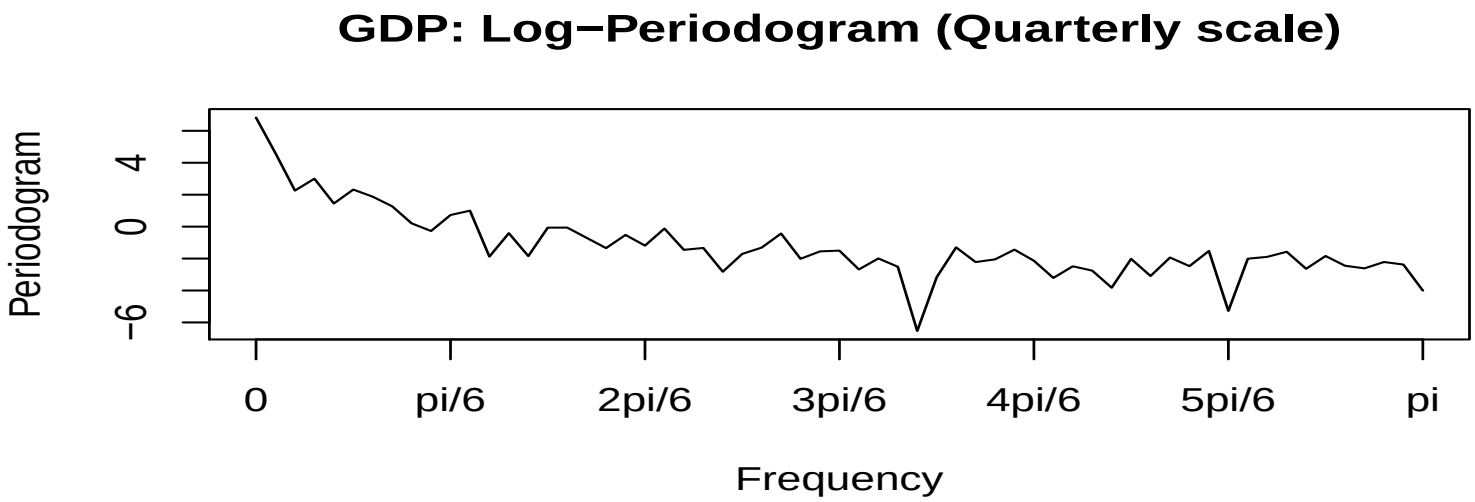
- Frequency-domain formulation

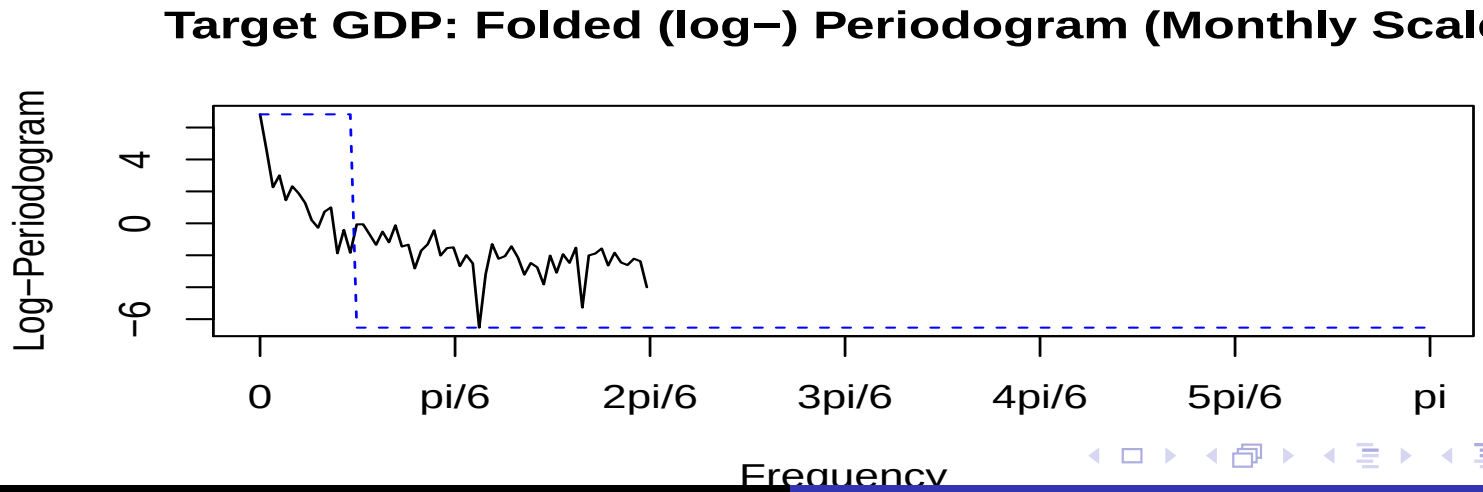
$$\left(\sum_{k=0}^{L^m} b_{k,j}^m \exp(-ik\omega)\right) \Xi_{Tm}(\omega) + \exp(-ij\omega) \left(\sum_{k=0}^{L^q} b_{k,j}^q \exp(-i3k\omega)\right) \Xi_{Tq}(\omega$$

where

- $\Xi_{Tm}(\omega)$ and $\Xi_{Tq}(\omega)$ are the DFTs of the data
- $\exp(-ij\omega)$ shifts the quarterly data by j months relative to the monthly series

Target Specification (GDP)





Optimization Criterion (Toy-Example)

- Target: $\Gamma(\omega_n)\Xi_{Tq}(\omega_n)$
  - $\Gamma(\omega)$ is the target filter: typically symmetric low-pass/band-pass
- MSE-criterion for month $j = 0, 1, 2$ in the quarter

$$\sum_n \left| \Gamma(\omega_n)\Xi_{Tq}(\omega_n) - \left(\sum_{k=0}^{L^m} b_{k,j}^m \exp(-ik\omega_n) \right) \Xi_{Tm}(\omega_n) \right.$$
$$\left. - \exp(-ij\omega_n) \left(\sum_{k=0}^{L^q} b_{k,k}^q \exp(-i3k\omega_n) \right) \Xi_{Tq}(\omega_n) \right|^2 \rightarrow \min_{\mathbf{b}_j,\ j=0,1,2}$$

- The negative lag j in $\exp(-ij\omega_n)$ means a (fractional) shift of the quarterly data into the past by $j = 0, 1, 2$ months

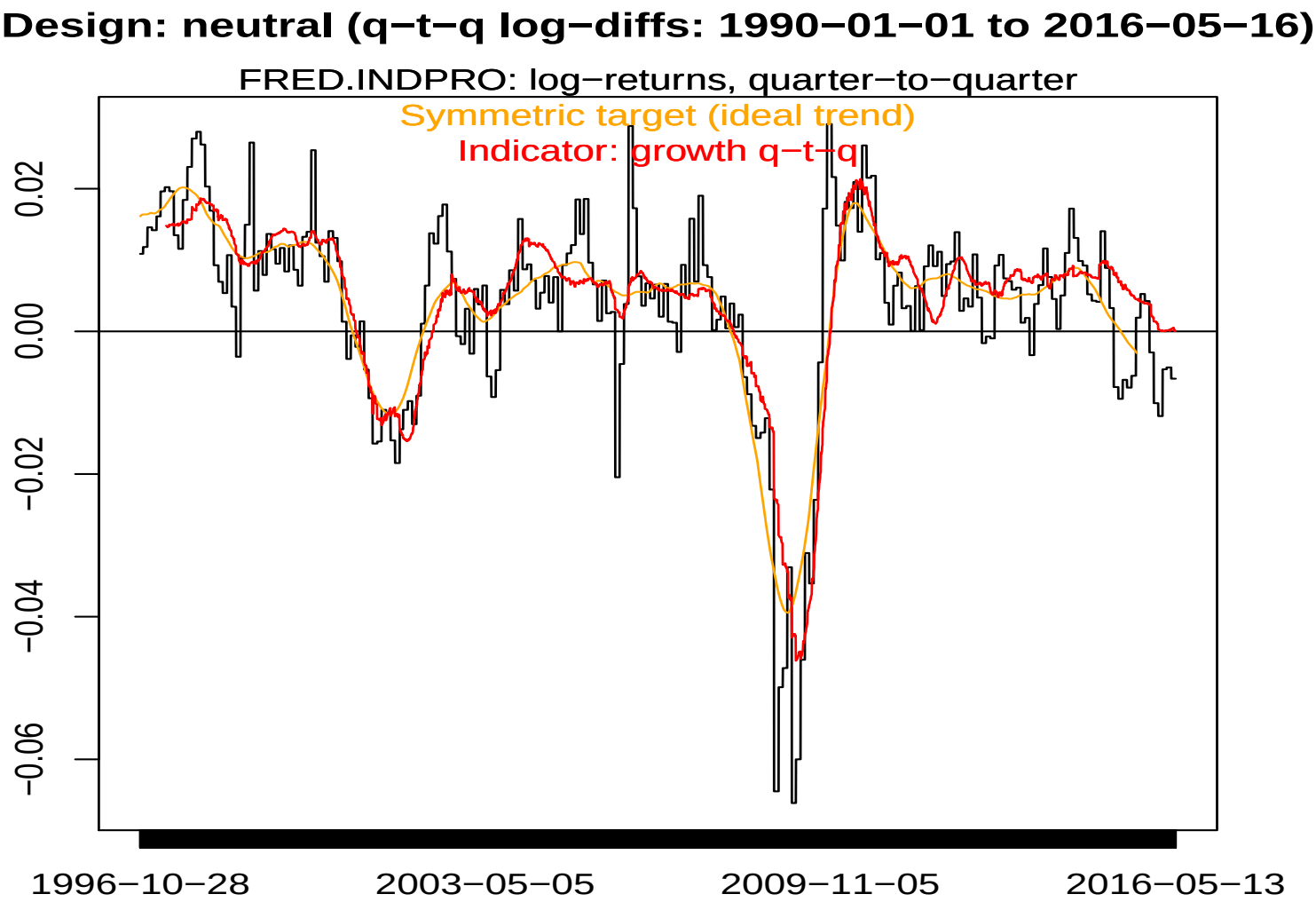
Summary

- Optimal (MSE-) design
- No explicit data-interpolation: 'fractional' shift applied to low-frequency data
- No data-partitioning: each filter set $\mathbf{b}_j$, $j = 0, 1, 2$ relies on the whole data
- Target is tracked on the high-frequency scale to obtain a *high-frequency* real-time estimate
- Concepts straightforwardly extend to mixing data of arbitrary sampling frequencies (in the examples below: quarterly, monthly, weekly and daily)

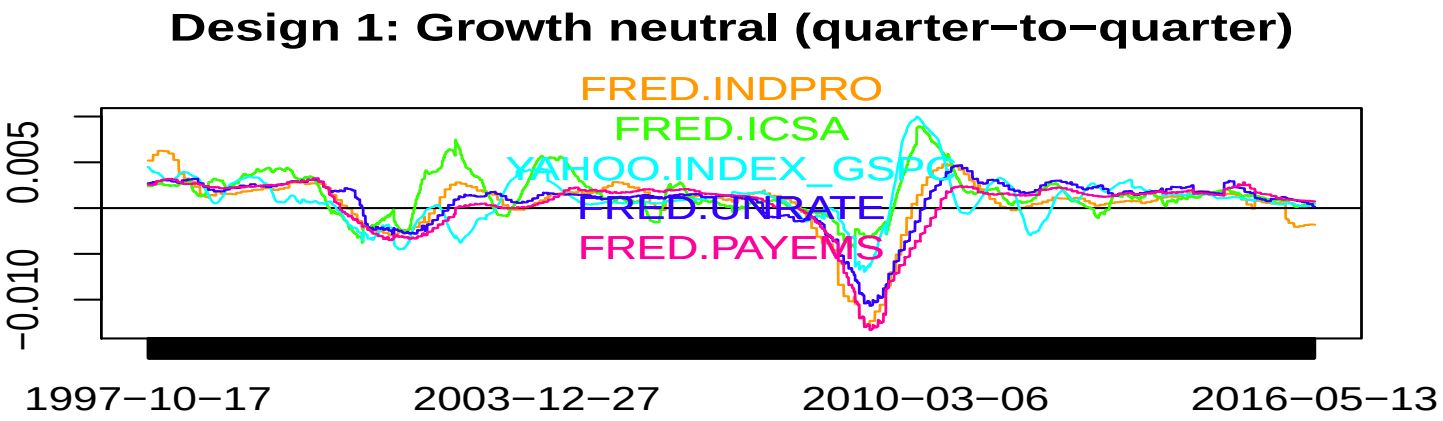
Empirical Example: Daily Indicator Design IPI

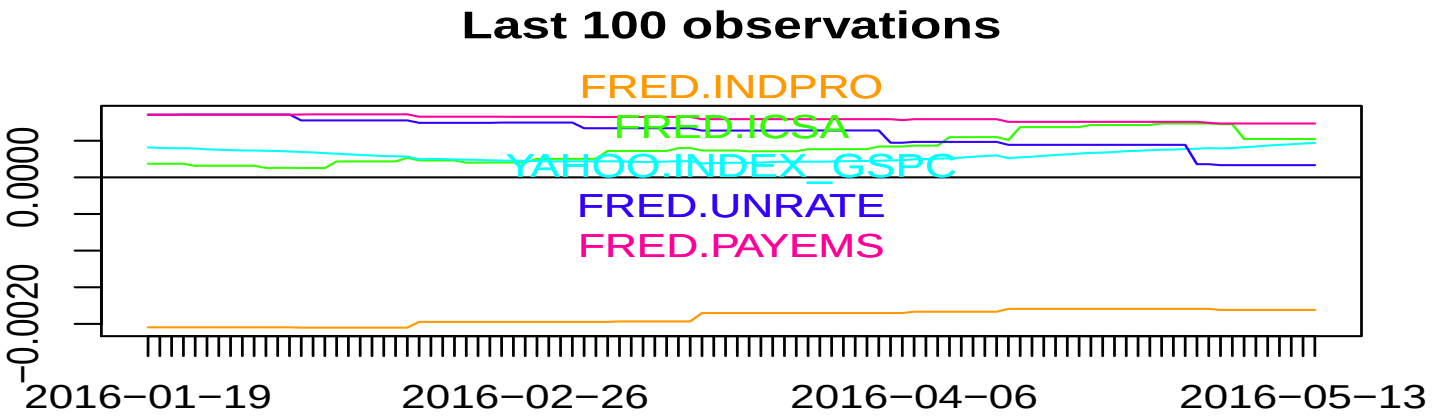
- Target: IPI (cutoff=pi/12 or 24 months)
- Explaining series: **monthly** (IPI,NFP,UNEM), **weekly** (ICSA) and **daily** (SP500, VIX)

IPI: Daily Indicator (Design 1)

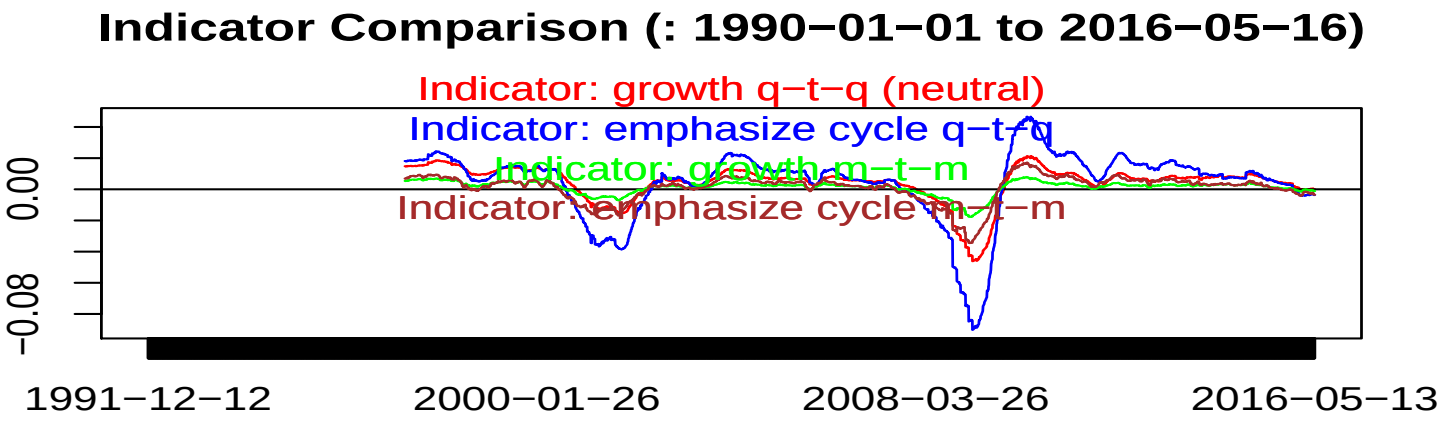


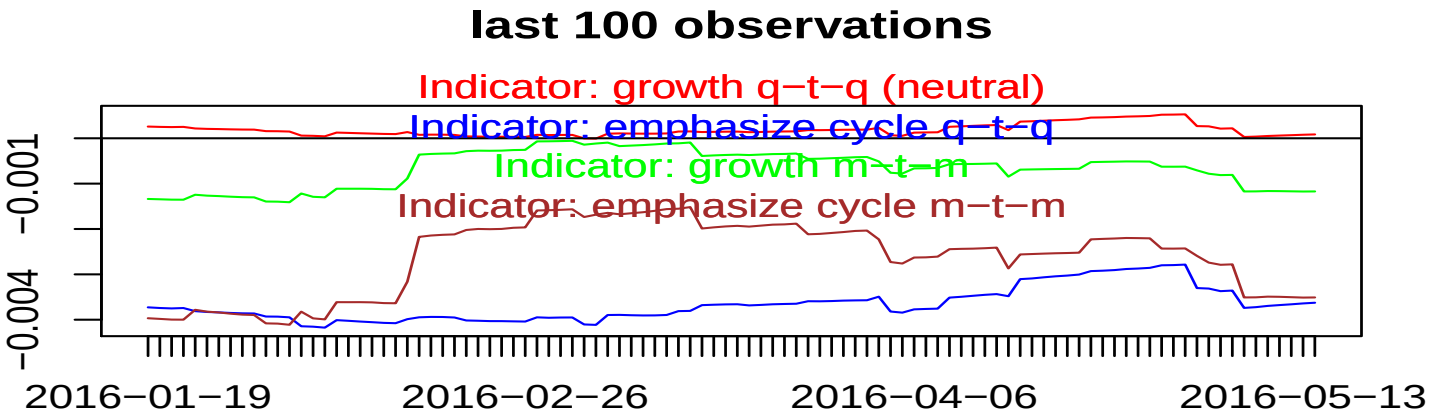
Additional Information: which Sub-series Trigger Alarm





IPI: System of Daily Indicators (Designs 1-4)

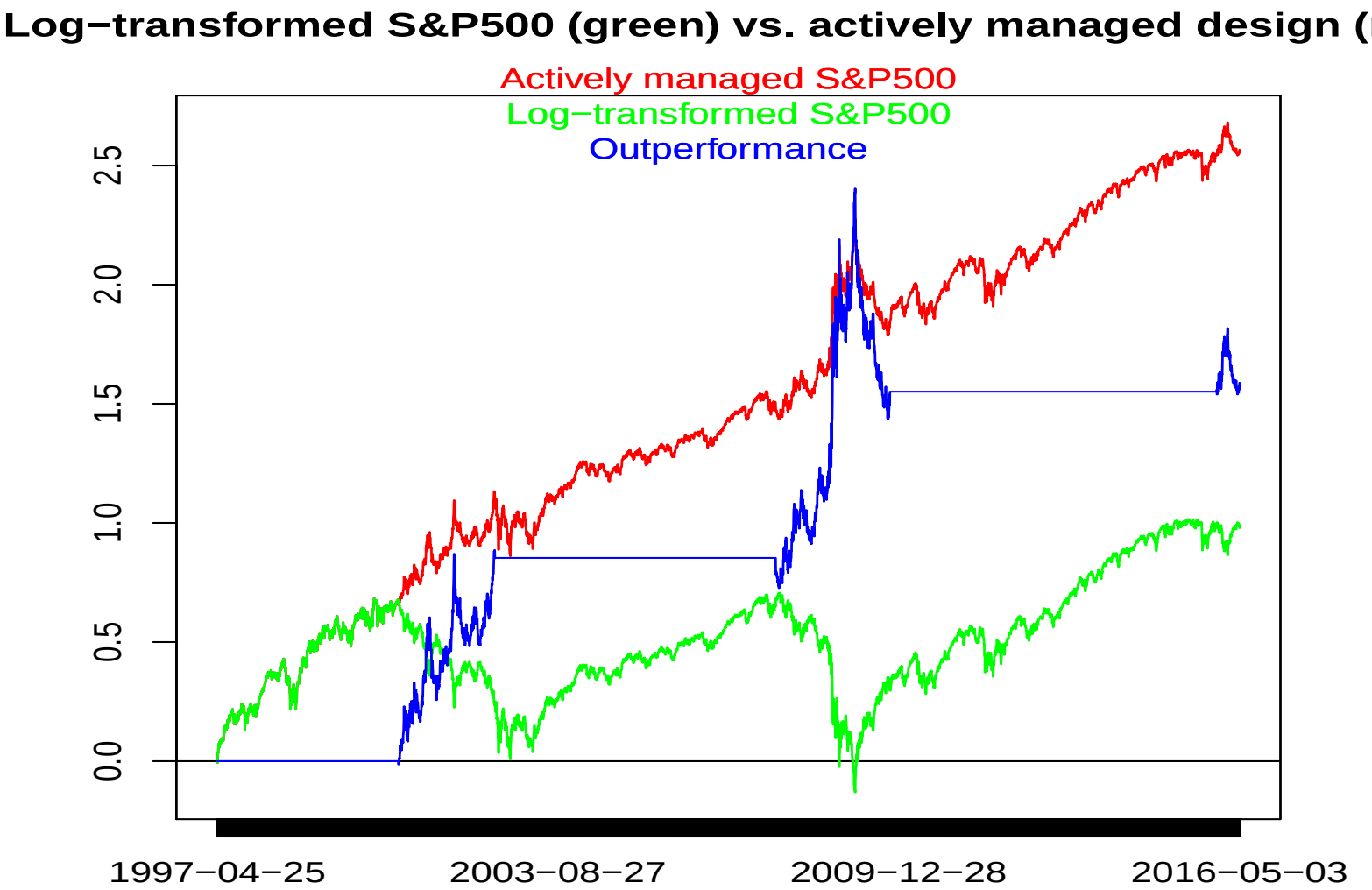




Typical 'Patterns' (Dynamics) at the Onset of Severe Contractions (Recessions)

- Single indicator: draw-downs observed across all sub-series
- System of indicators: draw-downs observed in all indicators
- Timing/Chronology: faster/leading series anticipate coincident/lagging series: *crossings* of the faster indicators below (above) the slower indicators

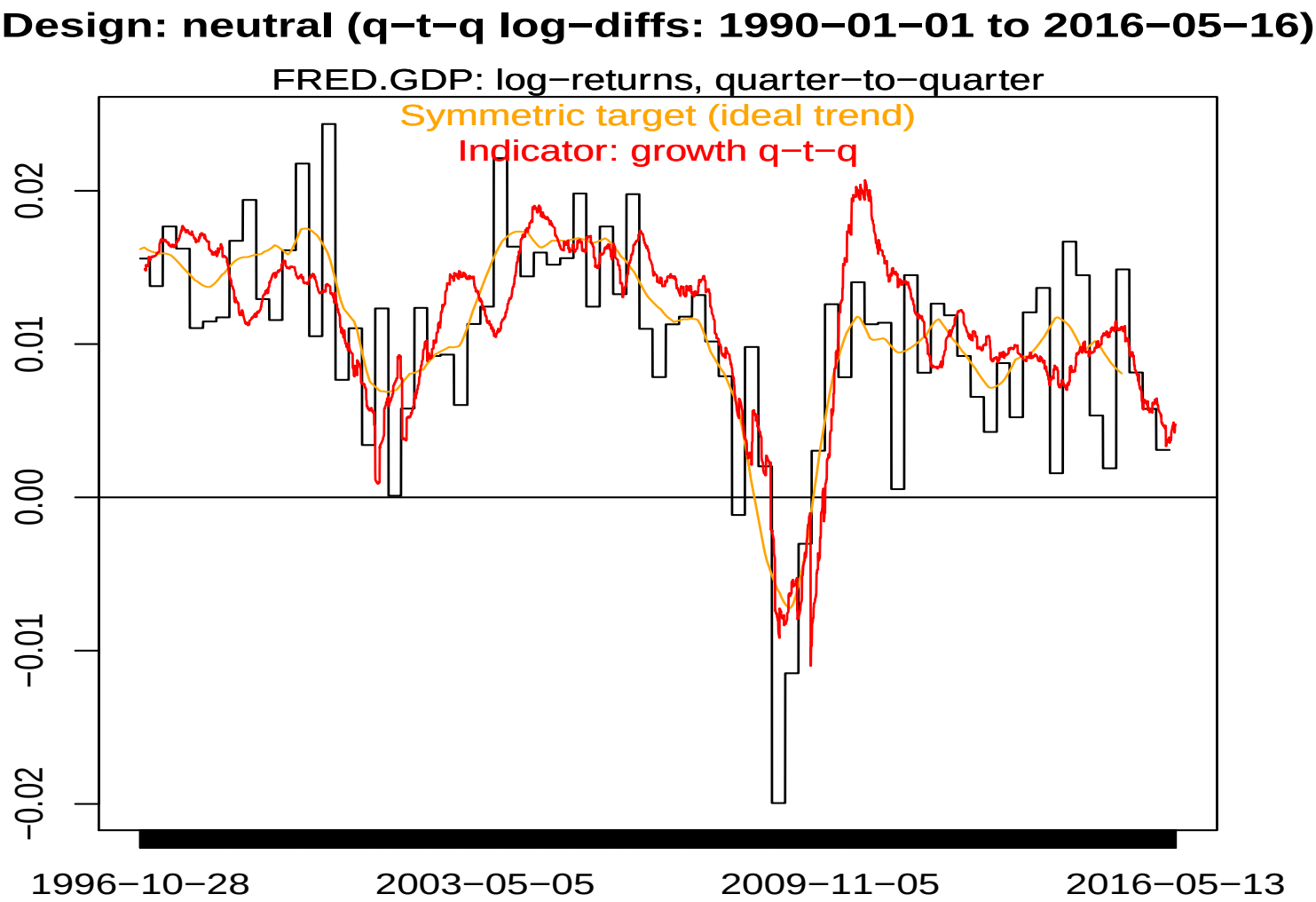
Trading Opportunities (S&P500)



Indicator Design: Daily GDP Indicator

- Target: smoothed **quarterly** GDP (cutoff=pi/4 or 8 Quarters)
- Explaining series: **monthly** (IPI,NFP,UNEM), **weekly** (ICSA) and **daily** (SP500, VIX)
  - GDP is **not** an explaining variable (publication lag, revisions)
- Estimation of filter coefficients accounts for publication lag and revisions

Estimation Lag 3*75 Days (3 Quarters)



Summary/Outlook

- Daily real-time indicators
  - Fast: Improved timing abilities due to the inclusion of high-frequency data
  - Smooth: strong noise suppression
  - Small revisions: real-time filters, GDP is not an explanatory variable
- Recession tracking based on 'typical patterns': exploit
  - Sub-series
  - Indicator system
  - Chronology of events (lead/lag patterns)
- Macro-trading: indicator-system generates views for a (more sophisticated) multi-period optimization tool (current industry-project)

Generic MDFA-framework

- AST-trilemma and customization
- Regularization
- Replicate and customize classic maximum likelihood approaches

Outlook

- Track performances and adjust order-sizing of a low-frequency trading algorithm (for example a daily system) based on high-frequency (intraday) data-flow.