Modeling Divergence Swap Rates

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From VIX to power divergence (Schneider and Trojani (2015))

CBOE (2000) calculates the Volatility Index...

$$VIX_t^2 = \frac{2}{T}\sum_j \frac{\Delta K_j}{K_j^2} Q_t(K_j) \approx \frac{2}{T}\int_0^\infty \frac{Q_t(K)}{K^2} dK = -2\mathbb{E}_t^{\mathbb{Q}}\left[\ln\frac{F_T}{F_t} + \sum_{\tau=1}^{T}\delta_{\tau-1}(F_\tau - F_{\tau-1})\right]$$

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- Family of power divergence functions D_p defines family of (first-order) quadratic variation swaps. Compare with Bondarenko (2014); Martin (2012); Lee (2010).
- Weights $\frac{\partial^2}{\partial K^2} D_p(K,x) = \frac{1}{K^p}$.

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- Weights $\frac{\partial^2}{\partial K^2} D_p(K,x) = \frac{1}{K^p}$.
- HF limits:

$$\frac{1}{F_t^p}\sum_{\tau=1}^{T} D_p(F_\tau, F_{\tau-1}) \to \frac{1}{2}\int_t^T \left(\frac{F_s}{F_t}\right)^p \sigma_s^2 ds + \sum_{t\leq s\leq T} \frac{D_p(F_s, F_{s-})}{F_t}.$$

- $1/F^p$ scaling essential $\to$ notation: $\bar{D}_p = \frac{D_p(F_s,F_t)}{F_t^p}$

Higher-order swaps

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- Variation in p in divergence swaps $\rightarrow$ another dimension for fitting.

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- Take a difference...

$$\frac{1}{2\varepsilon}\left(\frac{D_{p_0+\varepsilon}(F_T, F_t)}{F_t^{p_0+\varepsilon}} - \frac{D_{p_0-\varepsilon}(F_T, F_t)}{F_t^{p_0-\varepsilon}}\right) = O\left(\ln\frac{F_T}{F_t}\right)^3$$

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- ... or differentiate.

$$\frac{\partial}{\partial p} \frac{D_p(F_T, F_t)}{F_t^p} = O\left(\ln \frac{F_T}{F_t}\right)^3$$

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- Rinse and repeat.

$$\frac{\partial^2}{\partial p^2} \frac{D_p(F_T, F_t)}{F_t^p} = O\left(\ln \frac{F_T}{F_t}\right)^4$$

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$$S_p := \frac{\partial}{\partial p} \frac{D_p(F_T, F_t)}{F_t^p} \qquad Q_p := \frac{\partial^2}{\partial p^2} \frac{D_p(F_T, F_t)}{F_t^p}$$
- **Exactly** tradable with static option portfolio and forward trading under any dynamics.

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- Corresponding realized measures:

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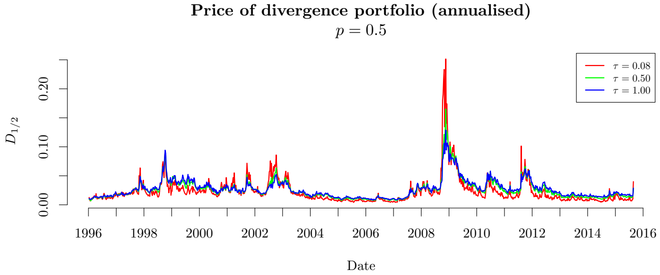
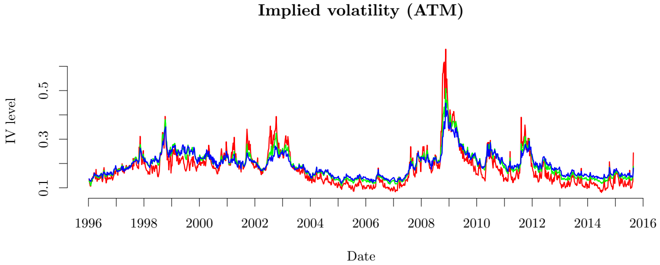
- Fully replicable realized measures allow to define higher-order risk premia.
- D_p, S_p and Q_p highly correlated $\rightarrow$ standardise:

$$\bar{S}_p := \frac{S_p}{D_p^{3/2}} \qquad \bar{Q}_p := \frac{S_p}{D_p^2}$$

Empirical motivation – S&P 500 options





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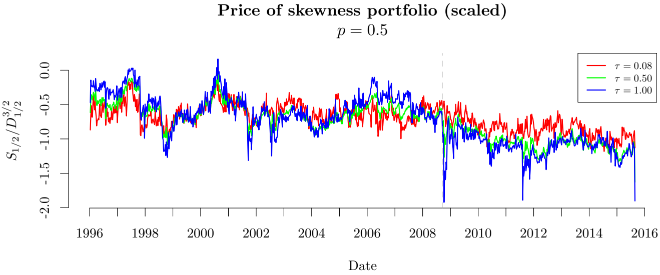

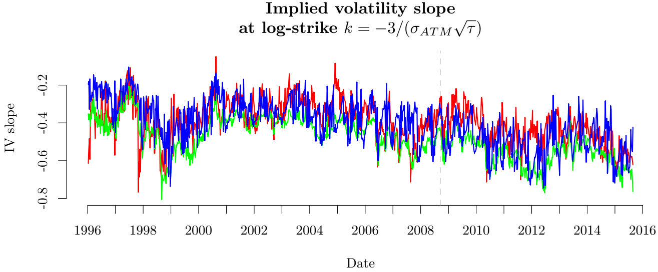

Empirical motivation – S&P 500 options

- Prices of variation swaps at range of p and τ contain the same information as the IV surface;
- Forming option portfolios might alleviate measurement error, particularly for deep OTM contracts;
- Variation swaps are exactly tradable. IV is not. Who knows what IV is anyway?

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Divergence in Affine Jump Diffusions

- Affine Jump Diffusion models are a popular (and somewhat successful) modeling tool.
- State variables: $X_t = [\ln S_t/S_{t-s} \quad V_{1t} \quad \ldots \quad V_{Mt}]$, $V_t := X_t[-1]$

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- Latent volatility factor dynamics:

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- Unscented Kalman Filter – counterfactual assumption of $W_t \sim N(0, \mathbf{I}_{M\times M})$;
- Specification of model under $\mathbb{Q} \rightarrow$ Risk Premia $\rightarrow$ specification under $\mathbb{P}$;
- Reasonable performance in simulated settings.

Preliminary results

- First estimations with $p = 1/2$ (symmetric swaps), maturities 1M, 6M;
- In three factor specification possible to identify:
  - a variance-level factor,
  - a common factor for $\bar{S}$ and $\bar{Q}$,
  - a term-structure factor driving the slopes of the $\bar{S}$ and $\bar{Q}$ TS.
- Existing challenges:
  - Improvements in fit of $\bar{S}$ and $\bar{Q}$ at the price of bad $\bar{D}$ fit;
  - $\rightarrow$ not enough flexibility in the model's pricing properties.
- Key for improvement:
  - Careful modeling of jump distributions (Exp-Laplace co-jumps, but see Bollerslev and Todorov (2014));
  - Inclusion of purely diffusive vol factor that does not drive jumps;
- Further work:
  - Implementing more flexible jump specifications;
  - Estimation with a greater number of swap contracts (e.g. $p = 0$, more maturities);
  - Inference about risk premia.

R packages www.github.com/piotrek-orlowski

affineModelR

Numeric backend for handling $\mathbb{P}$ and $\mathbb{Q}$ Characteristic Functions of (almost) arbitrarily specified Affine Jump-Diffusion models. Semi-closed form solutions for up to 3rd derivative wrt the stock price argument. Solutions for $\mathbb{E}_s^{\mathbb{P}}\left[\left(\frac{\Delta S_{t+1}}{S_t}\right)^k (\Delta V_j)^m\right]$ for $m+k \leq 2$, $m, k > 0$. Use with package transformOptionPricer for vanilla options.

divergenceModelR

Model-based pricing of divergence and higher-order swaps. Unscented Kalman Filters for estimation of AJD models. Builds against affineModelR and ukfRcpp.

ukfRcpp

Rcpp implementation of an Unscented Kalman Filter class. Users have to write C++ functions for handling the state dynamics and observation equation, then write an Rcpp function to create an ukfClass object and filter to return states or likelihood value.

Rlibcmaes by András Sali

R bindings for the libcmaes optimisation library.

Section 1

References

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