Modeling Divergence Swap Rates

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From VIX to power divergence (Schneider and Trojani (2015))

CBOE (2000) calculates the Volatility Index...

$$VIX_t^2 = \frac{2}{T} \sum_j \frac{\Delta K_j}{K_j^2} Q_t(K_j) \approx \frac{2}{T} \int_0^\infty \frac{Q_t(K)}{K^2} dK = -2\mathbb{E}_t^Q \left[ \ln \frac{F_T}{F_t} + \sum_{\tau=1}^T \delta_{\tau-1}(F_T - F_{\tau-1}) \right]$$

$$D_p(y, x) = \frac{y^p - x^p}{p(p-1)} - \frac{x^{p-1}(y - x)}{p-1}$$

$$D_0 = \lim_{p \to 0} D_p$$

$$\forall p \in \mathbb{R} \quad D_p(y, x) = O \left( \ln \frac{y}{x} \right)^2 \quad \text{for} \quad \ln \frac{y}{x} \to 0$$
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- Family of power divergence functions \( D_p \) defines family of (first-order) quadratic variation swaps. Compare with Bondarenko (2014); Martin (2012); Lee (2010).
- Weights \( \frac{\partial^2}{\partial K^2} D_p(K, x) = \frac{1}{K^p} \).
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- HF limits:

\[ \frac{1}{F_t^p} \sum_{\tau=1}^{T} D_p(F_{\tau}, F_{\tau-1}) \rightarrow \frac{1}{2} \int_t^T \left( \frac{F_s}{F_t} \right)^p \sigma_s^2 ds + \sum_{t \leq s \leq T} \frac{D_p(F_s, F_{s-})}{F_t} \]

- \( 1/F^p \) scaling essential → notation: \( \bar{D}_p = \frac{D_p(F_s, F_t)}{F_t^p} \).
Higher-order swaps

- AJD models not bad at fitting TS of Variance Swaps $\rightarrow$ how many SV factors? Gruber et al. (2015); Andersen et al. (2015).
- Variation in $p$ in divergence swaps $\rightarrow$ another dimension for fitting.

Fully replicable realized measures allow to define higher-order risk premia. $D_p, S_p$ and $Q_p$ highly correlated $\rightarrow$ standardise:

$$S_p := \frac{D_p}{2}$$

$$Q_p := \frac{S_p}{2}$$
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- Take a difference...

\[
\frac{1}{2\varepsilon} \left( \frac{D_{p_0+\varepsilon}(F_T, F_t)}{F_{t_0+\varepsilon}} - \frac{D_{p_0-\varepsilon}(F_T, F_t)}{F_{t_0-\varepsilon}} \right) = O \left( \ln \frac{F_T}{F_t} \right)^3
\]
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- ... or differentiate.

$$\frac{\partial}{\partial p} \frac{D_p(F_T, F_t)}{F_t^p} = O \left( \ln \frac{F_T}{F_t} \right)^3$$

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$$\bar{S}_p := S_p \frac{D_p}{2}$$

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- Rinse and repeat.

\[
\frac{\partial^2}{\partial p^2} \frac{D_p(F_T, F_t)}{F_t^p} = O \left( \ln \frac{F_T}{F_t} \right)^4
\]

Exactly tradable with static option portfolio and forward trading under any dynamics. Corresponding realized measures:

\[
\frac{1}{T} \sum_{\tau=1}^{T} s \cdot (p(F_{\tau}, F_{\tau-1}) - 1) + \frac{1}{2} \int_T^T s \cdot \left( F_s F_t \right) p \ln F_s F_t \sigma_s^2 ds + \sum_{t \leq s \leq T} (p(F_s - F_t) - D_p(F_s, F_{s-1}))
\]

\[
\frac{1}{T} \sum_{\tau=1}^{T} Q_p(F_{\tau}, F_{\tau-1}) \rightarrow \frac{1}{2} \int_T^T s \cdot \left( F_s F_t \right) p \ln 2 F_s F_t \sigma_s^2 ds + \sum_{t \leq s \leq T} (p(F_s - F_t) - D_p(F_s, F_{s-1}))
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\[
\bar{S}_p := S_p \cdot \frac{3}{2} p \quad \bar{Q}_p := S_p \cdot \frac{2}{p}
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- $D_p$, $S_p$ and $Q_p$ highly correlated → standardise:

$$\bar{S}_p := \frac{S_p}{D_p^{3/2}} \quad \quad \bar{Q}_p := \frac{S_p}{D_p^2}$$
Empirical motivation – S&P 500 options

**Price of divergence portfolio (annualised)**

\[ p = 0.5 \]

**Implied volatility (ATM)**
Empirical motivation – S&P 500 options

Price of skewness portfolio (scaled)

$p = 0.5$

Implied volatility slope at log-strike $k = -3/\left(\sigma_{ATM} \sqrt{\tau}\right)$
Empirical motivation – S&P 500 options

- Prices of variation swaps at range of \( p \) and \( \tau \) contain the same information as the IV surface;
- Forming option portfolios might alleviate measurement error, particularly for deep OTM contracts;
- Variation swaps are exactly tradable. IV is not. Who knows what IV is anyway?
Empirical motivation – S&P 500 options

- Variation swaps are exactly tradable. IV is not. Who knows what IV is anyway?
Affine Jump Diffusion models are a popular (and somewhat successful) modeling tool.

State variables: \( X_t = [\ln S_t / S_{t-s}, V_{1t}, \ldots, V_{M_t}] \), \( V_t := X_t[-1] \)

\[
E^M_t \left[ e^{u \cdot X_{t+s}} \right] = e^{\alpha(s,u) + \beta(s,u) \cdot V_t}, \quad u \in \mathbb{C}^{M+1}
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Easily calculate \( \bar{D}_{p,t,s}, \bar{S}_{p,t,s}, \bar{Q}_{p,t,s} \) in model (MGF and derivatives).
Divergence in Affine Jump Diffusions

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- Easily calculate \( \bar{D}_{p,t,s}, \bar{S}_{p,t,s}, \bar{Q}_{p,t,s} \) in model (MGF and derivatives).
- Observable quantities: \( \frac{\Delta S_{t+1}}{S_t}, \ \bar{D}_{p,t+1,s}, \ \bar{S}_{p,t+1,s}, \ \bar{Q}_{p,t+1,s} \): functions of \([V_t \ V_{t+1}]\)
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- Latent volatility factor dynamics:
  \[ \Delta V_{t+1} = \mu_{t+1}^V + \Sigma_{t+1}^V \cdot W_{t+1}, \quad \mu_{t+1}^V, \Sigma_{t+1}^V : \text{non-linear functions of } V_t \]
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Observable quantities: \( \Delta S_{t+1} / S_t, \bar{D}_{p,t+1,s}, \bar{S}_{p,t+1,s}, \bar{Q}_{p,t+1,s} \): functions of \([V_t \ V_{t+1}]\).

Latent volatility factor dynamics:

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Unscented Kalman Filter – counterfactual assumption of \( W_t \sim N(0, I_{M \times M}) \);

Specification of model under \( \mathbb{Q} \rightarrow \text{Risk Premia} \rightarrow \text{specification under } \mathbb{P} \);

Reasonable performance in simulated settings.
Preliminary results

- First estimations with $p = 1/2$ (symmetric swaps), maturities 1M, 6M;
- In three factor specification possible to identify:
  - a variance-level factor,
  - a common factor for $\tilde{S}$ and $\tilde{Q}$,
  - a term-structure factor driving the slopes of the $\tilde{S}$ and $\tilde{Q}$ TS.
- Existing challenges:
  - Improvements in fit of $\tilde{S}$ and $\tilde{Q}$ at the price of bad $\tilde{D}$ fit;
  - $\rightarrow$ not enough flexibility in the model’s pricing properties.
- Key for improvement:
  - Careful modeling of jump distributions (Exp-Laplace co-jumps, but see Bollerslev and Todorov (2014));
  - Inclusion of purely diffusive vol factor that does not drive jumps;
- Further work:
  - Implementing more flexible jump specifications;
  - Estimation with a greater number of swap contracts (e.g. $p = 0$, more maturities);
  - Inference about risk premia.
**affineModelR**

Numeric backend for handling $\mathbb{P}$ and $\mathbb{Q}$ Characteristic Functions of (almost) arbitrarily specified Affine Jump-Diffusion models. Semi-closed form solutions for up to 3rd derivative wrt the stock price argument. Solutions for $\mathbb{E}_s^\mathbb{P} \left[ \left( \frac{\Delta S_{t+1}}{S_t} \right)^k (\Delta V_j)^m \right]$ for $m + k \leq 2$, $m, k > 0$. Use with package `transformOptionPricer` for vanilla options.

**divergenceModelR**

Model-based pricing of divergence and higher-order swaps. Unscented Kalman Filters for estimation of AJD models. Builds against `affineModelR` and `ukfRcpp`.

**ukfRcpp**

Rcpp implementation of an Unscented Kalman Filter class. Users have to write C++ functions for handling the state dynamics and observation equation, then write an Rcpp function to create an ukfClass object and filter to return states or likelihood value.

**Rlibcmaes by András Sali**

R bindings for the `libcmaes` optimisation library.
Section 1

References


