It's Madness!

(Multivariate Automatic Differentiation)

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May 20, 2016

What is it?

It's madness:

- Package for 'multivariate' automatic differentiation.
- Available on CRAN and github.
- 'Multivariate': supports array to array operations.
  If `v <- f(x)`, then `dim(dvdx(f)) == c(length(v),length(x))`.
- Should act just like a numeric, with derivatives computed for you.
- Derivatives via forward accumulation and chain rule.
  No fancy tricks like NSE.
- S4 object with lots of overloaded methods.

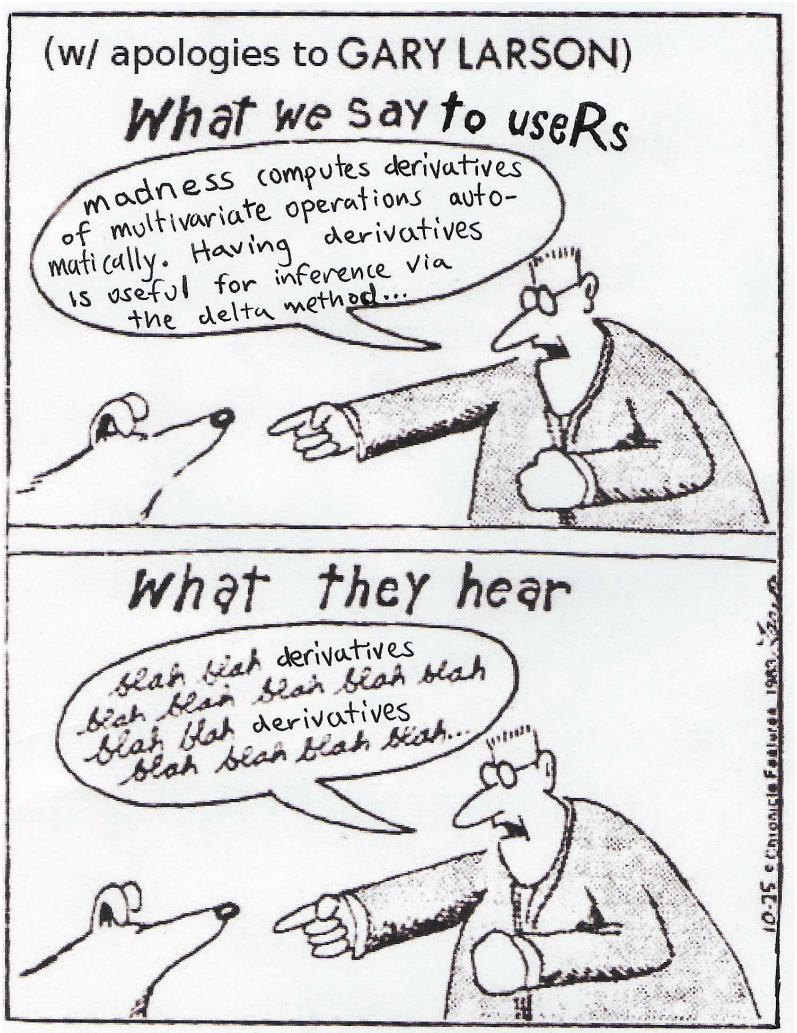
Motivation

- Primary use case is multivariate delta method.

$$\left\{\sqrt{n}\left(\bar{X}-X\right) \rightsquigarrow \mathcal{N}\left(0,\Omega\right)\right\} \Rightarrow$$
$$\left\{\sqrt{n}\left(f\left(\bar{X}\right)-f\left(X\right)\right) \rightsquigarrow \mathcal{N}\left(0,\frac{\mathrm{d}f}{\mathrm{d}X}\Omega\frac{\mathrm{d}f}{\mathrm{d}X}^{\top}\right)\right\}$$

- Allow user to transform estimates via f, automatically compute $\frac{\mathrm{d}f}{\mathrm{d}X}$.
- Maybe also useful for constructing objective functions.

blah blah derivatives



Basic use I

Initialize with independent variable:

```
X_independent <- array(1:4, dim = c(1, 4))
X_mad <- madness(X_independent)
show(X_mad)

## class: madness
##          d X_independent
##  calc: -----------------
##          d X_independent
##   val: 1 2 3 4 ...
##  dvdx: 1 0 0 0 ...
##  varx:  ...

show(diag(dvdx(X_mad)))

## [1] 1 1 1 1
```

Basic use II

Initialize with independent variable:

```
X_independent <- array(1:4, dim = c(1, 4))
X_mad <- madness(X_independent)
show(X_mad^2)

## class: madness
##          d (X_independent ^ numeric)
##  calc: ----------------------------
##                d X_independent
##   val: 1 4 9 16 ...
##  dvdx: 2 0 0 0 ...
##  varx:  ...

show(diag(dvdx(X_mad^2)))

## [1] 2 4 6 8
```

Basic use III

```
X_mad <- madness(array(1:4, dim = c(1, 4)))
show(diag(dvdx(abs(cos(X_mad))^sqrt(X_mad))))

## [1] -1.01  0.80  0.24 -1.03

show(dvdx(prod(X_mad)))

##      [,1] [,2] [,3] [,4]
## [1,]   24   12    8    6

show(dvdx(colSums(crossprod(X_mad))))

##      [,1] [,2] [,3] [,4]
## [1,]   11    1    1    1
## [2,]    2   12    2    2
## [3,]    3    3   13    3
## [4,]    4    4    4   14
```

Intermediate use I

`as.madness` promotes objects with `coef` and optional `vcov` methods:

```
set.seed(12345)  # mental note: change gmail password
adf <- data.frame(x = rnorm(1000), y = rnorm(1000))
alm <- lm(y ~ x, data = adf)
amad <- as.madness(alm)
show(cov2cor(vcov(amad^2)))
```

```
##       [,1]  [,2]
## [1,] 1.000 0.046
## [2,] 0.046 1.000
```

Intermediate use II

`twomoments` computes first and second moments from data as `madness`:

```
set.seed(12345)  # mental note: change gmail password
X <- matrix(rnorm(1000 * 2), ncol = 2)
two <- twomoments(X, diag.only = FALSE)
# symmetric square root of covariance:
show(cov2cor(vcov(sqrtm(two$Sigma))))
```

```
##       [,1]  [,2]  [,3] [,4]
## [1,] 1.000 0.029 0.029 0.02
## [2,] 0.029 1.000 1.000 0.11
## [3,] 0.029 1.000 1.000 0.11
## [4,] 0.020 0.110 0.110 1.00
```

```
# Hotelling's statistic, up to scaling:
show(vcov(t(two$mu) %*% solve(two$Sigma, two$mu))[1])
```

```
## [1] 1.3e-05
```

Markowitz portfolio

```
library(dplyr)
ff3 <- Quandl("KFRENCH/FACTORS_W", order = "asc") %>%
    select(starts_with("Mkt"), SMB, HML)
# compute first and second moments:
two <- twomoments(ff3, diag.only = FALSE)
the_mp <- solve(two$Sigma, two$mu)
show(val(t(the_mp)))
```

```
##       [,1]  [,2]  [,3]
## [1,] 0.013 0.013 0.022
```

```
show(vcov(the_mp))
```

```
##           [,1]      [,2]      [,3]
## [1,]  3.0e-05 -4.9e-06 -1.4e-05
## [2,] -4.9e-06  9.9e-05  6.9e-07
## [3,] -1.4e-05  6.9e-07  7.5e-05
```

What's next?

To learn more:

- Check the github project page.
- Read the vignette.
- Comb through the unit tests.
- Ask me questions.

Thank you.

Common Questions I

Why another AD package?

I needed it for the multivariate delta method, and suspected that numerical approximations would be too slow.

Can it compute Hessians or higher order derivatives?

Not yet. That would be very complicated. You might be better off using the numDeriv package.

How are derivatives computed?

I took a 'high level' approach, overloading as many methods as possible. A 'low level' approach that overloads only addition, multiplication, exponentiation, and so on, would not work for computations performed in *e.g.*, C or C++ code.

Common Questions II

I am getting an error.

Not everything works yet. File an issue on github. In the interim, you can use the `madness::numderiv` method for numerical approximation, or instead use the numDeriv package.

What about symmetry?

The `twomoments` and `theta` methods deal with symmetry correctly. Be wary about constructing your own `madness` objects where the matrix must be symmetric. To 'force' matrix symmetry, use *e.g.*,
`forcesym <- function(x) { 0.5 * (x+t(x)) }`.