R package **mcrp**: Multiple criteria risk contribution optimization

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Contents

Overview

- The concept of risk parity (*aka* ERC) is due to Qian (2005, 2006, 2011) (see also Maillard et al., 2010; Roncalli, 2013).
- Hereby, the contributions with respect to the portfolio standard deviation risk of the constituents are equal.
- Recently, this concept has been extended to include the higher-moment risk contributions (see Baitinger et al., 2017), *i.e.*, skewness and kurtosis.
- In this talk:
    1. Optimization problem of multiple criteria risk contributions.
    2. Structure of the R package **mcrp**.
    3. Empirical application to a multi-asset class portfolio.

Multiple criteria risk optimization

Higher Moments: Definitions

Definitions provided in Jondeau and Rockinger (2006):

$$\sigma_p^2 = \mathrm{E}\left[\sum_{i=1}^{n} \omega_i (R_i - \mu_i)(r_p - \mu_p)\right] = \omega' \Sigma_p$$
$$= \omega' M_2 \omega$$

$$\mathrm{s}_p^3 = \mathrm{E}\left[\sum_{i=1}^{n} \omega_i (R_i - \mu_i)(r_p - \mu_p)^2\right] = \omega' \mathrm{S}_p$$
$$= \omega' M_3 (\omega \otimes \omega)$$

$$\kappa_p^4 = \mathrm{E}\left[\sum_{i=1}^{n} \omega_i (R_i - \mu_i)(r_p - \mu_p)^3\right] = \omega' \mathrm{K}_p$$
$$= \omega' M_4 (\omega \otimes \omega \otimes \omega)$$

Multiple criteria risk optimization

Higher Moments: Partial Derivatives

$$\mathrm{MRC}_2 = \frac{\delta \sigma_p^2}{\delta \omega} = 2 M_2 \omega \qquad \text{and} \quad \mathrm{ARC}_2 = \mathrm{MRC}_2 \times \omega$$
$$\mathrm{MRC}_3 = \frac{\mathrm{s}_p^3}{\delta \omega} = 3 M_3 (\omega \otimes \omega) \qquad \text{and} \quad \mathrm{ARC}_3 = \mathrm{MRC}_3 \times \omega$$
$$\mathrm{MRC}_4 = \frac{\kappa_p^4}{\delta \omega} = 4 M_4 (\omega \otimes \omega \otimes \omega) \qquad \text{and} \quad \mathrm{ARC}_4 = \mathrm{MRC}_4 \times \omega$$

Matrices $(n \times n)$ M_2, $(n \times n^2)$ M_3, $(n \times n^3)$ M_4 are the centred (tensor) product moment matrices:

$$M_2 = \mathrm{E}\left[(R-\mu)(R-\mu)'\right] \qquad = \{\sigma_{ij}\}$$
$$M_3 = \mathrm{E}\left[(R-\mu)(R-\mu)' \otimes (R-\mu)'\right] \qquad = \{\mathrm{s}_{ijk}\}$$
$$M_4 = \mathrm{E}\left[(R-\mu)(R-\mu)' \otimes (R-\mu)' \otimes (R-\mu)'\right] \qquad = \{\kappa_{ijkl}\}$$

Multiple criteria risk optimization

Problem formulation

$$
\begin{aligned}
\text{minimize } & F(\omega) = \lambda_1 \mathrm{VAR}(ARC_2) + \lambda_2 \mathrm{VAR}(ARC_3) + \lambda_3 \mathrm{VAR}(ARC_4) \\
\text{subject to } & \sum_{i=1}^{n} \omega_i = 1 \\
& 0 \leq \omega_i \leq 1 \;\; \text{for } i = 1, \ldots, n\,.
\end{aligned}
$$

R package **mcrp**
Structure

- Package is purely written in R.
- Dependencies to **FRAPO** (see Pfaff, 2016) and suggests **testthat** (see Wickham, 2011) for unit testing.
- Optimization conducted with `stats::nlmnib()`.
- Core function: `mcrp()`.
- Auxilliary functions: `Portfoo()`, `PortfooDeriv()`, `PortfooContrib()` with `foo = {Risk, Skew, Kurt}`.
- Available on GitHub: https://github.com/bpfaff/mcrp

R package **mcrp**

Core function: Input

Function `mcrp()`

```
> args(mcrp)

function (start, returns, lambda = c(1, 1, 1), ...)
NULL
```

Arguments

- `start`: vector of starting values.
- `returns`: matrix of assets' returns.
- `lambda`: selection/weighting of sub-objectives; set element(s) to `NA` for exclusion.
- `...`: ellipsis argument is passed down to `stats::nlmnib()`.

R package **mcrp**

Core function: Output

S4-Class: PortSol from package **FRAPO**

```
> showClass("PortSol")

Class "PortSol" [package "FRAPO"]

Slots:

Name:    weights       opt      type      call
Class:   numeric      list character     call

Known Subclasses: "PortCdd", "PortAdd", "PortMdd"
```

Slots

- `weights`: portfolio weight vector.
- `opt`: list object returned by `stats::nlmnib()`.
- `type`: decsription of portfolio problem.
- `call`: the call to `mcrp()` (used for `stats::update()`-method).

Empirical application

Specification

- Data set `MultiAsset` contained in **FRAPO**: month-end data from 11/2004 until 11/2011.
- Fix-me strategy:
  1. 1st sub-sample until 12/2008 used for optimization.
  2. 2nd sub-sample from 01/2009 for backtest.
- Portfolio optimizations with respect to:
  1. single criteria: contrib. to risk.
  2. single criteria: contrib. to skewness.
  3. single criteria: contrib. to kurtosis.
  4. multiple criteria: contrib. to risk and skewness.
  5. multiple criteria: contrib. to risk and kurtosis.
  6. multiple criteria: contrib. to risk, skewness and kurtosis.

Empirical application

R code: Data and descriptive statistics

```
> library(mcrp)
> data(MultiAsset)
> P <- as.timeSeries(MultiAsset[, c("GSPC", "GDAXI", "FTSE", "EEM", "GLD")])
> R <- returns(P) * 100
> K <- ncol(R)
> ew <- rep(1 / K, K) ## equal weight / starting values
> ans <- cbind(apply(R, 2,  sd),
+              apply(R, 2,  skewness),
+              apply(R, 2,  kurtosis, method = "moment"))
```

Index	Risk	Skewness	Kurtosis
GSPC	4.835	−0.897	4.876
GDAXI	5.980	−1.002	5.096
FTSE	4.377	−0.704	3.581
EEM	8.079	−0.744	4.380
GLD	5.474	−0.485	3.852

Table: Empirical higher-moments

Empirical application

R code: Portfolio optimizations

```
> ## single criteria: contrib. to risk
> p100 <- mcrp(ew, R, lambda = c(1, NA, NA), lower = 0)
> ## single criteria: contrib. to skewness
> p010 <- mcrp(ew, R, lambda = c(NA, 1, NA), lower = 0)
> ## single criteria: contrib. to kurtosis
> p001 <- mcrp(ew, R, lambda = c(NA, NA, 1), lower = 0)
> ## multiple criteria: contrib. to risk and skewness
> p110 <- mcrp(ew, R, lambda = c(1, 1, NA), lower = 0)
> ## multiple criteria: contrib. to risk and kurtosis
> p101 <- mcrp(ew, R, lambda = c(1, NA, 1), lower = 0)
> ## multiple criteria: contrib. to risk, skewness and kurtosis
> p111 <- mcrp(ew, R, lambda = c(1, 1, 1), lower = 0)
> ## aggregating results to list object
> popt <- list(p100, p010, p001, p110, p101, p111)
> pn <- length(popt)
> pnames <- c("p100", "p010", "p001", "p110", "p101", "p111")
> names(popt) <- pnames
> ## checking solutions
> (sopt <- unlist(lapply(popt, function(x) Solution(x)$convergence)))

p100 p010 p001 p110 p101 p111
   0    0    0    0    0    0
```

Empirical application

R code: Allocations

```
> wopt <- lapply(popt, Weights)
> wmat <- matrix(unlist(wopt), nrow = K, ncol = pn)
> colnames(wmat) <- pnames
> rownames(wmat) <- colnames(P)
> ans <- round(wmat * 100, 2)
```

Index	p100	p010	p001	p110	p101	p111
GSPC	19.47	17.43	18.16	18.09	18.77	18.07
GDAXI	17.24	21.43	21.45	21.42	21.01	21.46
FTSE	22.13	31.99	30.13	30.27	28.76	30.21
EEM	10.99	12.18	11.60	12.42	11.87	12.01
GLD	30.17	16.97	18.66	17.80	19.58	18.25

Table: Optimal allocations

Empirical application

R code: In-sample characteristics, part I

```
> ## Function for higher moments and portfolio contributions
> momis <- function(w, r = R){
+     a1 <- PortRisk(r, w)
+     a2 <- PortSkew(r, w)
+     a3 <- PortKurt(r, w)
+     a4 <- sd(PortRiskContrib(r, w))
+     a5 <- sd(PortSkewContrib(r, w))
+     a6 <- sd(PortKurtContrib(r, w))
+     a7 <- mean(c(a4, a5, a6))
+     a <- c(a1, a2, a3, a4, a5, a6, a7)
+     a
+ }
> ans <- matrix(unlist(lapply(wopt, momis)), ncol = pn)
> colnames(ans) <- pnames
> rownames(ans) <- c("Risk", "Skewness", "Kurtosis",
+                     "Sd Risk ctrb.", "Sd Skew ctrb.", "Sd Kurt ctrb.",
+                     "Average of Sd")
```

Empirical application

R code: In-sample characteristics, part II

Measure	p100	p010	p001	p110	p101	p111
Moments						
Risk	17.022	19.652	19.160	19.580	19.047	19.363
Skewness	−1.042	−0.889	−0.908	−0.900	−0.920	−0.904
Kurtosis	5.961	4.589	4.740	4.684	4.849	4.712
Contributions						
Sd Risk ctrb.	0.000	0.088	0.078	0.081	0.071	0.080
Sd Skew ctrb.	0.120	0.000	0.018	0.012	0.029	0.014
Sd Kurt ctrb.	0.100	0.017	0.000	0.009	0.012	0.005
Average of Sd	0.073	0.035	0.032	0.034	0.037	0.033

Table: In-sample characteristics

Summary

- Approach for an extended portfolio risk balancing with respect to higher moments.
- Weighting between (higher) moment risk contributions is possible by setting of λ, and hereby allowing selection of certain kind of risk contributions as special cases.
- Empirical example: By considering higher moment risk contributions, overall dispersion is reduced compared to the ERC-only solution.
- Caveats:
  1. Casting of optimization problem not ideal, if constituents have differing signs for skewness.
  2. No guarantee that risk contributions of higher moment risks are the same; but at least a solution of least dispersed risk contributions is obtained.

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