A BAYESIAN MULTIVARIATE FUNCTIONAL DYNAMIC LINEAR MODEL

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Joint work with David S. Matteson and David Ruppert
Install and load the package **FDLM** from GitHub:

```r
devtools::install_github("drkowal/FDLM")
library(FDLM)
```
MULTIECONOMY YIELD CURVES:
A MULTIVARIATE TIME SERIES OF FUNCTIONAL DATA
FUNCTIONAL: $y(\tau)$
FUNCTIONAL ($\tau$) + TIME SERIES ($t$): $y_t(\tau)$
FUNCTIONAL (τ) + TIME SERIES (t) + MULTIVARIATE (c): \( y_{t}^{(c)}(\tau) \)


Maturity (years)
Yield (%)
FED: 11/08
FED: 11/15
BOE: 11/08
BOE: 11/15
Dominant structural features in the data:

1. Functional
2. Time-dependence: time-ordered observations
3. Contemporaneous dependence: multivariate observations
A MULTIVARIATE FUNCTIONAL DYNAMIC LINEAR MODEL
A FUNCTIONAL DYNAMIC LINEAR MODEL (FDLM)

Functional Dynamic Linear Model (FDLM) [Kowal et al., 2016]:

\[ y_t(\tau) = \sum_{k=1}^{K} f_k(\tau) \beta_{k,t} + \epsilon_t(\tau), \quad \tau \in \mathcal{T}, \quad t = 1, \ldots, T \]

Decompose a functional time series \( y_t \) into

- **functional component** \( \{ f_k(\tau) \}_{k=1}^{K} \)
- **time series component** \( \{ \beta_{k,t} \}_{k=1}^{K} \)

We model \( \beta_t = (\beta_{1,t}, \ldots, \beta_{K,t})' \) as the state vector in a DLM
Functional Dynamic Linear Model (FDLM) [Kowal et al., 2016]:

$$y_t(\tau) = \sum_{k=1}^{K} f_k(\tau) \beta_{k,t} + \epsilon_t(\tau), \quad \tau \in \mathcal{T}, \quad t = 1, \ldots, T$$

Dynamic functional factor model:

- \(\{f_k\}\) are factor loading curves
- \(\{\beta_{k,t}\}\) are dynamic factors
- \(K = \text{number of factors}\)
Suppose we observe $y_t = (y_t(t_1), \ldots, y_t(t_M))'$

**Functional Dynamic Linear Model (FDLM):**

$$\begin{cases}
    y_t = F \beta_t + \epsilon_t, \\
    \beta_t = G \beta_{t-1} + \omega_t
\end{cases}, \quad \epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2_{\epsilon_t} I_M)$$

where $F = (f_1, \ldots, f_K)$, and $f_k = (f_k(t_1), \ldots, f_k(t_M))'$

**Extensions** for covariates, stochastic volatility, change points
Functional Dynamic Linear Model (FDLM):

\[
\begin{align*}
\mathbf{y}_t &= \mathbf{F}_t \mathbf{\beta}_t + \mathbf{\epsilon}_t, \\
\mathbf{\beta}_t &= \mathbf{G}_t \mathbf{\beta}_{t-1} + \mathbf{\omega}_t,
\end{align*}
\]

\[\mathbf{\epsilon}_t \overset{\text{indep}}{\sim} \mathcal{N}(\mathbf{0}, \sigma^2_{\mathbf{\epsilon}_t} \mathbf{I}_M)\]

\[\mathbf{\omega}_t \overset{\text{indep}}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{W}_t)\]

For R implementation, use a state space package, such as KFAS [Helske, 2017]:

\[
\text{SSModel}(\mathbf{y}_t \sim -1 + \text{SSMcustom}(Z = \mathbf{F}, T = \mathbf{G}, Q = \mathbf{W}_t), H = \text{sigma}_\mathbf{\epsilon}_t^2)
\]
Nonparametric regression model for each $f_k$:

$$f_k(\tau) = \phi'(\tau)d_k$$

- $\phi(\cdot)$ known vector of basis functions (splines)
- $d_k$ unknown vector of basis coefficients
- Include roughness penalty via the prior

$$d_k \sim N(0, \lambda_k^{-1}\Omega^{-1})$$

Flexible, computationally efficient, and smooth
MODEL IMPLEMENTATION:
MCMC SAMPLER
1. Sample the factors, $\{\beta_{k,t}\}$, using code from KFAS:

   \[
   \text{Beta} = \text{fdlm\_factor}(...) 
   \]

2. Sample the observation and evolution error variances:

   \[
   \text{sigma\_e} = 1/\text{sqrt}(\text{rgamma}(...)) \\
   \text{Wt} = \text{sample\_Wt}(...) 
   \]

3. Sample the factor loading curves, $\{f_k\}$

   \[
   \text{D} = \text{sampleFLC}(...) \quad \# \text{basis coefficients} 
   \]
FDLM CODE EXAMPLE: fdlm()

# Read in the FED yield curve data (Y, tau, dates):
data("US_Yields")
#data("UK_Yields")

# Restrict to dates since 2012:
Y = Y[which(dates > as.Date("2012-01-01")),];

# Run the MCMC:
mcmc_output = fdlm(Y, tau,
    K = 3,
    nsims = 2500, burnin = 2500,
    mcmc_params = list("beta", "fk",
                      "Yhat", "sigma_e"))
DYNAMIC FACTORS: FED

FED: Dynamic Factors

Dates

2012 2013 2014 2015 2016 2017

FED: Dynamic Factors

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FACTOR LOADING CURVES: BOE

BOE: Factor Loading Curves

Maturity (years)
BOE: Dynamic Factors
1. Hierarchical framework for modeling a multivariate time series of functional data
   - Covariates, stochastic volatility, change points

2. Joint estimation of model parameters; “exact” inference

3. Efficient Gibbs sampling algorithm; package FDLM

4. Applications: finance/economics and neuroscience; astronomy
*KFAS: Kalman Filter and Smoother for Exponential Family State Space Models.*  
R package version 1.2.6.

*A Bayesian multivariate functional dynamic linear model.*  
*Journal of the American Statistical Association.*  
(in press).