A BAYESIAN MULTIVARIATE FUNCTIONAL DYNAMIC LINEAR MODEL

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INSTALL THE PACKAGE FDLM

Install and load the package `FDLM` from GitHub:

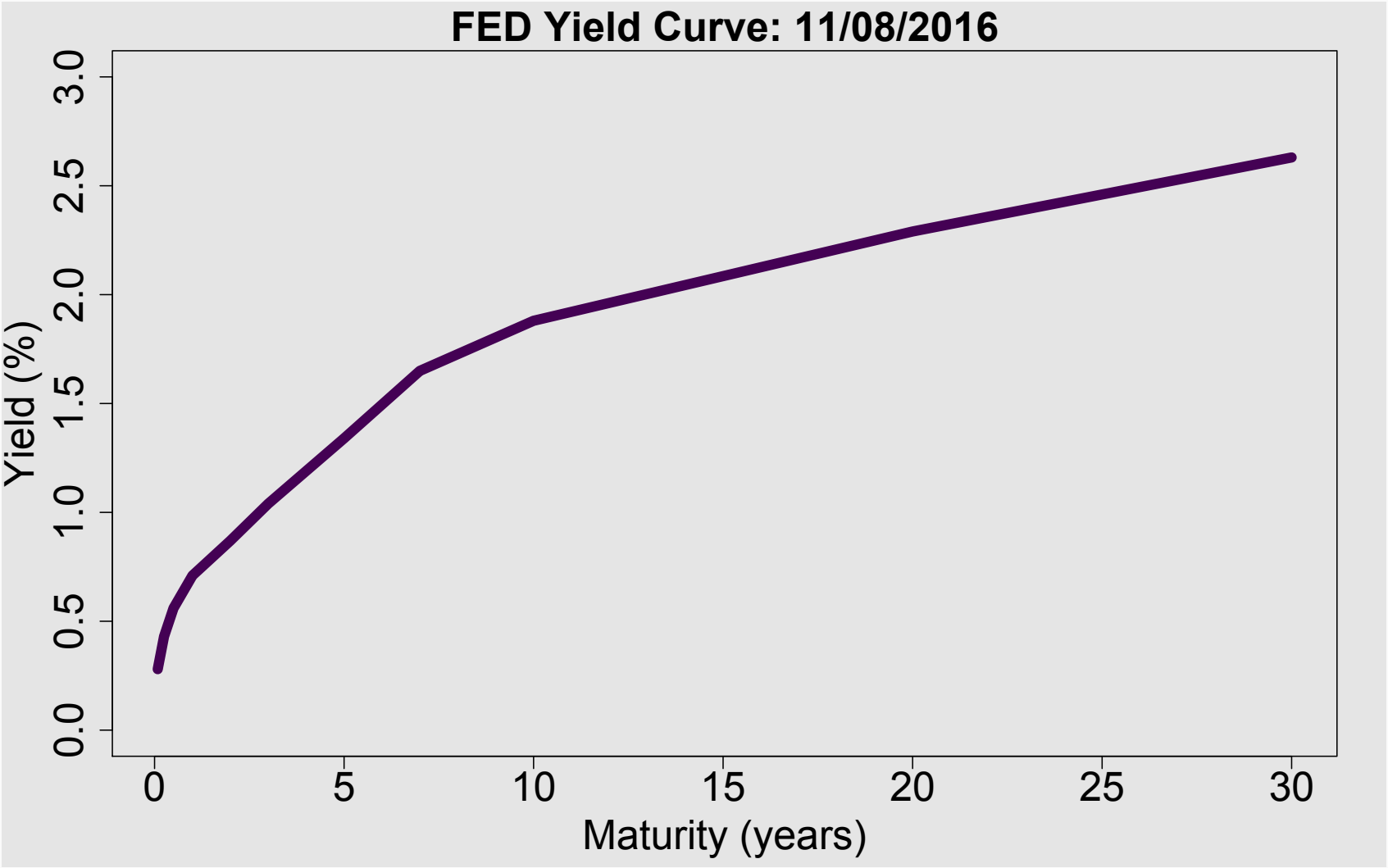
```
devtools::install_github("drkowal/FDLM")
```

```
library(FDLM)
```

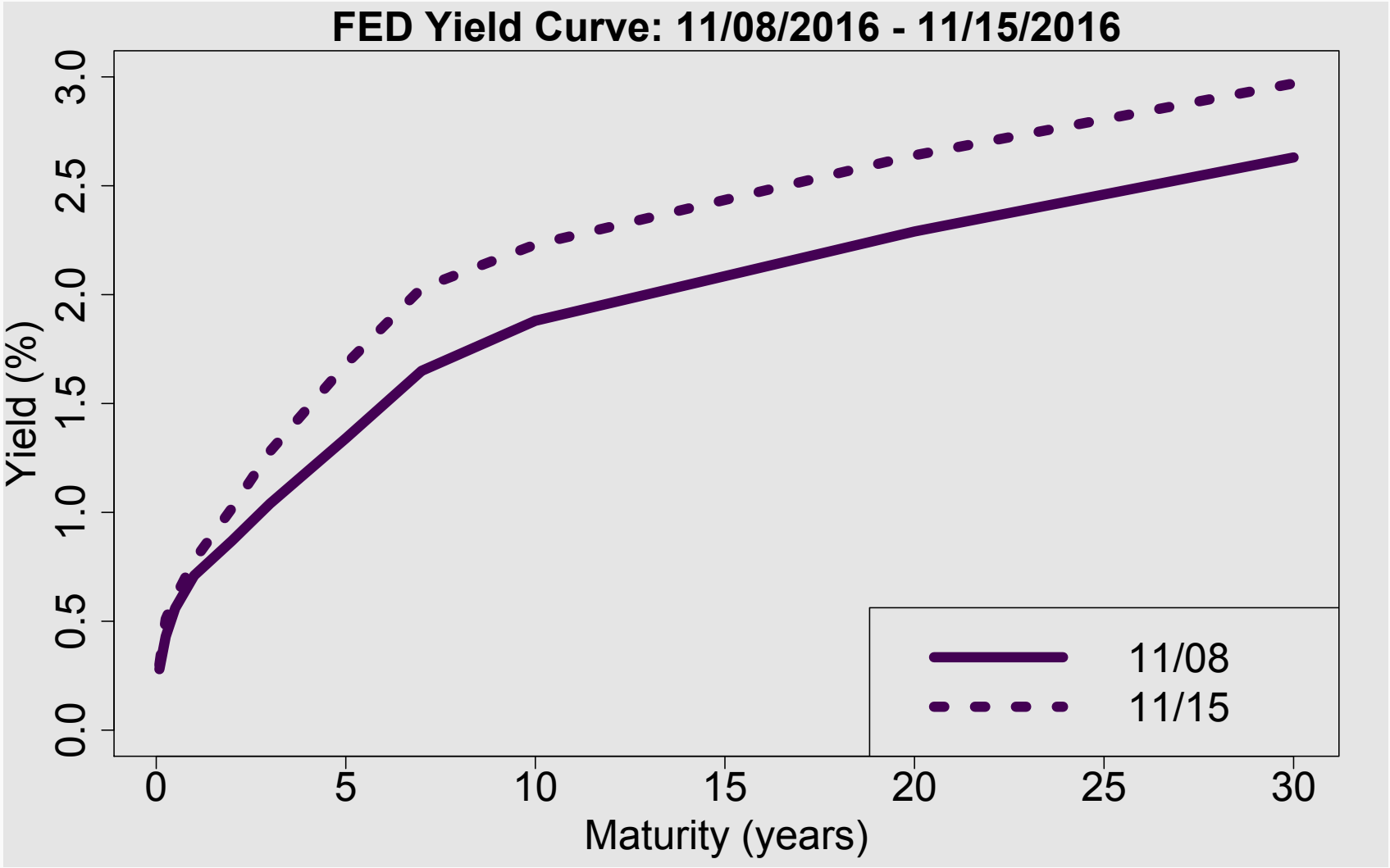
MULTIECONOMY YIELD CURVES:

A MULTIVARIATE TIME SERIES OF FUNCTIONAL DATA

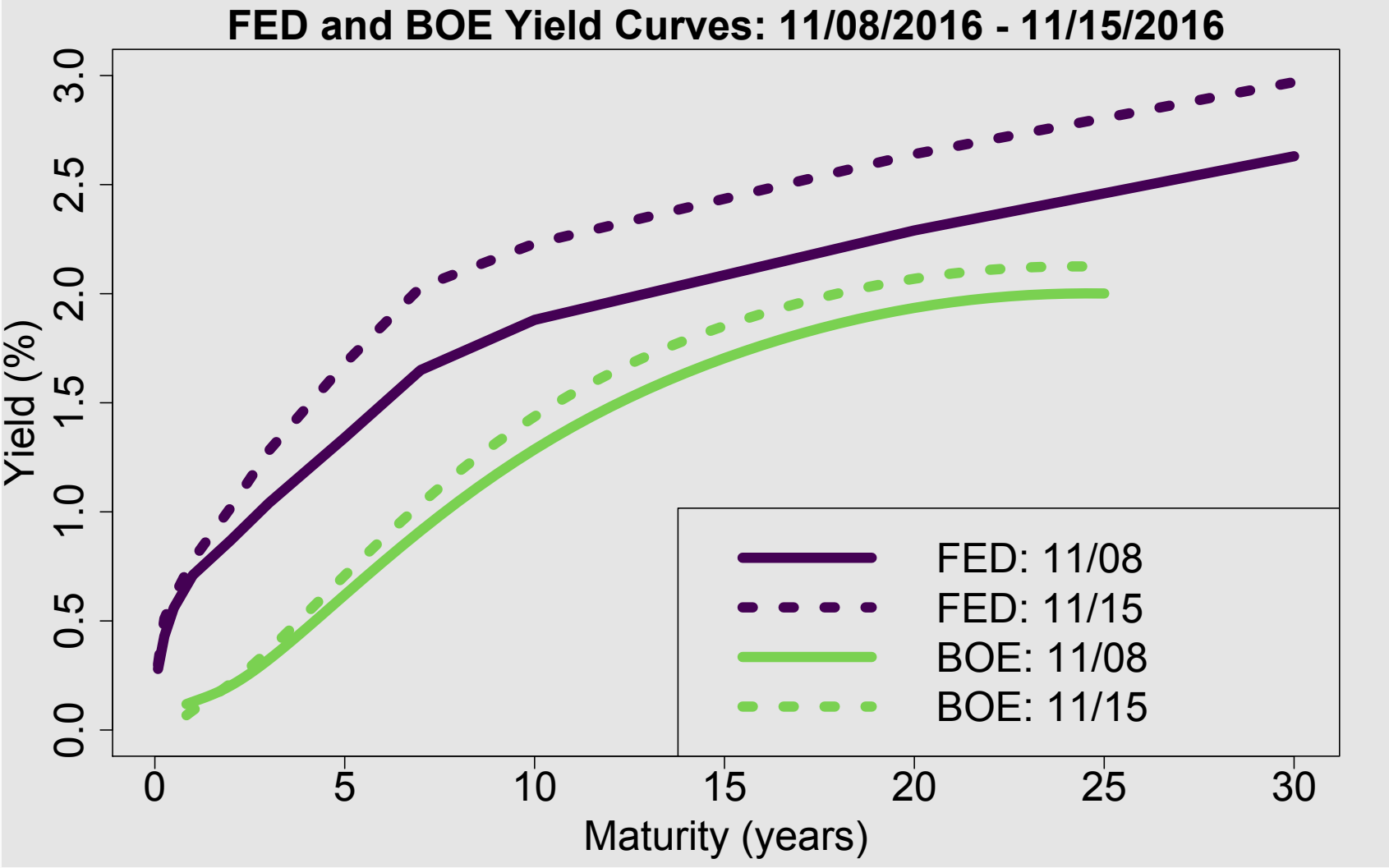
FUNCTIONAL: $y(\tau)$



FUNCTIONAL (τ) + TIME SERIES (t): $y_t(\tau)$



FUNCTIONAL (τ) + TIME SERIES (t) + MULTIVARIATE (c): $y_t^{(c)}(\tau)$



GENERAL SETTING: MULTIVARIATE TIME SERIES OF FUNCTIONAL DATA

Dominant structural features in the data:

1. Functional
2. Time-dependence: time-ordered observations
3. Contemporaneous dependence: multivariate observations

A MULTIVARIATE FUNCTIONAL DYNAMIC LINEAR MODEL

A FUNCTIONAL DYNAMIC LINEAR MODEL (FDLM)

Functional Dynamic Linear Model (FDLM) [Kowal et al., 2016]:

$$y_t(\tau) = \sum_{k=1}^{K} f_k(\tau)\beta_{k,t} + \epsilon_t(\tau), \quad \tau \in \mathcal{T}, \quad t = 1, \ldots, T$$

Decompose a functional time series y_t into

- *functional component* $\{f_k(\tau)\}_{k=1}^{K}$
- *time series component* $\left\{\beta_{k,t}\right\}_{k=1}^{K}$

We model $\boldsymbol{\beta}_t = (\beta_{1,t}, \ldots, \beta_{K,t})'$ as the state vector in a DLM

FDLM AS A FACTOR ANALYSIS

Functional Dynamic Linear Model (FDLM) [Kowal et al., 2016]:

$$y_t(\tau) = \sum_{k=1}^{K} f_k(\tau)\beta_{k,t} + \epsilon_t(\tau), \quad \tau \in \mathcal{T}, \quad t = 1, \ldots, T$$

Dynamic functional factor model:

- $\{f_k\}$ are *factor loading curves*
- $\{\beta_{k,t}\}$ are dynamic *factors*
- $K =$ number of factors

FDLM AS A DYNAMIC LINEAR MODEL

Suppose we observe $\boldsymbol{y}_t = (y_t(\tau_1), \ldots, y_t(\tau_M))'$

Functional Dynamic Linear Model (FDLM):

$$\begin{cases} \underbrace{\boldsymbol{y}_t}_{M\times 1} = \underbrace{\boldsymbol{F}}_{M\times K} \underbrace{\beta_t}_{K\times 1} + \underbrace{\epsilon_t}_{M\times 1}, & \epsilon_t \overset{indep}{\sim} N(\mathbf{o}, \sigma^2_{\epsilon_t} \boldsymbol{I}_M) \\ \\ \beta_t = \boldsymbol{G}\beta_{t-1} + \boldsymbol{\omega}_t, & \boldsymbol{\omega}_t \overset{indep}{\sim} N(\mathbf{o}, \boldsymbol{W}_t) \end{cases}$$

where $\boldsymbol{F} = (\boldsymbol{f}_1, \ldots, \boldsymbol{f}_K)$, and $\boldsymbol{f}_k = (f_k(\tau_1), \ldots, f_k(\tau_M))'$

Extensions for covariates, stochastic volatility, change points

Functional Dynamic Linear Model (FDLM):

$$\begin{cases} \underbrace{\boldsymbol{y}_t}_{M\times 1} = \underbrace{\boldsymbol{F}}_{M\times K}\underbrace{\boldsymbol{\beta}_t}_{K\times 1} + \underbrace{\boldsymbol{\epsilon}_t}_{M\times 1}, & \boldsymbol{\epsilon}_t \overset{indep}{\sim} N(\mathbf{o}, \sigma^2_{\epsilon_t}\boldsymbol{I}_M) \\ \\ \boldsymbol{\beta}_t = \boldsymbol{G}\boldsymbol{\beta}_{t-1} + \boldsymbol{\omega}_t, & \boldsymbol{\omega}_t \overset{indep}{\sim} N(\mathbf{o}, \boldsymbol{W}_t) \end{cases}$$

For `R` implementation, use a state space package, such as `KFAS` [Helske, 2017]:

```
SSModel(yt ~ -1 + SSMcustom(Z = F,
                            T = G,
                            Q = Wt), H = sigma_et^2)
```

ESTIMATING THE FACTOR LOADING CURVES f_k

Nonparametric regression model for each f_k:

$$f_k(\tau) = \phi'(\tau)\boldsymbol{d}_k$$

- $\phi(\cdot)$ *known* vector of basis functions (splines)
- $\boldsymbol{d}_k$ *unknown* vector of basis coefficients
- Include roughness penalty via the prior

$$\boldsymbol{d}_k \sim N\left(\mathbf{o}, \lambda_k^{-1}\boldsymbol{\Omega}^{-1}\right)$$

Flexible, computationally efficient, and smooth

MODEL IMPLEMENTATION: MCMC SAMPLER

FDLM SAMPLING ALGORITHM

1. Sample the factors, $\{\beta_{k,t}\}$, using code from `KFAS`:

```
Beta = fdlm_factor(...)
```

2. Sample the observation and evolution error variances:

```
sigma_e = 1/sqrt(rgamma(...)
Wt = sample_Wt(...)
```

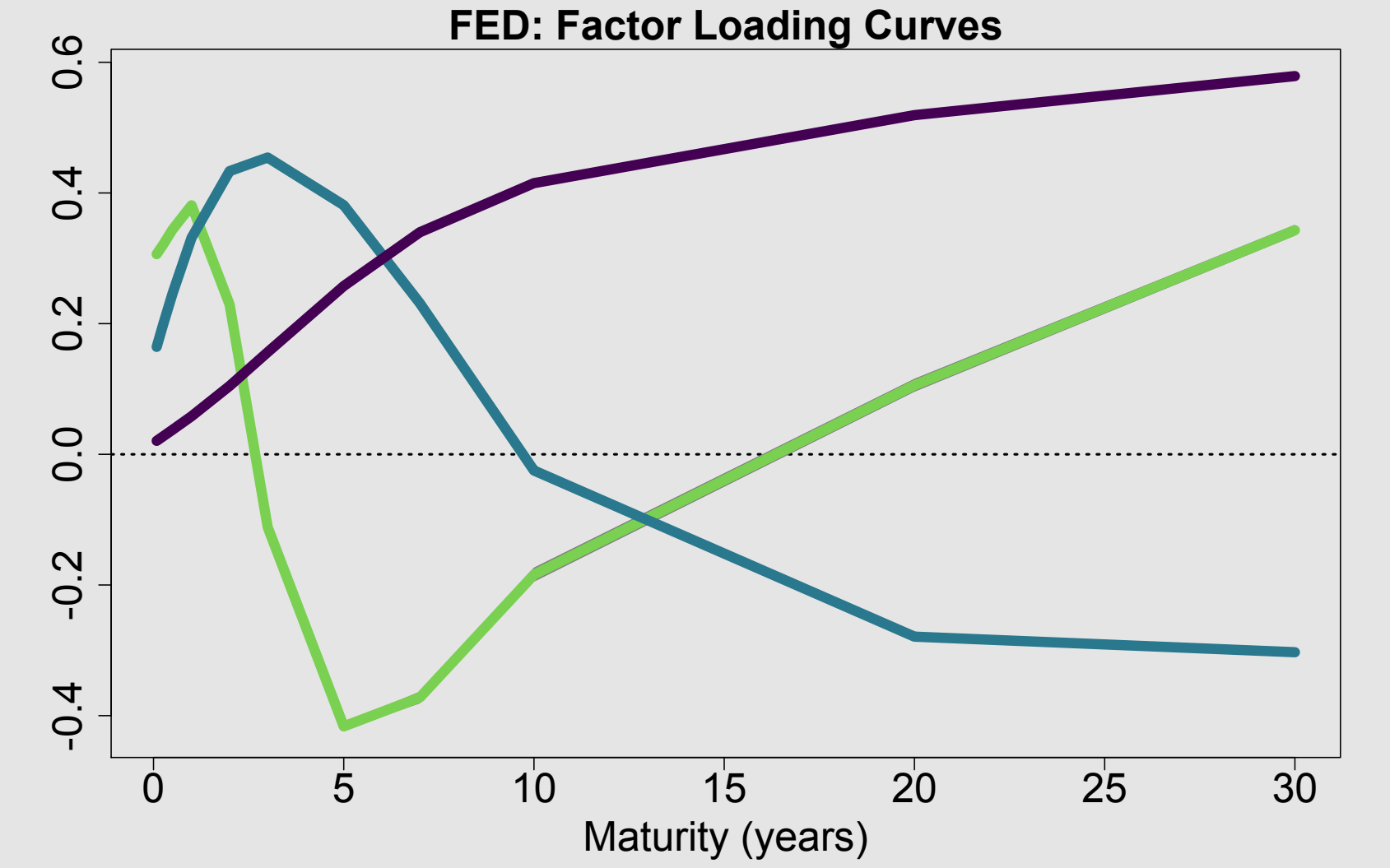
3. Sample the factor loading curves, $\{f_k\}$

```
D = sampleFLC(...)        # basis coefficients
```

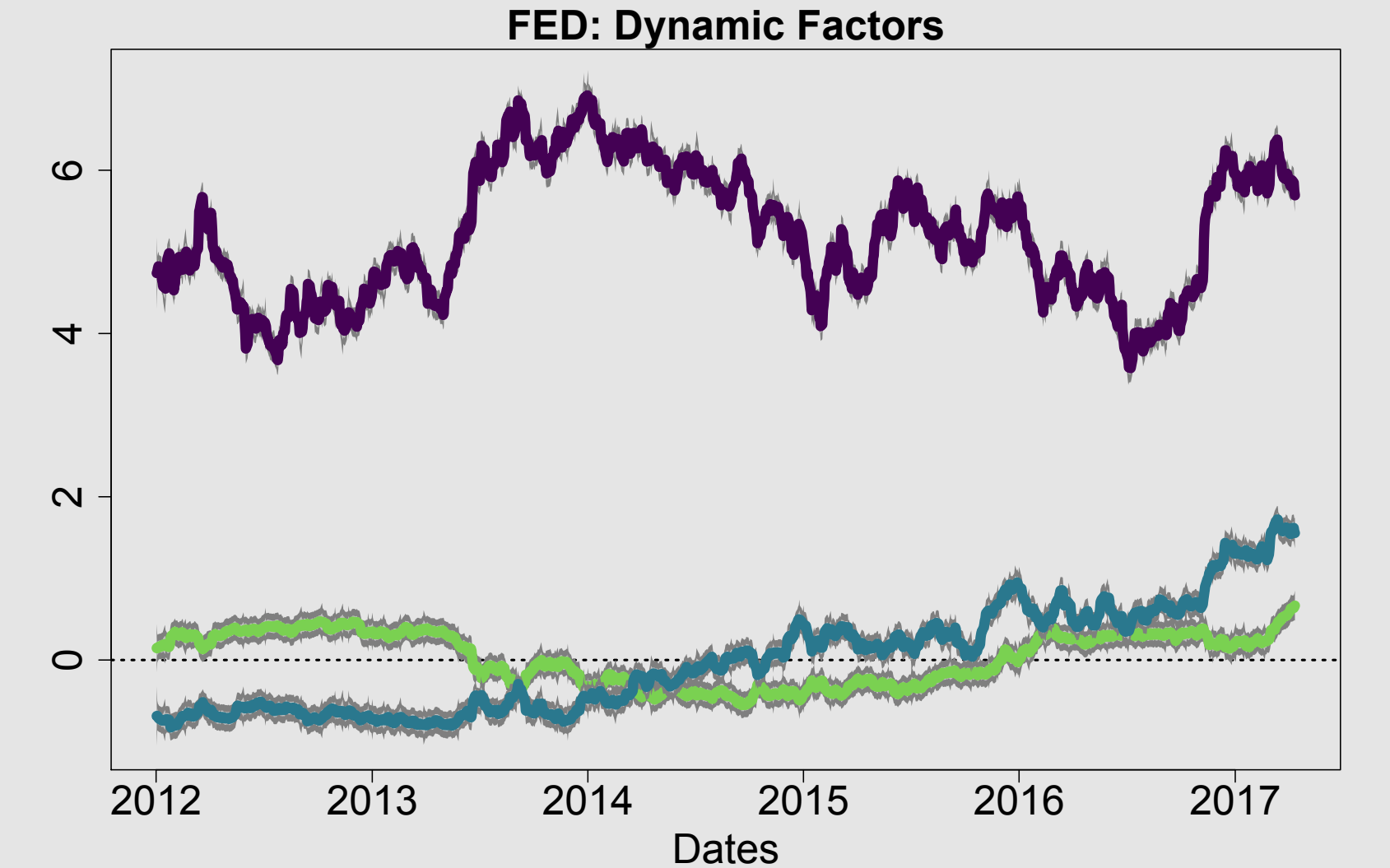
FDLM CODE EXAMPLE: fdlm()

```
# Read in the FED yield curve data (Y, tau, dates):
data("US_Yields")
#data("UK_Yields")

# Restrict to dates since 2012:
Y = Y[which(dates > as.Date("2012-01-01")),];

# Run the MCMC:
mcmc_output = fdlm(Y, tau,
                   K = 3,
                   nsims = 2500, burnin = 2500,
                   mcmc_params = list("beta", "fk",
                                      "Yhat", "sigma_e"))
```

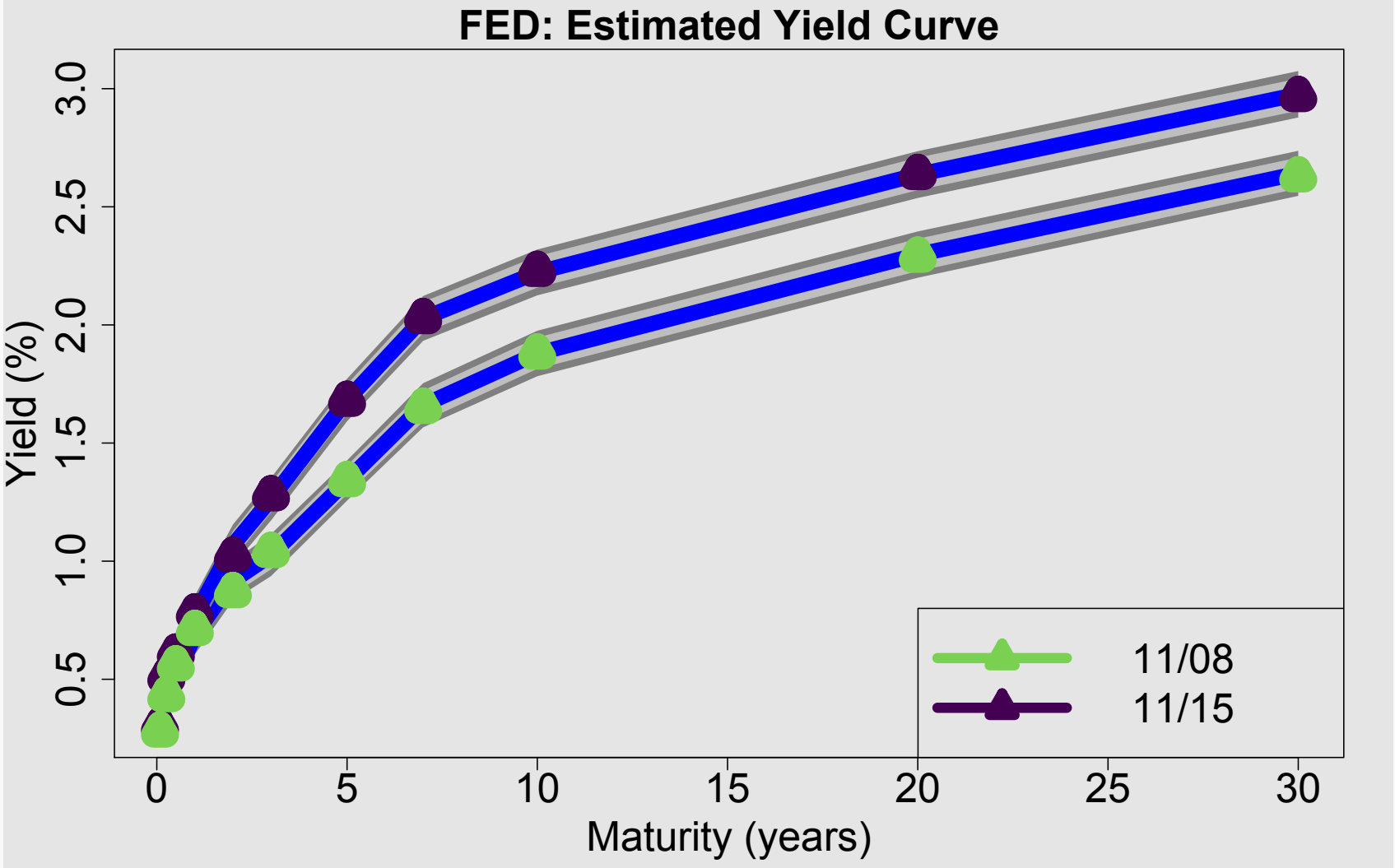
FACTOR LOADING CURVES: FED



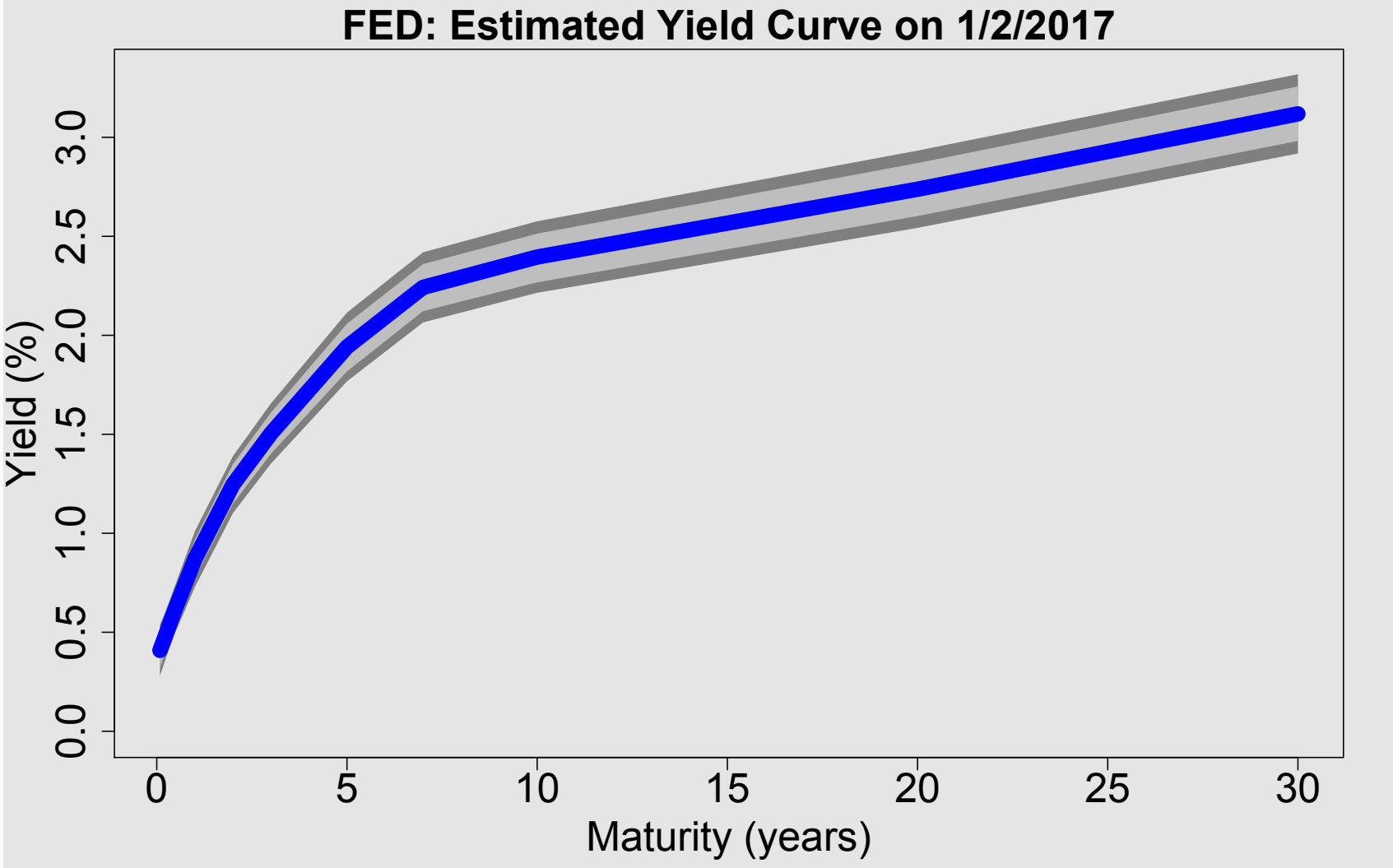
DYNAMIC FACTORS: FED



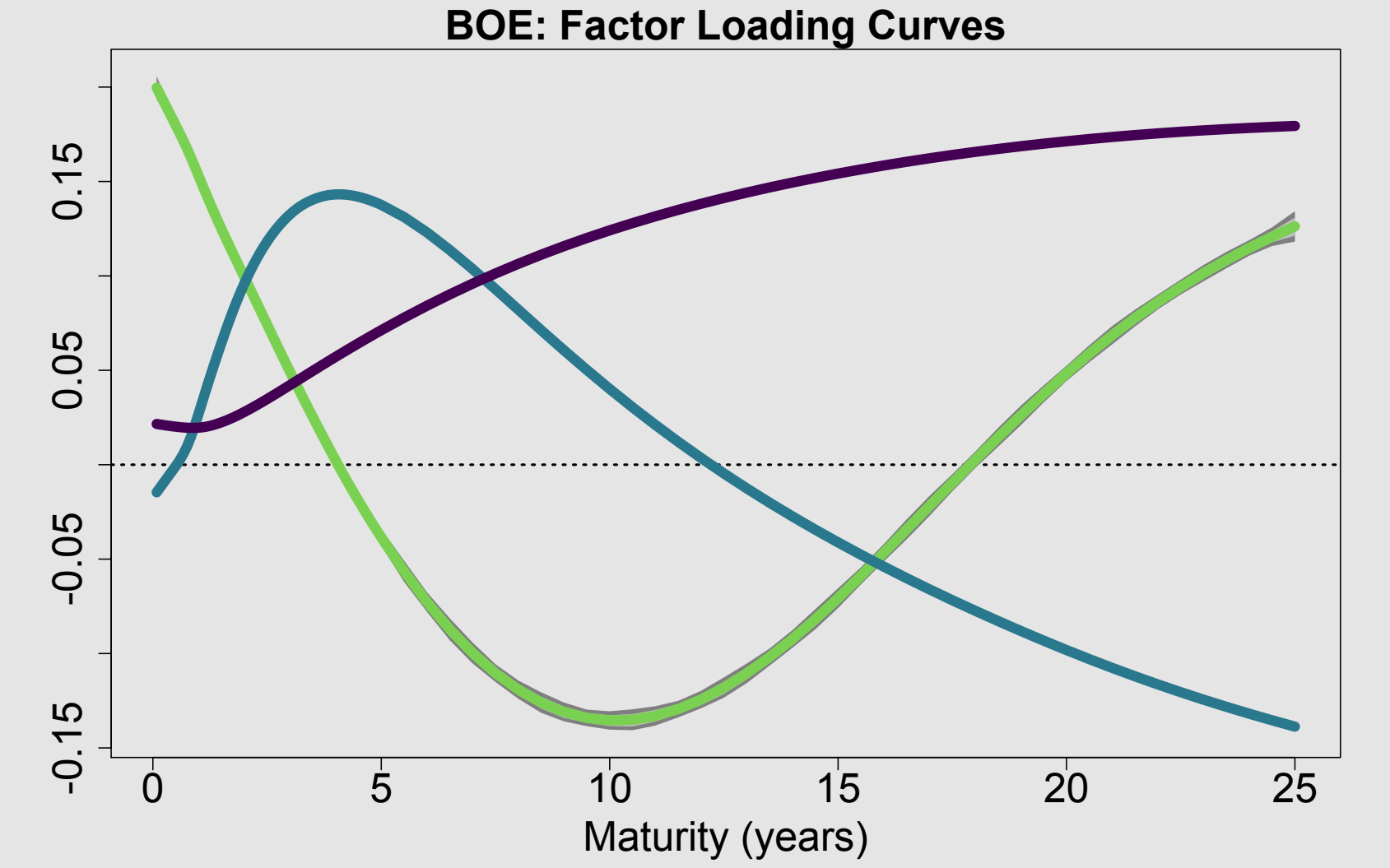
FITTED YIELD CURVE: FED



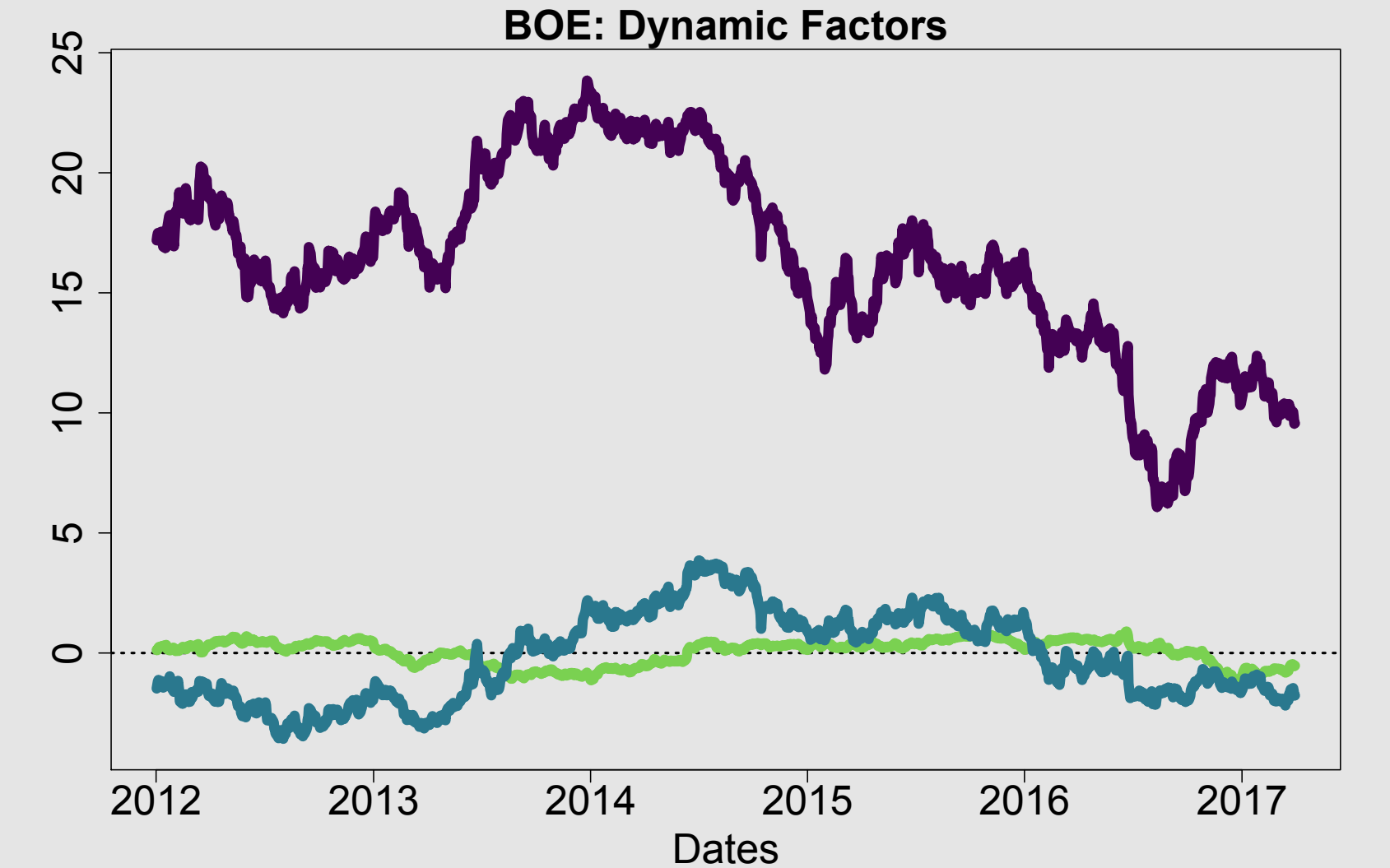
FITTED YIELD CURVE: FED



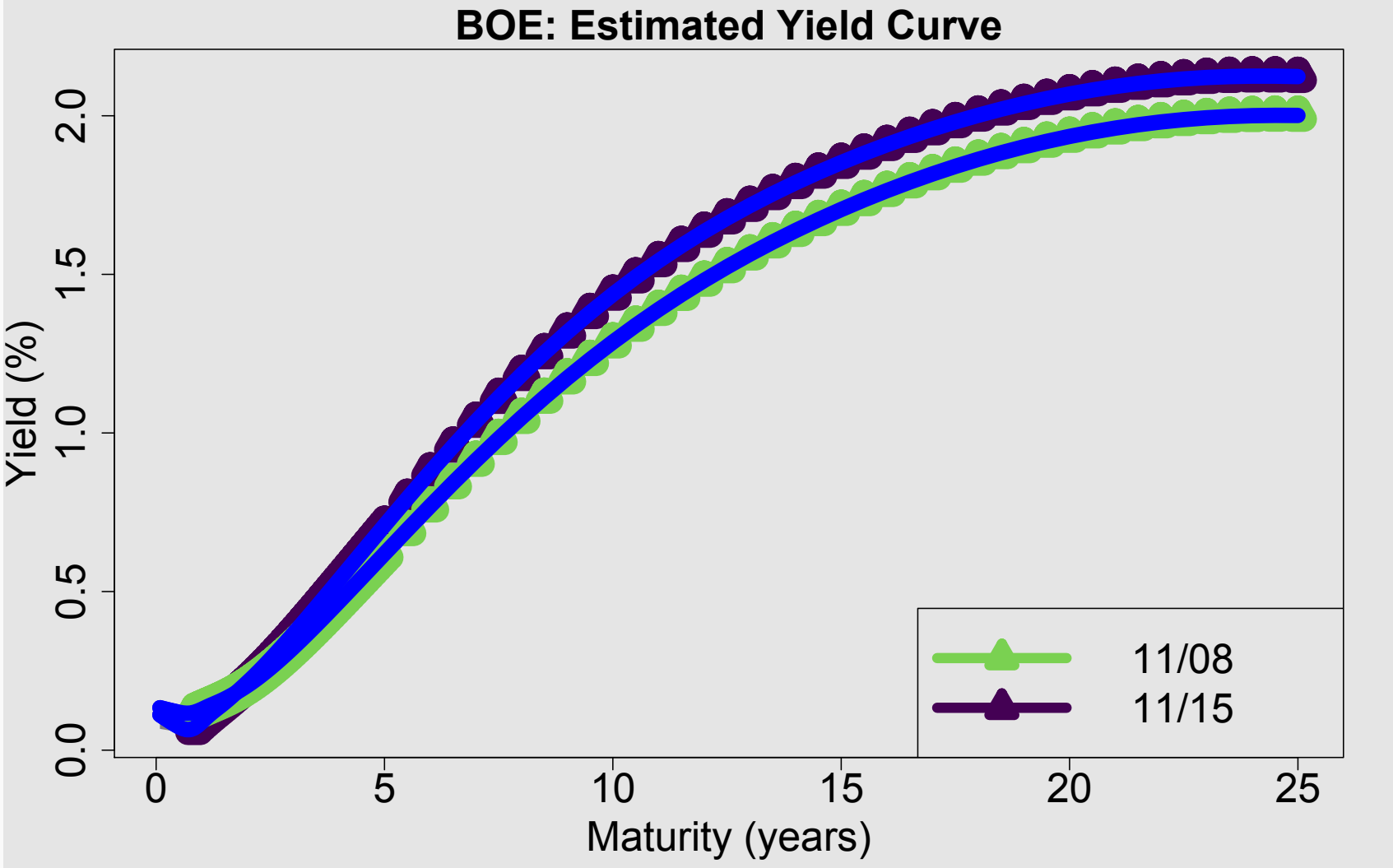
FACTOR LOADING CURVES: BOE



DYNAMIC FACTORS: BOE



FITTED YIELD CURVE: BOE



SUMMARY: CONTRIBUTIONS OF THE MFDLM

1. Hierarchical framework for modeling a multivariate time series of functional data
    - Covariates, stochastic volatility, change points
2. Joint estimation of model parameters; “exact” inference
3. Efficient Gibbs sampling algorithm; package `FDLM`
4. Applications: finance/economics and neuroscience; astronomy

REFERENCES

Helske, J. (2017).
KFAS: Kalman Filter and Smoother for Exponential Family State Space Models.
R package version 1.2.6.

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