Measuring Risk with COGARCH(p, q) Models

Joint work with Lorenzo Mercuri & Edit Rroji

Francesco Bianchi
francesco.bianchi04@icatt.it
Università Cattolica del Sacro Cuore, Milan
MDOTM, Milan

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Define \((r_n)_{n \in \mathbb{N}}\) as a GARCH \((p, q)\) process

\[
\begin{align*}
    r_n &= \sqrt{V_n} \epsilon_n \\
    V_n &= \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{n-i}^2 + \sum_{j=1}^{q} \beta_j V_{n-i}
\end{align*}
\]  

(1)

where \((V_n)_{n \in \mathbb{N}_0}\) is an ARMA\((q, p-1)\) process.

Define \((G_t)_{t \geq 0}\) as a COGARCH\((p, q)\) process

\[
dG_t = \sqrt{V_t} \, dL_t \quad \text{with} \quad G_0 = 0
\]

(2)

where \((V_t)_{t \geq 0}\) is a CARMA\((q, p-1)\) process driven by
the discrete part of the quadratic variation of the Lévy process \((L_t)_{t \geq 0}\).
Why choosing a *continuous* GARCH model?

As in GARCH models:

- ARCH Effect
- Heavy tails

Moreover:

- High frequency and *irregularly spaced* data management
- No missing values approximation
Consider a market composed by assets $P_{1,t}, \ldots, P_{\tilde{N},t}$

$$P_{i,t} = P_{i,0} \exp \left[ \mu_i t + X_{i,t} \right] \quad i = 1, \ldots, \tilde{N}.$$ 

$\mu_i \in \mathcal{R}$ and the vector process $X_t := \begin{bmatrix} X_{1,t} \ldots, X_{\tilde{N},t} \end{bmatrix}^\top$ is the ICA-COGARCH $(p, q)$ model defined as:

$$X_t = A S_t$$

where $A$ is a $\tilde{N} \times p$ matrix and $S_t = (S_{1,t}, \ldots, S_{p,t})^\top$ is a $p$-vector process that each entry $S_{i,t}$ is a COGARCH$(p, q)$ model defined in (2).
The optimization problem is defined as

$$\max_{c_1,\ldots,c_N} \mathbb{E}[G_T] - \lambda \rho(-G_t)$$

where $G_T = \sum_{i=1}^{\tilde{N}} c_i (P_{i,T} - P_{i,0})$ with no short selling.

- $\lambda$ is the risk-aversion coefficient
- $\rho$ is an homogeneous risk measure (VaR, ES)
Empirical Analysis

Two different datasets (APR 2010 - DEC 2015)

1. 100 members of the FTSE 100 Index
2. 154 European hedge funds tracking the FTSE 100 Index (UKX)

The analysis follows this structure:

- Leader selection combining cluster analysis and lead-lag estimation
- Portfolio’s weights based on the ICA-COGARCH model (for a single optimization)
- Portfolio out-of-sample performance (6 months forecast)
Out-of-Sample Performance

lambda = 0.5

lambda = 10

lambda = 1
R Package: COGARCH.rm

- **Market Data**
  - Dataset Download: COGgetdata

- **Univariate Risk Measure Analysis**
  - estCOGUniv
  - SimBoot
  - univariateRM

- **Leader Selection**
  - COGleader

- **Portfolio Optimization**
  - portfolioCOG

- **Univariate Out-of-sample Forecast**
  - forecastCOG

- **Multivariate Out-of-sample Forecast**
  - forecastCOG

Dependencies: **Yuima, fastICA, quantmod, cluster, rugarch**
Thank you!

Francesco Bianchi  
UCSC Milan  
MDOTM

Lorenzo Mercuri  
University of Milan  
CREST Agency

Edit Rroji  
University of Milano-Bicocca

References:


• Bianchi, Francesco and Mercuri, Lorenzo and Rroji, Edit (2017). COGARCH.rm: Portfolio selection with Multivariate COGARCH(p,q) models. R package version 0.1.0.
