Measuring Risk with COGARCH(p, q) Models

Joint work with Lorenzo Mercuri & Edit Rroji

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GARCH Model: Discrete and Continuous

Define $(r_n)_{n\in\mathbb{N}}$ as a GARCH (p, q) process

$$\begin{cases} r_n = \sqrt{V_n}\,\epsilon_n \\ V_n = \alpha_0 + \sum_{i=1}^{p} \alpha_i\, r_{n-i}^2 + \sum_{j=1}^{q} \beta_j\, V_{n-i} \end{cases} \tag{1}$$

where $(V_n)_{n\in\mathbb{N}_0}$ is an $ARMA(q, p-1)$ process.

Define $(G_t)_{t\geq 0}$ as a COGARCH(p, q) process

$$\mathrm{d}G_t = \sqrt{V_t}\,\mathrm{d}L_t \qquad \text{with} \qquad G_0 = 0 \tag{2}$$

where $(V_t)_{t\geq 0}$ is a $CARMA(q, p-1)$ process driven by the discrete part of the quadratic variation of the Lévy process $(L_t)_{t\geq 0}$.

COGARCH Model Key Features

Why choosing a *continuous* GARCH model?

As in GARCH models:

- ARCH Effect
- Heavy tails

Moreover:

- High frequency and irregularly spaced data management
- No missing values approximation

ICA-COGARCH

Consider a market composed by assets $P_{1,t}, \ldots P_{\bar{N},t}$

$$P_{i,t} = P_{i,0} \exp\left[\mu_i t + X_{i,t}\right] \quad i = 1, \ldots, \bar{N}.$$

$\mu_i \in \mathcal{R}$ and the vector process $X_t := \left[X_{1,t} \ldots, X_{\bar{N},t}\right]^\top$ is the ICA-COGARCH (p, q) model defined as:

$$X_t = A\, S_t$$

where A is a $\bar{N} \times p$ matrix and $S_t = (S_{1,t}, \ldots, S_{p,t})^\top$ is a p-vector process that each entry $S_{i,t}$ is a COGARCH(p, q) model defined in (2).

Portfolio Selection

The optimization problem is defined as

$$\max_{c_1,\ldots,c_{\bar{N}}} \mathbb{E}[\mathcal{G}_T] - \lambda\, \rho(-\mathcal{G}_t)$$

where $\mathcal{G}_T = \sum_{i=1}^{\bar{N}} c_i(P_{i,T} - P_{i,0})$ with no short selling.

- λ is the risk-aversion coefficient
- ρ is an homogeneous risk measure (VaR, ES)

Empirical Analysis

Two different datasets (APR 2010 - DEC 2015)

1. 100 members of the FTSE 100 Index
2. 154 European hedge funds tracking the FTSE 100 Index (UKX)

`COGgetdata`

The analysis follows this structure:

- Leader selection combining cluster analysis and lead-lag estimation

`COGleader`

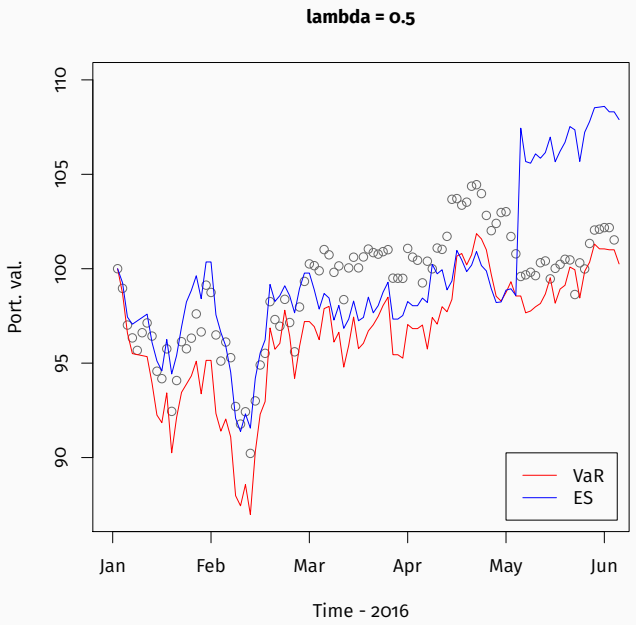
- Portfolio's weights based on the ICA-COGARCH model (for a single optimization)

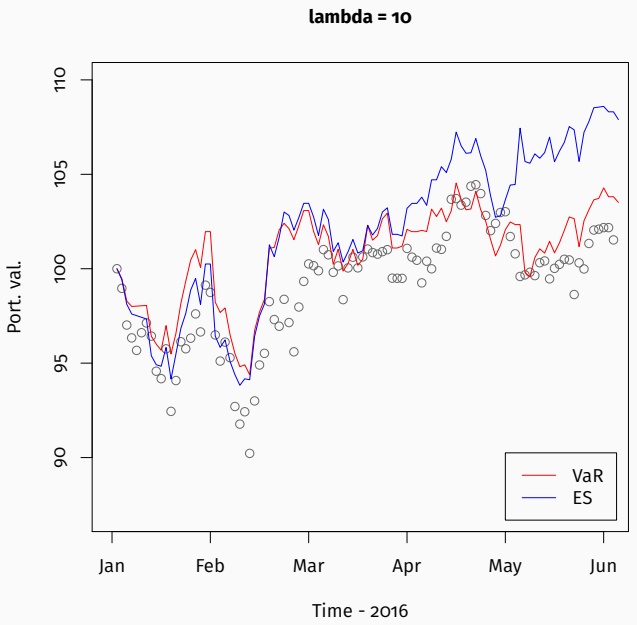
`portfolioCOG`

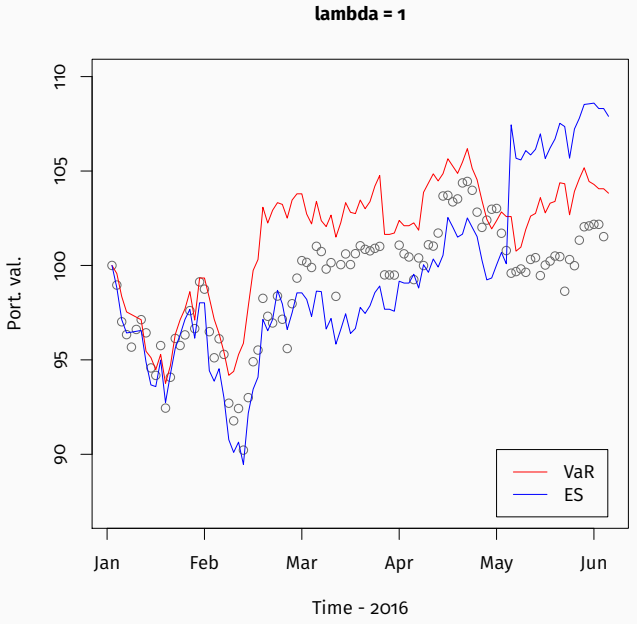
- Portfolio out-of-sample performance (6 months forecast)

`forecastCOG`

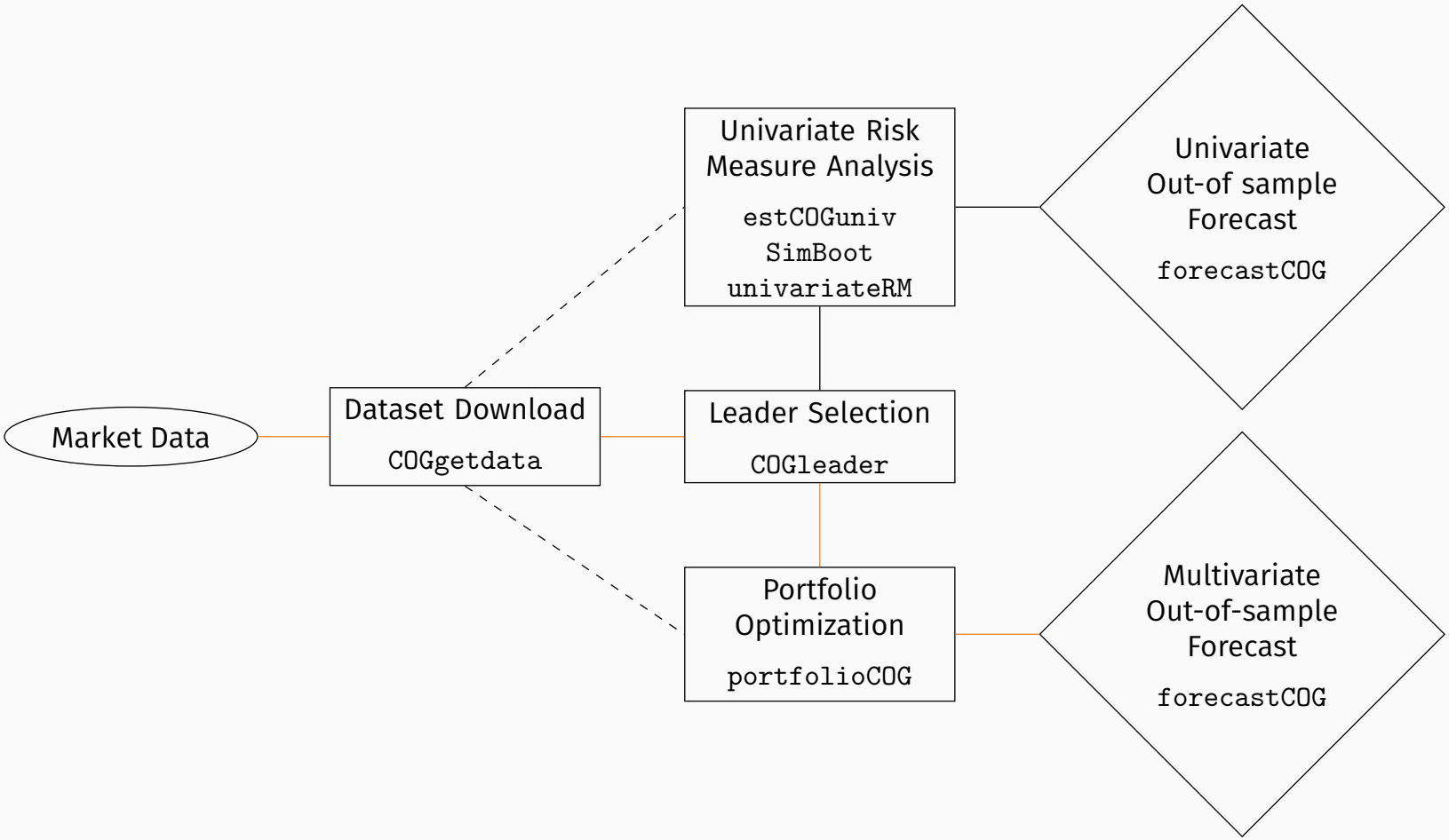
Out-of-Sample Performance







R Package: COGARCH.rm



Dependencies: Yuima, fastICA, quantmod, cluster, rugarch

Thank you!

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References:

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