

partialCI: An R Package for the Analysis of Partially Cointegrated Time Series

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Objectives and key issues to be covered today

What is partial cointegration?

How does it work?

How to use partialCI?

Where is it helpful?

What is *partial cointegration (PCI)*? – A weakening of cointegration allowing for transient and permanent components in the residual series ¹

Classic cointegration²

Partial cointegration

Residual series

Consists of a **stationary** mean-reverting component

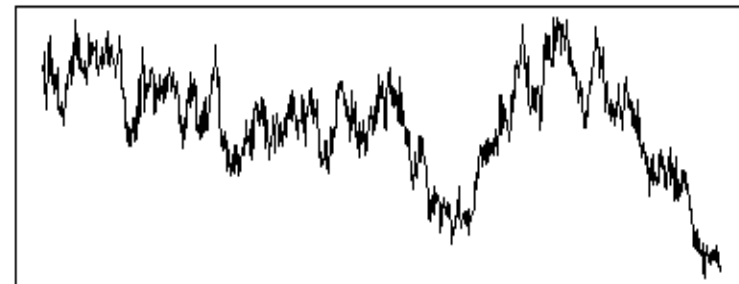
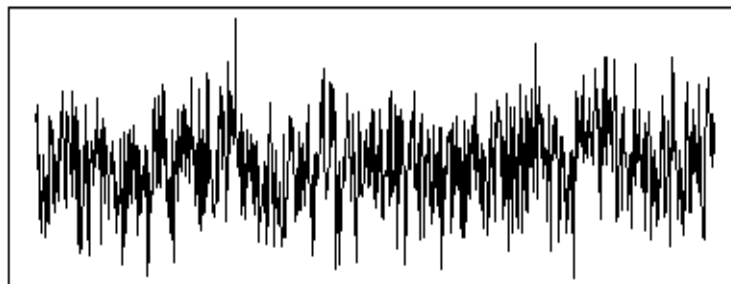
Consists of a sum of a **permanent** and a **stationary** mean-reverting component (partially autoregressive (PAR) ³ process)

Shocks

Are required to be **transient**

Are allowed to be **transient** and **permanent**

Visualization



How does it work? – A primer on the methodology

The partial cointegration framework⁴

- Y_t and $X_{j,t}$ are partially cointegrated, if a parameter vector $\iota = \{\beta, \rho, \sigma_M, \sigma_R\}$ exists so that:

$$Y_t = \beta_1 X_{1,t} + \beta_2 X_{2,t} + \dots + \beta_k X_{k,t} + W_t$$

$$W_t = M_t + R_t$$

$$M_t = \rho M_{t-1} + \varepsilon_{M,t}; \varepsilon_{M,t} \sim \mathcal{N}(0, \sigma_M^2)$$

$$R_t = R_{t-1} + \varepsilon_{R,t}; \varepsilon_{R,t} \sim \mathcal{N}(0, \sigma_R^2)$$

$$\beta_j \in \mathbb{R}; \rho \in (-1, 1); \sigma_M^2, \sigma_R^2 \in \mathbb{R}_0^+$$

- Proportion of variance attributable to mean reversion (PVMR):

$$\begin{aligned} R_{MR}^2 &= \frac{VAR[(1-B)M_t]}{VAR[(1-B)W_t]} \\ &= \frac{2\sigma_M^2}{2\sigma_M^2 + (1+\rho)\sigma_R^2} \end{aligned}$$

Notation / Assumptions

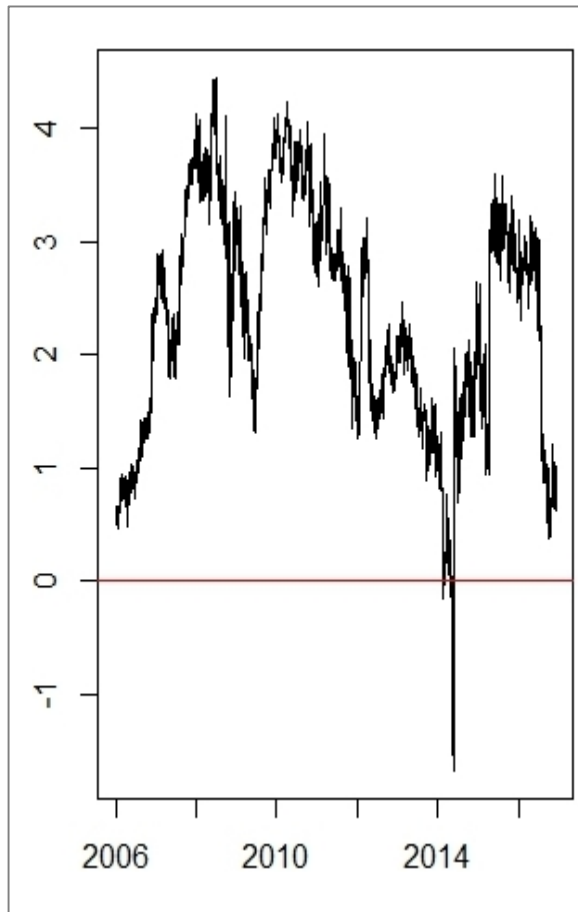
- Notation
 - Target time series: Y_t
 - Factor time series j : $X_{j,t}, j = 1, \dots, k$
 - Coefficient of factor time series j : β_j
 - PAR process: W_t
 - Mean-reverting component: M_t
 - Permanent component: R_t
 - Error terms M_t, R_t : $\varepsilon_{M,t}, \varepsilon_{R,t}$
 - Vector of factor coefficients: β
 - Coefficient of mean-reversion: ρ
 - PVMR: R_{MR}^2
- Assumptions: $\varepsilon_{M,t}$ and $\varepsilon_{R,t}$ are mutually independent normally distributed white noise processes with mean zero and variances σ_M^2 and σ_R^2

How to use *partialCI*? – An overview of the key functions

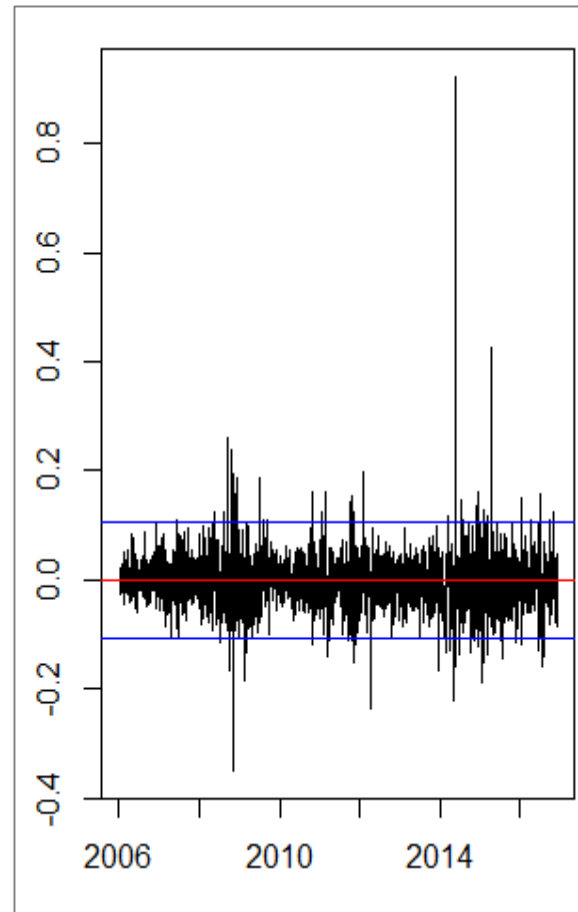
Function	Description	Code
<code>fit.pci</code>	Fits a partial cointegration model to a given collection of time series	
<code>test.pci</code>	Tests the goodness of fit of a PCI model	
<code>statehistory.pci</code>	Estimates and extracts the sequence of hidden states	
<code>hedge.pci</code>	Finds k factors from a predefined set of factors which yield the best fit to the target time series	

Where is it helpful (1/2)? – An example in pairs trading: RDS-A and RDS-B (2006-01-01 – 2016-01-12, daily prices)⁵

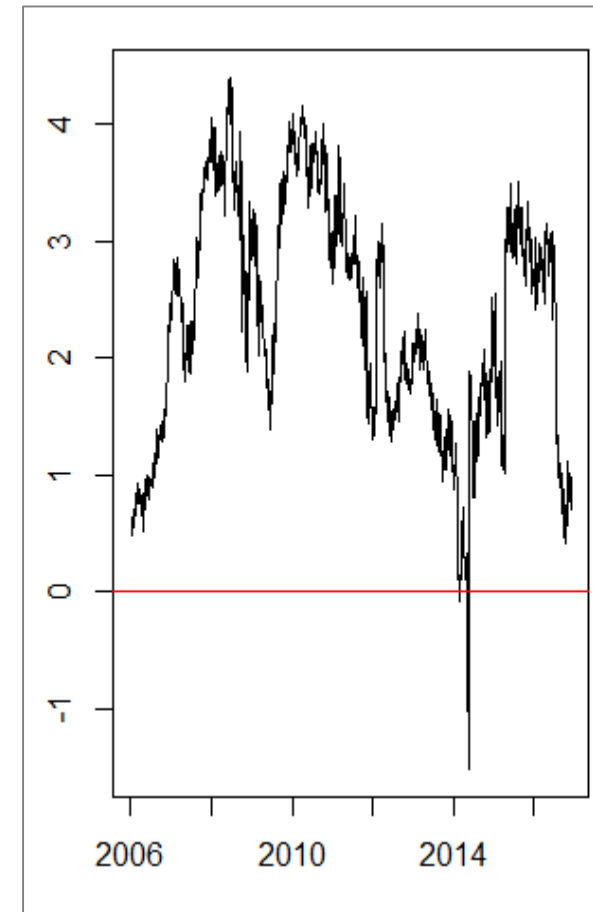
Spread: Classic cointegration



Spread: Transient component



Spread: Random walk component



⁵ Data and R-code: <https://github.com/jonasrende/Rfinance2017>

Where is it helpful (2/2)? – The test indicates that RDS-A and RDS-B are indeed partially cointegrated

Fit a PCI model

```

PCI_RDSA_RDSB<-fit.pci(RDSA, RDSB,
pci_opt_method = c("jp"),
par_model =c("par"), lambda = 0,
robust = FALSE, nu = 5,
include_alpha = FALSE))
  
```

```

Fitted values for PCI model
Y[t] = X[t] %*% beta + M[t] + R[t]
M[t] = rho * M[t-1] + eps_M [t],
      eps_M[t] ~ N(0, sigma_M^2)
R[t] = R[t-1] + eps_R [t],
      eps_R[t] ~ N(0, sigma_R^2)
  
```

	Estimate	Std. Err
beta_Close	0.9264	0.0038
rho	0.3959	0.0965
sigma_M	0.1063	0.0082
sigma_R	0.1174	0.0074

-LL = -1165.16, R²[MR] = 0.540

Test for PCI

```




test.pci(RDSA, RDSB, alpha = 0.05,
null_hyp = c("rw", "ar1"), robust =
FALSE, pci_opt_method = c("jp"))
  
```

Likelihood ratio test of [Random Walk or CI(1)] vs Almost PCI(1) (joint penalty method)

data: RDSA

Hypothesis	Statistic	p-value
Random Walk	-55.09	0.010
AR(1)	-52.88	0.010
Combined		0.010

Interested? – Further references

Paper / Package	Title	URL	QR code
Full paper	partialCI: An R package for the analysis of partially cointegrated time series	http://hdl.handle.net/10419/150014	
Slides	partialCI: An R package for the analysis of partially cointegrated time series	https://www.statistik.rwth-fau.de/files/2017/05/v01-2017.pdf	
R package (CRAN)	partialCI: Partial Cointegration	https://cran.r-project.org/web/packages/partialCI/index.html	
Initial show case for partial cointegration	Pairs trading with partial cointegration	http://hdl.handle.net/10419/140632	