MARKOV-SWITCHING GARCH MODELS IN R: THE MSGARCH PACKAGE

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joint work with:

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https://CRAN.R-project.org/package=MSGARCH

IN BRIEF

– **MSGARCH** implements Haas et al. (2004a) specification:

  1. K separate single-regime conditional variance processes.
  2. Possibly K separate conditional distributions.
  3. A Markov chain dictates the switches between regimes.
  4. Assumes a zero mean process.

– Core of the package is in C++ (thanks to **Rcpp**) to allow for fast and efficient computations.
– Easy estimation and specification creation (similar to **rugarch**).
– Functionality for visualization, simulation, model selection, and risk measure forecasting.

VOLATILITY MODELS

Conditional volatility models

GARCH model (`model = "sGARCH"`) — Bollerslev (1986)

$$h_t \equiv \alpha_0 + \alpha_1 y_{t-1}^2 + \beta h_{t-1}$$

EGARCH model (`model = "eGARCH"`) — Nelson (1991)

$$\ln(h_t) \equiv \alpha_0 + \alpha_1 \left(|y_{t-1}| - \mathsf{E}[|y_{t-1}|]\right) + \alpha_2 y_{t-1} + \beta \ln(h_{t-1})$$

GJR model (`model = "gjrGARCH"`) — Glosten et al. (1993)

$$h_t \equiv \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-1}^2 \mathbb{I}_{\{y_{t-1}<0\}} + \beta h_{t-1}$$

TGARCH model (`model = "tGARCH"`) — Zakoian (1994)

$$h_t^{1/2} \equiv \alpha_0 + \alpha_1 y_{t-1} \mathbb{I}_{\{y_{t-1}\geq 0\}} + \alpha_2 y_{t-1} \mathbb{I}_{\{y_{t-1}<0\}} + \beta h_{t-1}^{1/2}$$

GAS model (`model = "GAS"`) — Creal et al. (2013)

$$h_t \equiv \alpha_0 + \alpha_1 s_{t-1} + \beta h_{t-1},$$

$$s_{t-1} \equiv S_{t-1} \nabla_{t-1}, \qquad \nabla_{t-1} \equiv \frac{\partial \ln f(y_{t-1}|h_{t-1},\lambda)}{\partial h_{t-1}}, \qquad S_{t-1} \equiv \mathsf{E}[\nabla_{t-1} \nabla_{t-1}']^{-1}$$

CONDITIONAL DISTRIBUTIONS

Conditional distributions

Normal distribution (`distribution = "norm"`)

$$f_N(z) \equiv \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad z \in \mathbb{R}$$

Student–t distribution (`distribution = "std"`)

$$f_S(z;\nu) \equiv \sqrt{\frac{\nu}{\nu-2}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{z^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \qquad z \in \mathbb{R}$$

GED distribution (`distribution = "ged"`)

$$f_{\text{GED}}(z;\nu) \equiv \frac{\nu e^{-\frac{1}{2}|z/\lambda|^\nu}}{\lambda 2^{(1+1/\nu)}\Gamma(1/\nu)}, \qquad \lambda \equiv \left(\frac{\Gamma(1/\nu)}{4^{1/\nu}\Gamma(3/\nu)}\right)^{1/2}, \qquad z \in \mathbb{R}$$

- Skewed versions also available using the Fernández and Steel (1998) transformation.

CREATING A SPECIFICATION

– First step is to create a specification

```
> create.spec(model, distribution, do.skew, do.mix, do.shape.ind)
```

– Inputs:
  – `model`: “sGARCH”, “eGARCH”, “gjrGARCH”, “tGARCH”, “GAS”
  – `distribution`: “norm”, “std”,“ged”
  – `do.skew`: Skewed distribution Boolean.
  – `do.mix`: Mixture of GARCH specification of Haas et al. (2004b).
  – `do.shape.ind`: Make it so that only the conditional volatility models switches (distribution and shape parameter stays the same across regime).

EXAMPLES

– Simple GARCH(1,1) normal:

```
> spec = create.spec(model = "sGARCH", distribution = "norm")
```

– Two-state MSGARCH model with GARCH(1,1) normal in both regimes:

```
> spec = create.spec(model = c("sGARCH","sGARCH"),
+                    distribution = c("norm","norm"))
```

– Complex MSGARCH model:

```
> spec =create.spec(model = c("sGARCH", "tGARCH", "eGARCH"),
+                   distribution = c("norm", "std", "ged"),
+                   do.skew = c(TRUE, FALSE, TRUE))
```

WHAT IS INSIDE ?

```
> spec = create.spec(model = c("gjrGARCH", "gjrGARCH"),
+                    distribution = c("std", "std"),
+                    do.skew = c(TRUE, TRUE))
> summary(spec)
[1] "Specification Type: Markov-Switching "
[1] "Specification Name: gjrGARCH_student_skew gjrGARCH_student_skew"
[1] "Number of parameters in each variance model: 4 4"
[1] "Number of parameters in each distribution: 2 2"
[1] "Default parameters:"
     alpha0_1 alpha1_1 alpha2_1 beta_1 nu_1 xi_1 alpha0_2 alpha1_2 alpha2_2
[1,]      0.1     0.05      0.1    0.8   10    1      0.1     0.05      0.1
     beta_2 nu_2 xi_2   P   P
[1,]    0.8   10    1 0.5 0.5
```

– A specification is an S3 R class that gives you access to all the **MSGARCH** functionalities.

– Embedded C++ templated class inside. Why ?

  – C++ : Fast calculations.

  – Templated: Easy future extensions.

    – This means adding conditional volatility models and conditional distributions with minimal work (and debugging).

ILLUSTRATION – DATA

– SMI log-returns from 1990-11-12 to 2000-10-20.

```
> require(DEoptim)
> data(SMI)
> plot(SMI)
```

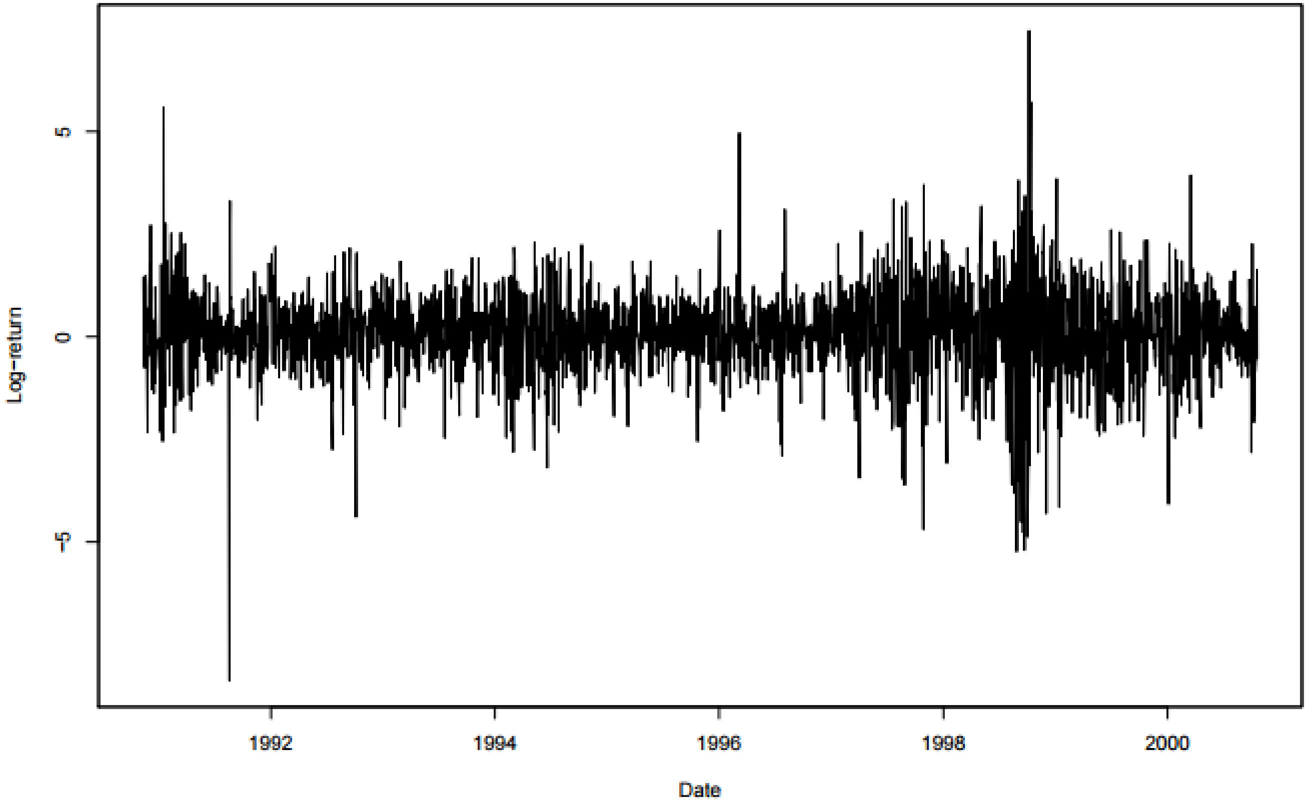


ILLUSTRATION – MLE ESTIMATION

– Make use of **DEoptim** (global) & nmkb from **dfoptim** (local)

```
> out.mle = fit.mle(spec = spec, y = SMI, ctr = list(do.init = TRUE))
> summary(out.mle)
[1] "DEoptim initialization: TRUE"
[1] "Fitted Parameters:"
       alpha0_1    alpha1_1 alpha2_1    beta_1     nu_1      xi_1   alpha0_2
[1,] 0.2226311 0.001360713 0.212886 0.5401085 5.943159 0.8521142 0.08298225
         alpha1_2  alpha2_2    beta_2    nu_2      xi_2         P           P
[1,] 0.006268897 0.1393344 0.8773668 20.0118 0.8581828 0.9980548 0.003125602
[1] "Transition matrix:"
                t = 1       t = 2
t + 1 = 1 0.998054778 0.003125602
t + 1 = 2 0.001945222 0.996874398
[1] "Stable probabilities:"
        Stable probabilities
State 1            0.5463842
State 2            0.4536158
[1] "Unconditional volatility:"
       State 1  State 2
[1,] 0.8101812 1.422818
Log-kernel:  -3364.587
AIC:  6687.677
BIC:  6769.213
```

ILLUSTRATION – BAYESIAN ESTIMATION

– Make use of **adaptMCMC**

```
> ctr.mcmc = list(N.burn = 5000, N.mcmc = 10000, N.thin = 10, theta0 = out.mle$theta)
> out.mcmc =  fit.bayes(spec = spec, y = SMI, ctr = ctr.mcmc)
  generate 15000 samples
  |=========================================================================| 100%
> summary(out.mcmc)
[1] "Bayesian posterior mean:"
    alpha0_1     alpha1_1     alpha2_1       beta_1         nu_1         xi_1
 0.213301726  0.018197992  0.221444331  0.535624781  5.962895732  0.862990160
    alpha0_2     alpha1_2     alpha2_2       beta_2         nu_2         xi_2
 0.073552525  0.015786970  0.133891540  0.878384572 19.989053308  0.852081140
           P            P
 0.996794482  0.004340049
[1] "Posterior mean transition matrix:"
                t = 1       t = 2
t + 1 = 1 0.996794482 0.004340049
t + 1 = 2 0.003205518 0.995659951
[1] "Posterior mean stable probabilities:"
        Stable probabilities
State 1            0.5399292
State 2            0.4600708
[1] "Posterior mean unconditional volatility:"
       State 1  State 2
[1,] 0.8124884 1.489994
Acceptance rate:  0.988
AIC:  6690.017
BIC:  6771.553
DIC:  6674.852
```

AND SO WHAT?

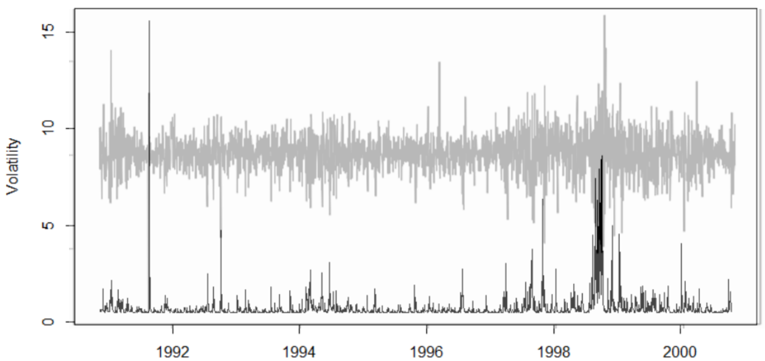
- Available functionalities:
  - Filtered volatilities.
  - Filtered probabilities.
  - 1-step ahead simulation.
  - Predictive density.
  - Risk measures (VaR and ES).
  - Information criteria.
  - And more !

- All functionalities are compatible for both MLE and Bayesian estimation.

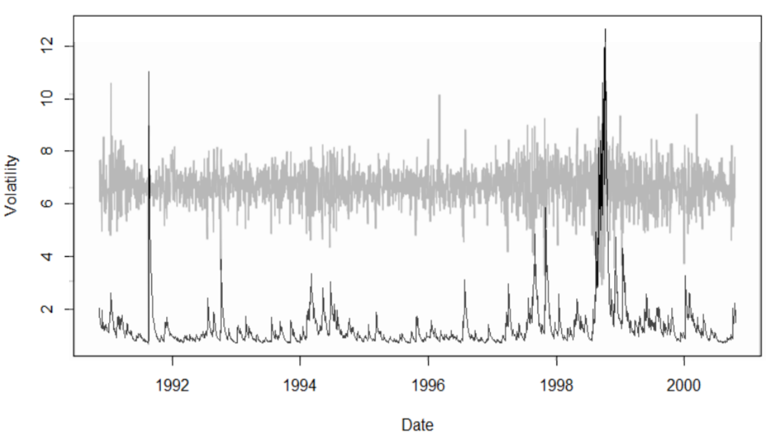
ILLUSTRATION – VOLATILITIES & STATE

```
> vol = ht(out.mle)
> plot(vol, date = index(SMI))
```

Conditional volatility of state 1

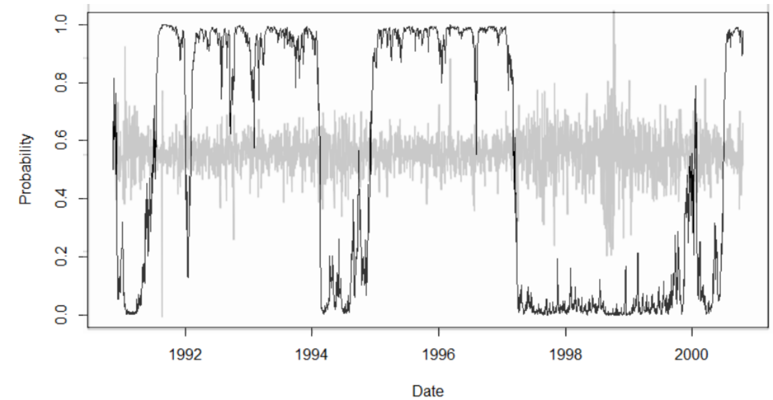


Conditional volatility of state 2

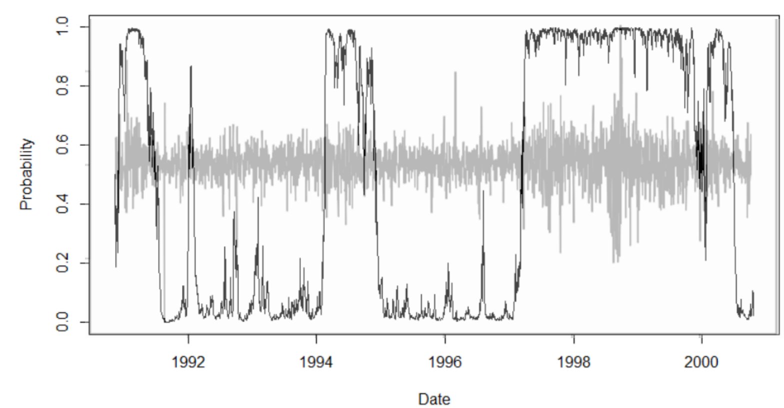



```
> state = Pstate(out.mle)
> plot(state, date = index(SMI))
```

Filtered probability to be in State 1



Filtered probability to be in State 2



PREDICTIVE DENSITY

```
> pred(object, x, theta, y, log, do.its)
```

1. `Object` can take a specification:
   1. In case of a specification, `theta` and `y` must be provided.
   2. Useful when using the same fitted model on new data `y`.
2. `Object` can take a fitted model:
   1. No need to input `theta` and `y`.
   2. Useful shortcut.
3. The variable `x` are what we want to evaluate.
4. If `do.its = TRUE`, `x` is not needed as we evaluated the function with the in-sample observation (in-sample).
5. If `do.its = FALSE`, `x` is evaluated as a 1-step ahead draws.

Log-likelihood function:

```
> log_like = pred(object = out.mle,
+                 log = TRUE,do.its = TRUE)
> sum(log_like$pred, na.rm = TRUE) #first observation = NA
[1] -3329.838
```

Use `kernel()` to include the priors:

```
> kernel(out.mle)
[1] -3364.587
```

ILLUSTRATION – PREDICTIVE DENSITY

```
> grid = seq(-4,4,length.out = 1000)
> pred_dist = pred(out.mle, x = grid,
+                  log = FALSE, do.its = FALSE)
> plot(pred_dist)
```

```
> grid = seq(-4,4,length.out = 1000)
> pred_dist = pred(out.mcmc, x = grid,
+                  log = FALSE, do.its = FALSE)
> plot(pred_dist)
```

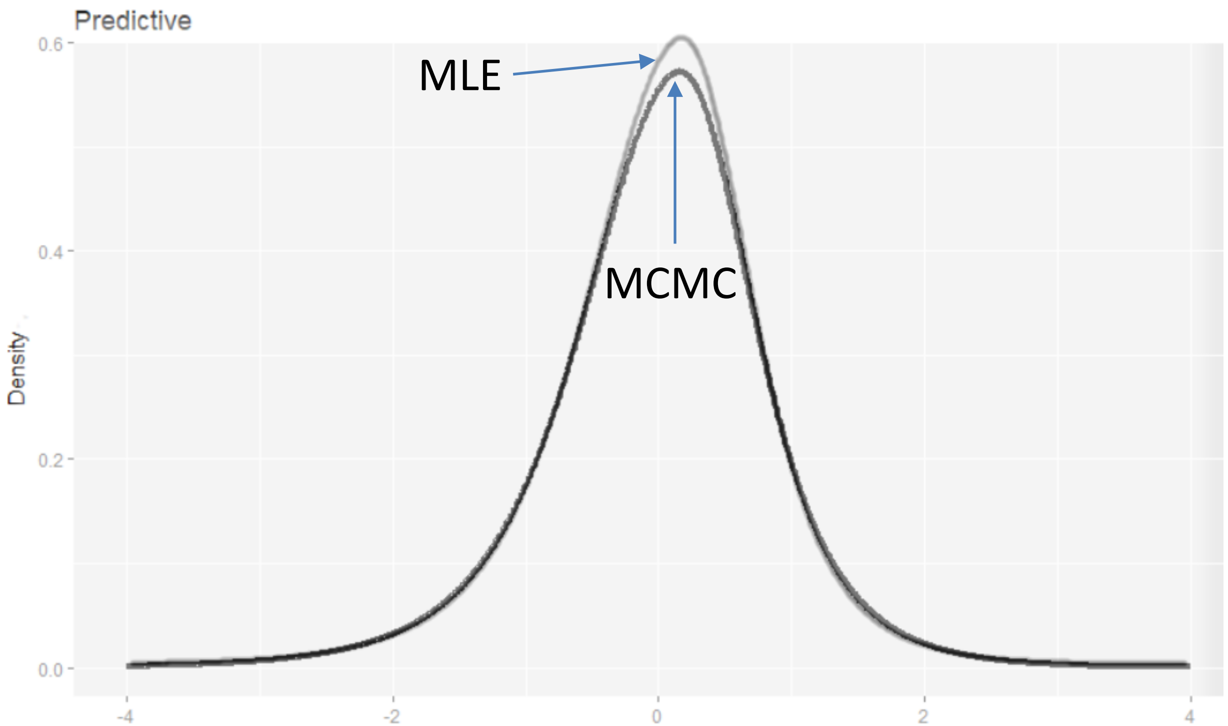
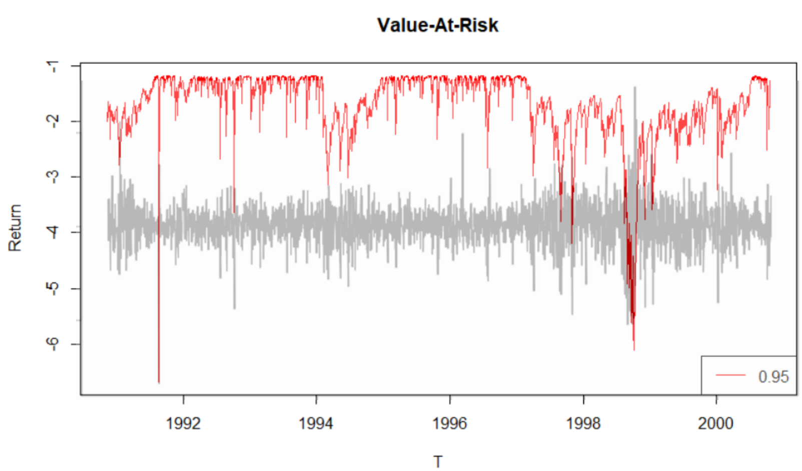


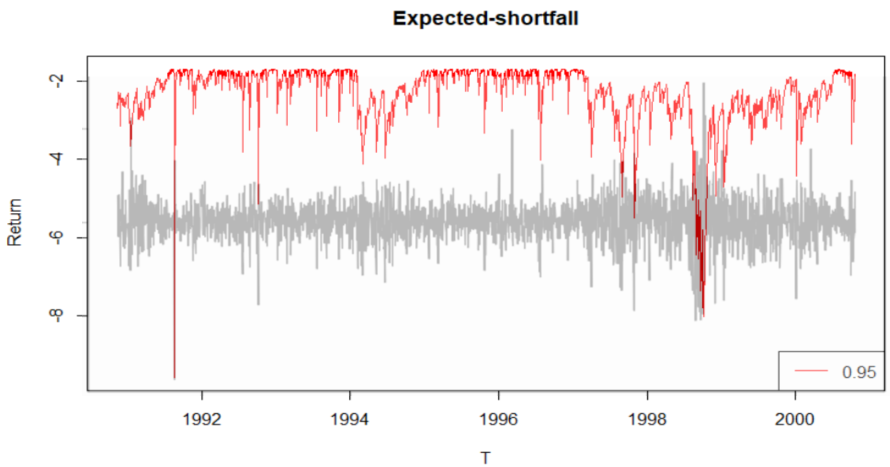
ILLUSTRATION – RISK MEASURES

```
> risk(object, theta, y, level, ES, do.its, ctr)
```

- The `risk` function works similarly to the `pred` function.
- It also leverages the `pred` function to calculate risk measures.
- `do.its = TRUE` will calculate the in-sample risk measures for all dates.
- `do.its = FALSE` will calculate the one-step ahead risk measures.

```
> VAR95 = risk(out.mle.2,level = c(0.95), ES = TRUE, do.its = TRUE)
> plot(VAR95,date = index(SMI))
```





WHAT NEXT?

- Google Summer of Code 2017 (Leopoldo Catania).
- Wish list:
  - Improved starting value strategy for faster optimization.
  - Multi-step ahead forecasts (by simulation).
  - Parameters constraints.
  - Standard errors of the estimates (MLE).
  - Custom MLE and MCMC optimizers (including custom priors).
  - Multivariate model with regime-switching copulas.
  - And more!

Some are currently implemented in **MSGARCH** 0.18.4 (available on GitHub)!

Thanks for your attention and hope you’ll enjoy our package!!

https://CRAN.R-project.org/package=MSGARCH

https://github.com/keblu/MSGARCH