Generalized Autoregressive Score Models in R: The GAS Package

David Ardia  Kris Boudt  Leopoldo Catania†

†University of Rome, “Tor Vergata”

R/Finance 2017
Chicago, IL
May 20, 2017
Assume you are employed in the risk management unit of a financial institution. You are required to:

1) Develop internal models.
2) Predict future Value–at–Risk (VaR) levels.
3) Respect capital requirements.

Mostly used internal models for VaR prediction are:

1) Historical simulation: does not work.
2) Exponential Weighed Moving Averages: does not work.
3) GARCH: generally works.
4) Stochastic Volatility (SV): generally works.

What about the Generalized Autoregressive Score (GAS) models of Creal et al. (2013) and Harvey (2013)?
GAS or GARCH?

Taken from Ardia et al. (2017)

Figure: One–step ahead VaR forecasts for General Electric (GE) at the $\alpha = 1\%$ confidence level for the GAS–$N$ (solid) and GAS–$ST$ (dotted) models.
Figure: Absolute Apple returns and estimated volatility for GARCH and GAS with Student’s t distribution.
GAS or GARCH?

Taken from www.gasmodel.com

Figure: GAS estimated volatility for Nordpool electricity prices. Gaussian GAS is equivalent to Gaussian GARCH.
The plain GARCH(1,1) model:

\[ y_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim iid (0, 1) \]

\[ \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \]

can be written as:

\[ \sigma_t^2 = \omega + \phi \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 u_{t-1} \]

where \( \phi = \alpha + \beta \) and \( u_t = y_t^2 / \sigma_t^2 - 1 \) is a Martingale Difference (MD). A GAS model with Student’s t distribution with \( \nu \) degrees of freedom would imply:

\[ u_t = \frac{(\nu + 1)y_t^2}{(\nu - 2)\sigma_t^2 + y_t^2} - 1 \]
Does it really matter? Yes.

Figure: Impact of $u_t$ for $\nu = 3$ (thick), $\nu = 6$ (medium dashed), $\nu = 10$ (thin) and $\nu = \infty$ (dashed).
Generalized Autoregressive Score (GAS) models have been introduced by Creal et al. (2013) and Harvey (2013). They assume:

\[ y_t | \mathcal{F}_{t-1} \sim \mathcal{D}(\theta_t, \psi) \]
\[ \theta_t = \Lambda(\tilde{\theta}_t) \]
\[ \tilde{\theta}_t = \kappa + A\tilde{u}_{t-1} + B\tilde{\theta}_{t-1} \]

where

\[ \tilde{u}_t \propto \frac{\partial \log p(y_t; \theta, \psi)}{\partial \theta} \bigg|_{\theta = \theta_t} \]

Estimation of GAS models is easily done via Maximum Likelihood.
The GAS package of Catania et al. (2016) permits to: i) simulate, ii) estimate, and iii) make predictions using GAS models.

- **GAS** can deal with univariate and multivariate models.
- Mostly written in C++.
- Works in parallel.
- It is part of a 2016 Google Summer of Code project.
- It is available from CRAN and GitHub: https://github.com/LeopoldoCatania/GAS

The two papers: Ardia et al. (2016) and Ardia et al. (2017) describe the main functionalities.

**GAS models are not only for financial applications!!!** Visit www.gasmodel.com.


