Generalized Autoregressive Score Models in R: The GAS Package

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A Quantitative Risk Management Problem

Assume you are employed in the risk management unit of a financial institution. You are required to:

1) Develop internal models.
2) Predict future Value–at–Risk (VaR) levels.
3) Respect capital requirements.

Mostly used internal models for VaR prediction are:

1) Historical simulation: **does not work**.
2) Exponential Weighed Moving Averages: **does not work**.
3) GARCH: **generally works**.
4) Stochastic Volatility (SV): **generally works**.

What about the Generalized Autoregressive Score (GAS) models of Creal et al. (2013) and Harvey (2013)?

GAS or GARCH ?

Taken from Ardia et al. (2017)

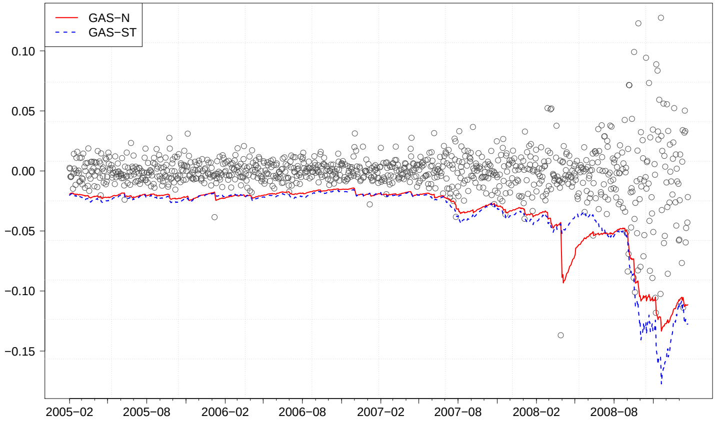

Figure: One–step ahead VaR forecasts for General Electric (GE) at the $\alpha = 1\%$ confidence level for the GAS–$\mathcal{N}$ (*solid*) and GAS–$\mathcal{ST}$ (*dotted*) models.

GAS or GARCH ?

Taken from Harvey (2013)

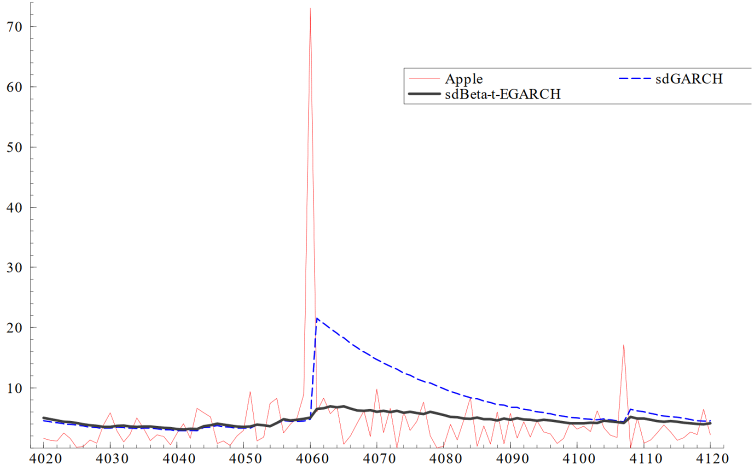

Figure: Absolute Apple returns and estimated volatility for GARCH and GAS with Student's t distribution.

Taken from www.gasmodel.com

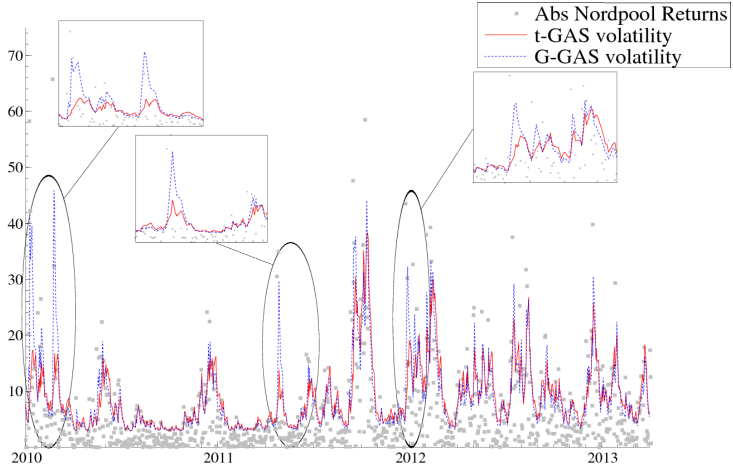

Figure: GAS estimated volatility for Nordpool electricity prices. Gaussian GAS is equivalent to Gaussian GARCH.

The plain GARCH(1,1) model:

$$y_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} (0,1)$$
$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$$

can be written as:

$$\sigma_t^2 = \omega + \phi \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 u_{t-1}$$

where $\phi = \alpha + \beta$ and $u_t = y_t^2/\sigma_t^2 - 1$ is a Martingale Difference (MD). A GAS model with Student's t distribution with ν degrees of freedom would imply:

$$u_t = \frac{(\nu + 1) y_t^2}{(\nu - 2)\sigma_t^2 + y_t^2} - 1$$

Does it really matter? Yes.

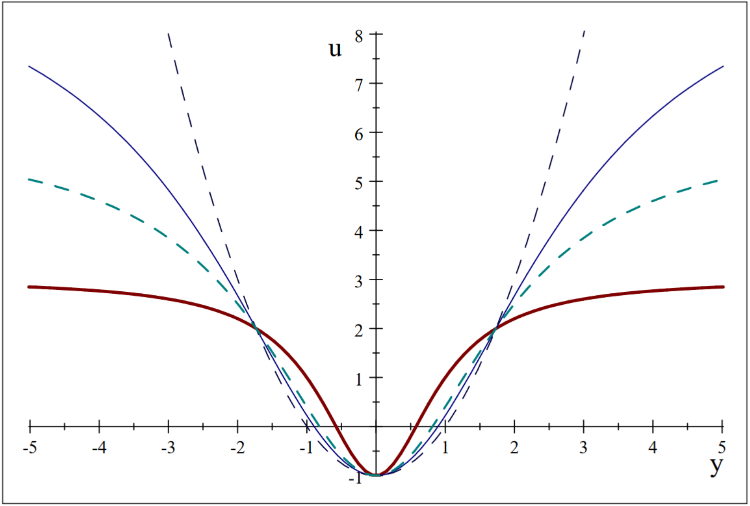

Figure: Impact of u_t for $\nu = 3$ (thick), $\nu = 6$ (medium dashed), $\nu = 10$ (thin) and $\nu = \infty$ (dashed).

GAS models

Generalized Autoregressive Score (GAS) models have been introduced by Creal et al. (2013) and Harvey (2013). They assume:

$$
\begin{aligned}
\mathbf{y}_t | \mathcal{F}_{t-1} &\sim \mathcal{D}(\boldsymbol{\theta}_t, \boldsymbol{\psi}) \\
\boldsymbol{\theta}_t &= \Lambda(\widetilde{\boldsymbol{\theta}}_t) \\
\widetilde{\boldsymbol{\theta}}_t &= \boldsymbol{\kappa} + \mathbf{A}\widetilde{\mathbf{u}}_{t-1} + \mathbf{B}\widetilde{\boldsymbol{\theta}}_{t-1}
\end{aligned}
$$

where

$$
\widetilde{\mathbf{u}}_t \propto \frac{\partial \log p(\mathbf{y}_t; \boldsymbol{\theta}, \boldsymbol{\psi})}{\partial \boldsymbol{\theta}}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_t}
$$

Estimation of GAS models is easily done via Maximum Likelihood.

The GAS package for R

The **GAS** package of Catania et al. (2016) permits to: i) simulate, ii) estimate, and iii) make predictions using GAS models.

- **GAS** can deal with univariate and multivariate models.
- Mostly written in C++.
- Works in parallel.
- It is part of a 2016 Google Summer of Code project.
- It is available from CRAN and GitHub: https://github.com/LeopoldoCatania/GAS

The two papers: Ardia et al. (2016) and Ardia et al. (2017) describe the main functionalities.

GAS models are not only for financial applications!!! Visit www.gasmodel.com.

References

Ardia, D., Boudt, K., and Catania, L. (2016). Generalized Autoregressive Score Models in R: The **GAS** Package. *Available at SSRN: https://ssrn.com/abstract=2871444*.

Ardia, D., Boudt, K., and Catania, L. (2017). Value-at-Risk Prediction in R with the **GAS** Package. *Available at SSRN: https://ssrn.com/abstract=2871444*.

Catania, L., Boudt, K., and Ardia, D. (2016). ***GAS**: Generalized Autoregressive Score Models*. R package version 0.2.0.

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized Autoregressive Score Models With Applications. *Journal of Applied Econometrics*, 28(5):777–795.

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*, volume 52. Cambridge University Press.