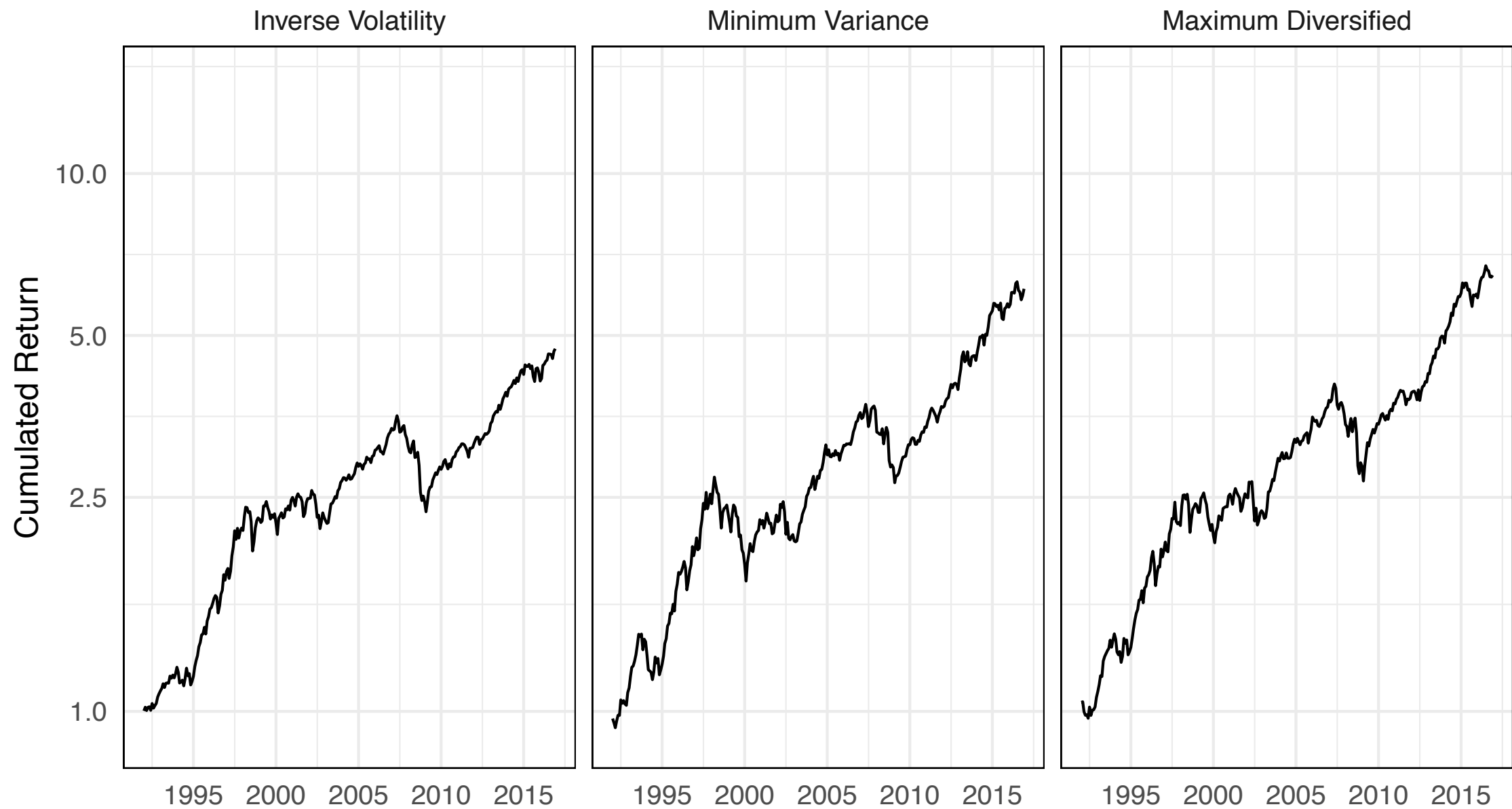


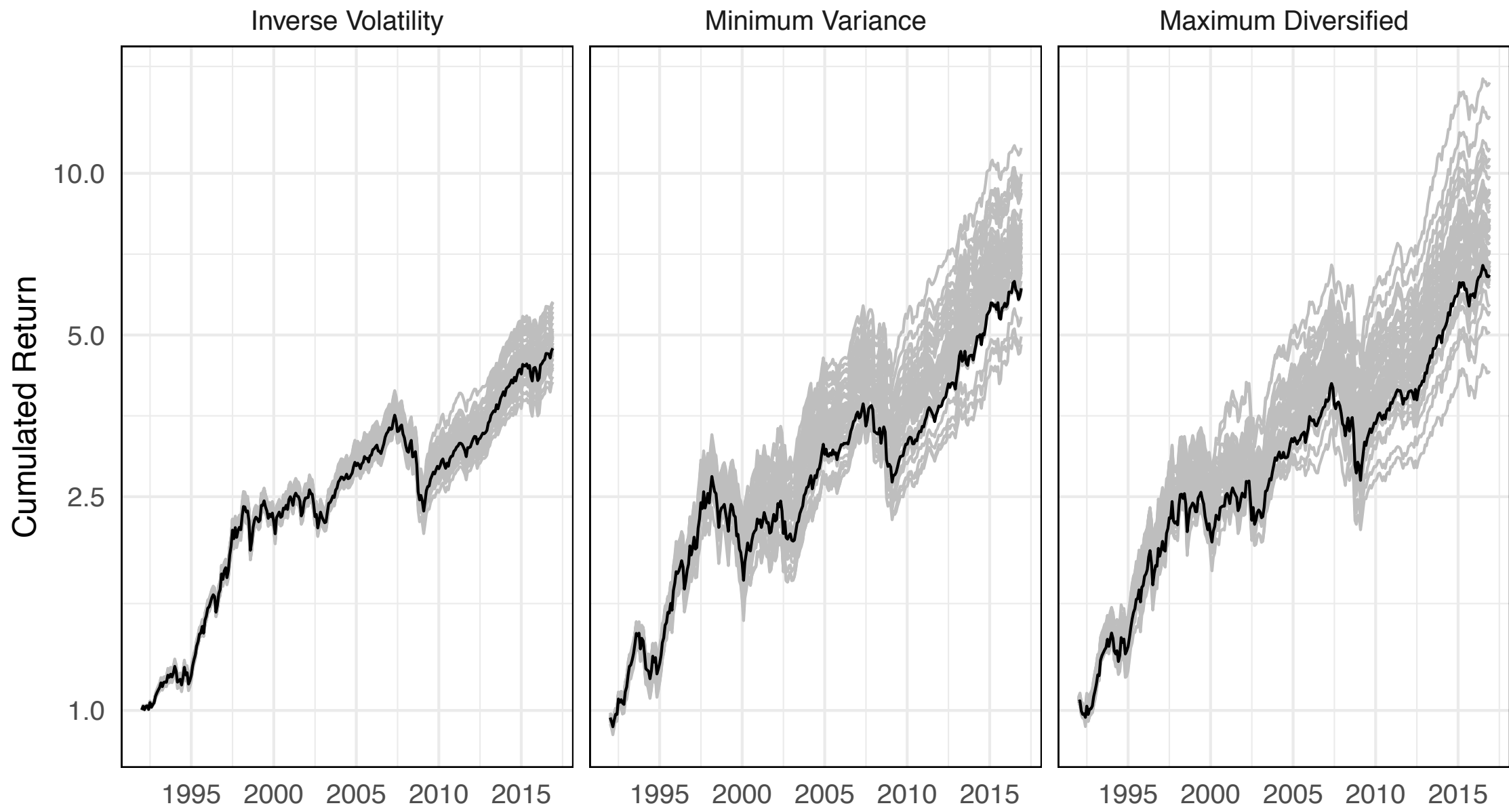
Risk-Based Portfolios under Parameter Uncertainty

R/Finance
May 20, 2017
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Which risk based portfolio strategy offers best out of sample performance ...



Which risk based portfolio strategy offers best out of sample performance ... at smallest sensitivity to parameter estimation.



Sensitivity to Calibration

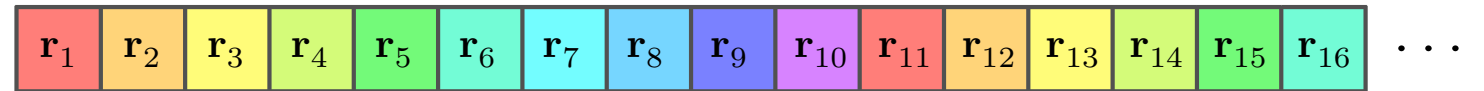
- High dispersion of out-of-sample performance of portfolios calibrated with bootstrapped sample periods indicates elevated sensitivity to parameter estimation.
- Bootstrap does not require distributional assumptions and maintains (nonlinear, higher moment) dependence information.
- Block-bootstrap with monthlong blocks maintains serial dependence.

“Financial variables are intrinsically linked in complicated ways.”

Harvey and Liu (2015)

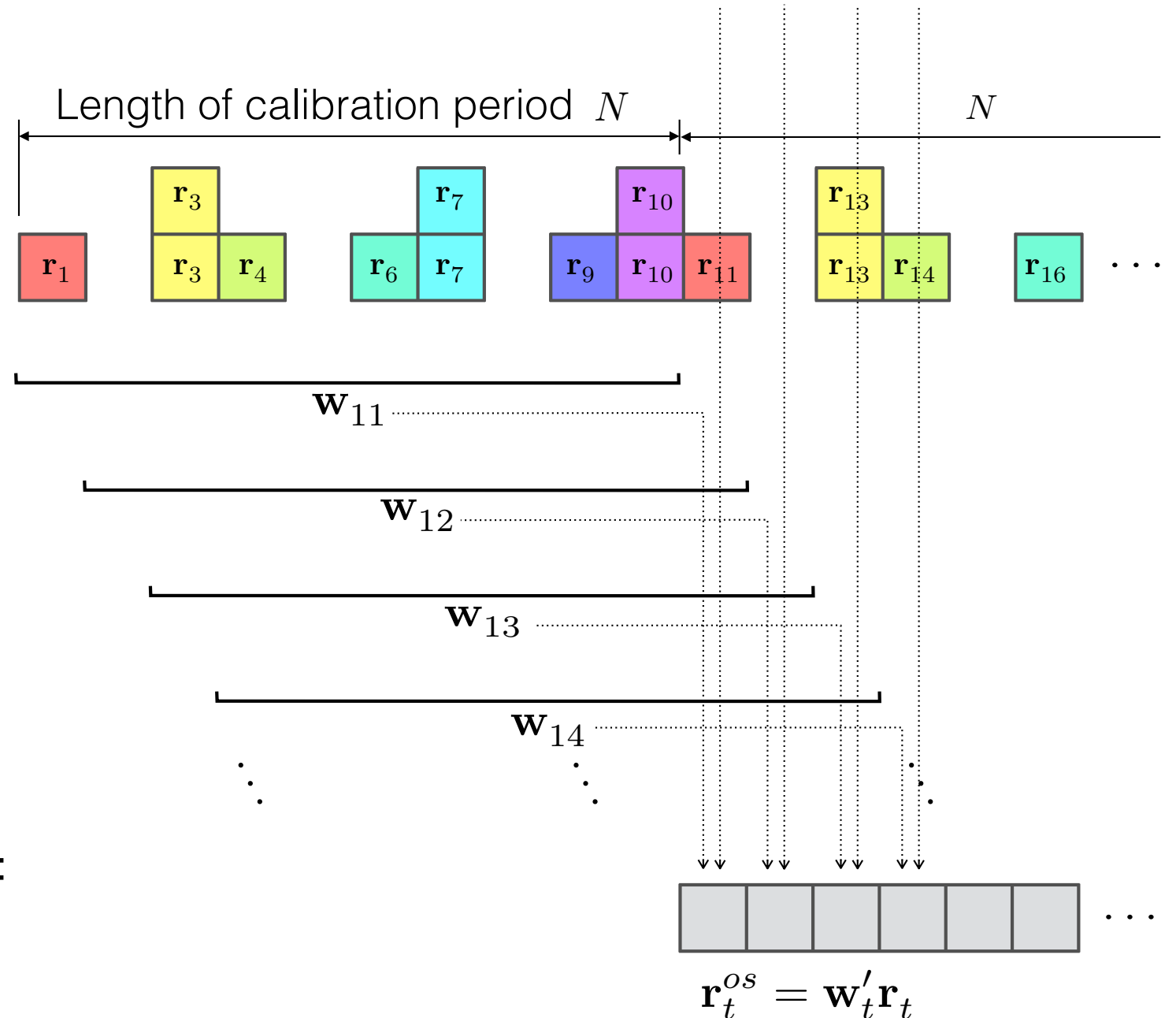
Bootstrap Procedure

Monthlong blocks of returns:



Bootstrap sample:

1. Randomly draw N blocks with replacement from $r_1 \dots r_N$
 2. Recycle random sequence over entire data set
- ⇒ overlapping parts of calibration periods remain unchanged.
 ⇒ Each calibration period contains N observations.



Calibrate portfolios based on overlapping bootstrap samples

Apply out-of-sample to full return series:

Asset Universe

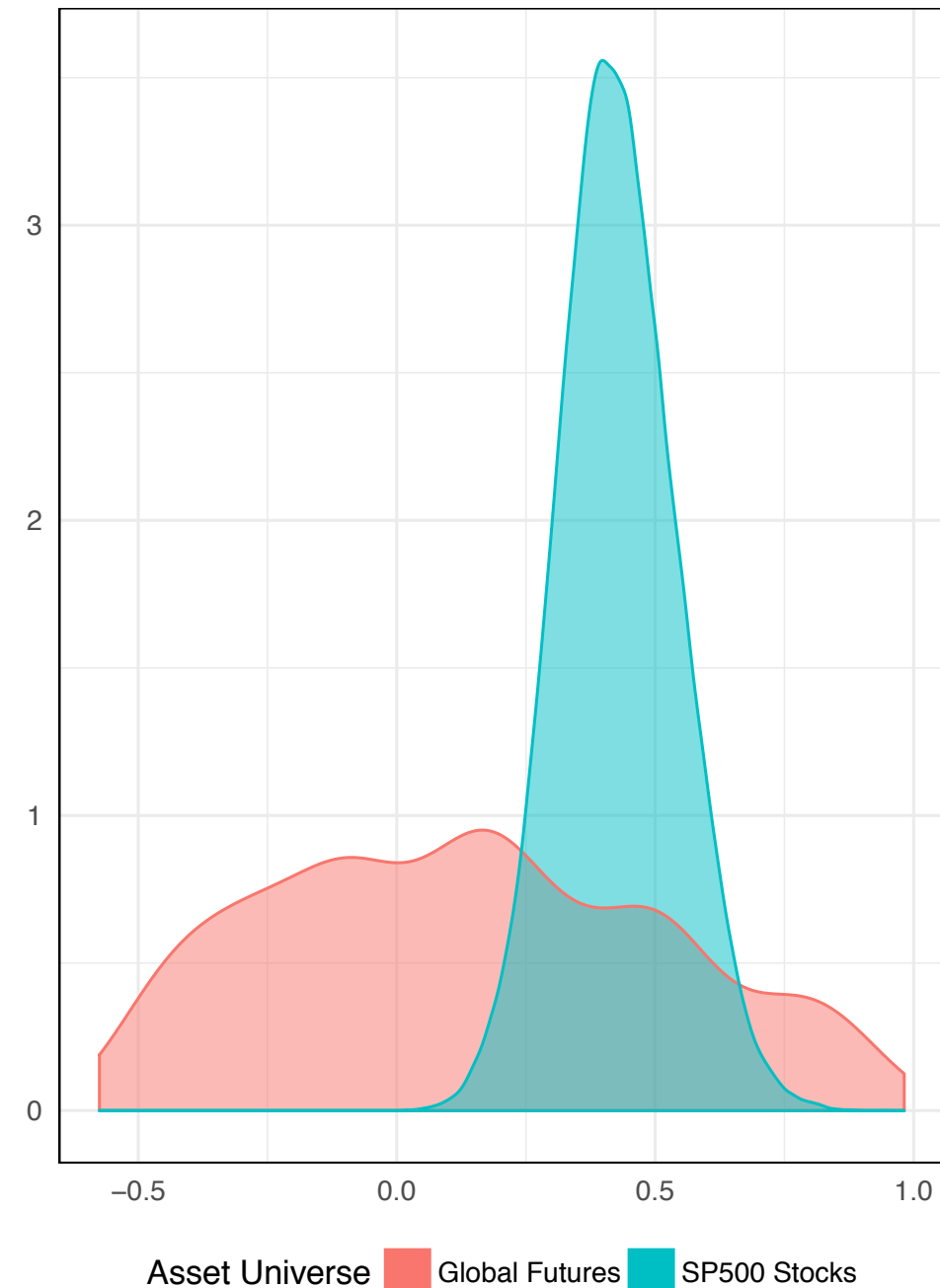
S&P 500 Stocks

- Historical index members
- Homogeneous correlations
- Daily CRSP data
- 27 years of data 1990 - 2016

Global Futures

- 42 liquid rate, bond, equity, volatility, commodity futures
- Heterogeneous correlations
- 17 years of data 1999 - 2016
- Intra-day data from tickdata.com for concurrent return observations
- Define rolling dates and account for rolling costs

Density of Pairwise Correlation Coefficients
Jan 2010 – Dec 2014



Cumulated Returns

S&P500 Stocks

Global Futures

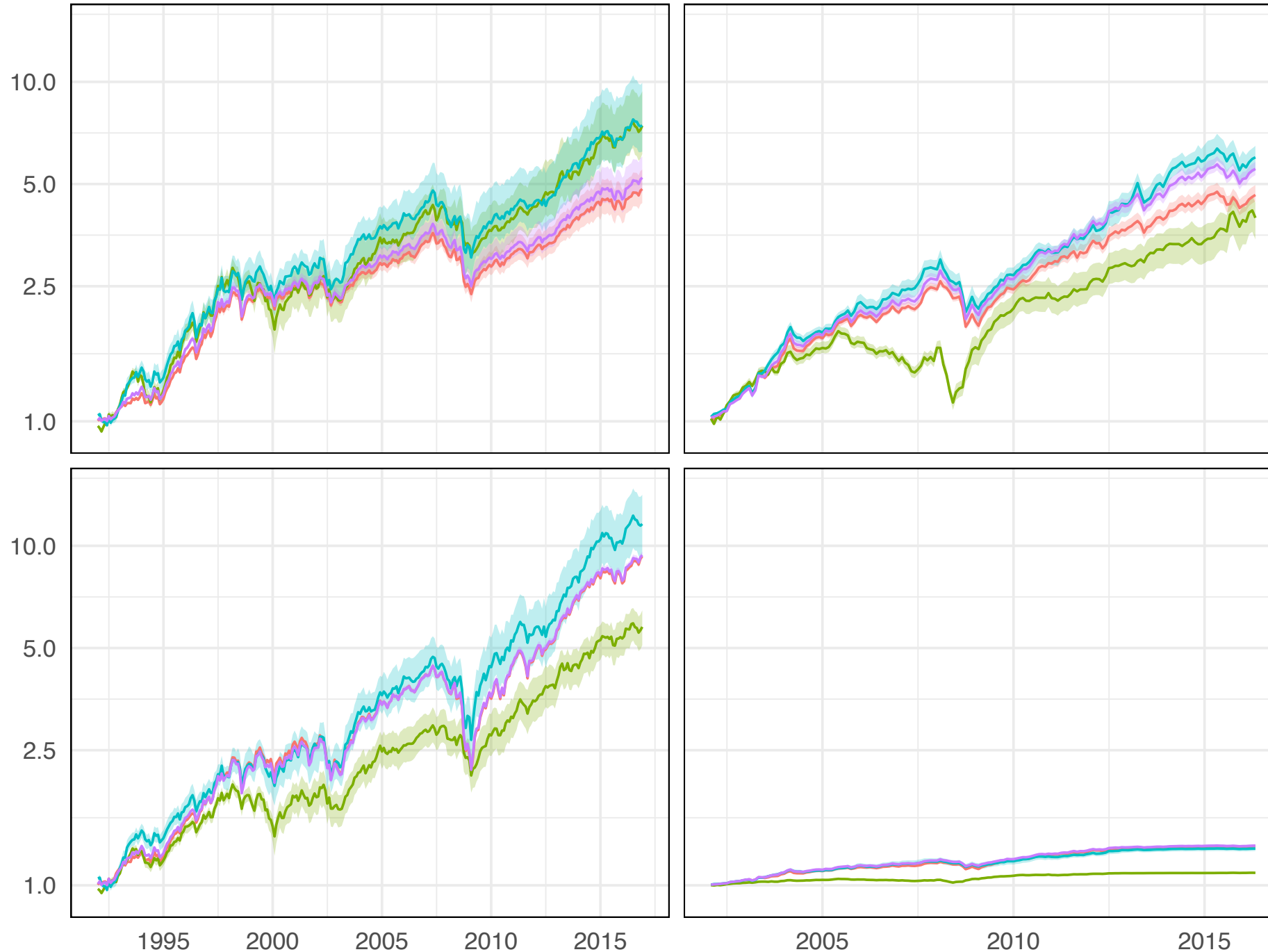
Strategy

- Inverse Volatility
- Minimum Variance
- Maximum Diversified
- Equal Risk Contribution

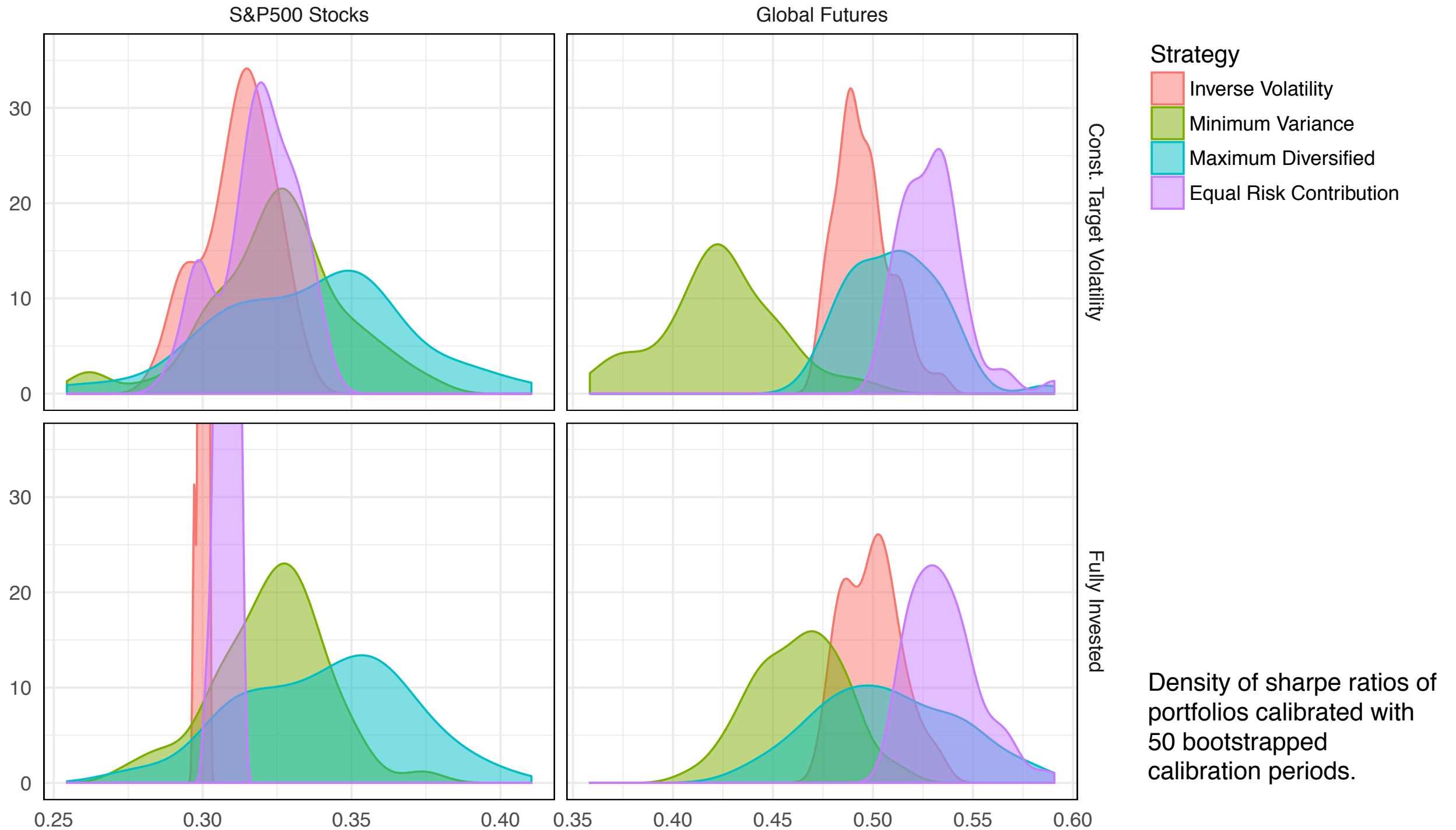
Const. Target Volatility

Fully Invested

Solid line represents median performance, shaded area spans from 10% to 90% quantile of performances of 50 portfolios calibrated with bootstrapped samples.



Sharpe Ratio Distribution



Density of sharpe ratios of portfolios calibrated with 50 bootstrapped calibration periods.

Take-Out

- Portfolios differ both in size and dispersion of sharpe ratios.
- Maximum Diversified portfolio reaches highest sharpe ratio in single US stocks / second highest sharpe ratio in global futures over study period.

$$\max_{\mathbf{w} \geq 0} \frac{\mathbf{w}' \sigma_r}{\sqrt{\mathbf{w}' \Sigma \mathbf{w}}}$$

- It pays out to use a more complex portfolio strategy in terms of number of parameters. This also leads to higher sensitivity to parameter estimation.
- New question: How to mitigate impact of parameter estimation.

Thanks to authors of PortfolioAnalytics and FRAPo packages
for great R implementations of portfolio strategies!

Annexes

Portfolio Objectives

- Inverse Volatility ¹: $w_i = \frac{1}{\sigma_r}$
- Minimum Variance ²: $\min_{w \geq 0} \mathbf{w}' \Sigma \mathbf{w}$
- Maximum Diversified ³: $\max_{\mathbf{w} \geq 0} \frac{\mathbf{w}' \sigma_r}{\sqrt{\mathbf{w}' \Sigma \mathbf{w}}}$
- Equal Risk Contribution ⁴: $\text{RC}_j = \text{RC}_i = \frac{1}{2} w_i (\Sigma \mathbf{w})_i$

References

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