Risk-Based Portfolios under Parameter Uncertainty

R/Finance
May 20, 2017
Lukas Elmiger
Which risk based portfolio strategy offers best out of sample performance …
Which risk based portfolio strategy offers best out of sample performance ... at smallest sensitivity to parameter estimation.
Sensitivity to Calibration

- High dispersion of out-of-sample performance of portfolios calibrated with bootstrapped sample periods indicates elevated sensitivity to parameter estimation.

- Bootstrap does not require distributional assumptions and maintains (nonlinear, higher moment) dependence information.

- Block-bootstrap with monthlong blocks maintains serial dependence.

“Financial variables are intrinsically linked in complicated ways.”

Harvey and Liu (2015)
Bootstrap Procedure

Monthlong blocks of returns:

Bootstrap sample:
1. Randomly draw $N$ blocks with replacement from $r_1 \ldots r_N$
2. Recycle random sequence over entire data set
   ⇒ overlapping parts of calibration periods remain unchanged.
   ⇒ Each calibration period contains $N$ observations.

Calibrate portfolios based on overlapping bootstrap samples

Apply out-of-sample to full return series:

$$ r_{t}^{os} = w' r_t $$
Asset Universe

S&P 500 Stocks
• Historical index members
• Homogeneous correlations
• Daily CRSP data
• 27 years of data 1990 - 2016

Global Futures
• 42 liquid rate, bond, equity, volatility, commodity futures
• Heterogeneous correlations
• 17 years of data 1999 - 2016
• Intra-day data from tickdata.com for concurrent return observations
• Define rolling dates and account for rolling costs
Cumulated Returns

Solid line represents median performance, shaded area spans from 10% to 90% quantile of performances of 50 portfolios calibrated with bootstrapped samples.
Density of sharpe ratios of portfolios calibrated with 50 bootstrapped calibration periods.
Take-Out

• Portfolios differ both in size and dispersion of sharpe ratios.

• Maximum Diversified portfolio reaches highest sharpe ratio in single US stocks / second highest sharpe ratio in global futures over study period.

\[
\max_{w \geq 0} \frac{w' \sigma_r}{\sqrt{w' \Sigma w}}
\]

• It pays out to use a more complex portfolio strategy in terms of number of parameters. This also leads to higher sensitivity to parameter estimation.

• New question: How to mitigate impact of parameter estimation.
Thanks to authors of PortfolioAnalytics and FRAPO packages for great R implementations of portfolio strategies!
Annexes
Portfolio Objectives

• Inverse Volatility ¹:
  \[ w_i = \frac{1}{\sigma_r} \]

• Minimum Variance ²:
  \[ \min_{w \geq 0} w'\Sigma w \]

• Maximum Diversified ³:
  \[ \max_{w \geq 0} \frac{w'\sigma_r}{\sqrt{w'\Sigma w}} \]

• Equal Risk Contribution ⁴:
  \[ RC_j = RC_i = \frac{1}{2} w_i (\Sigma w)_i \]
References


