Practical Options Modeling with the `sn` Package, Fat Tails, and How to Avoid the Ultraviolet Catastrophe

Oliver M. Haynold
CME Group

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Practical Options Modeling

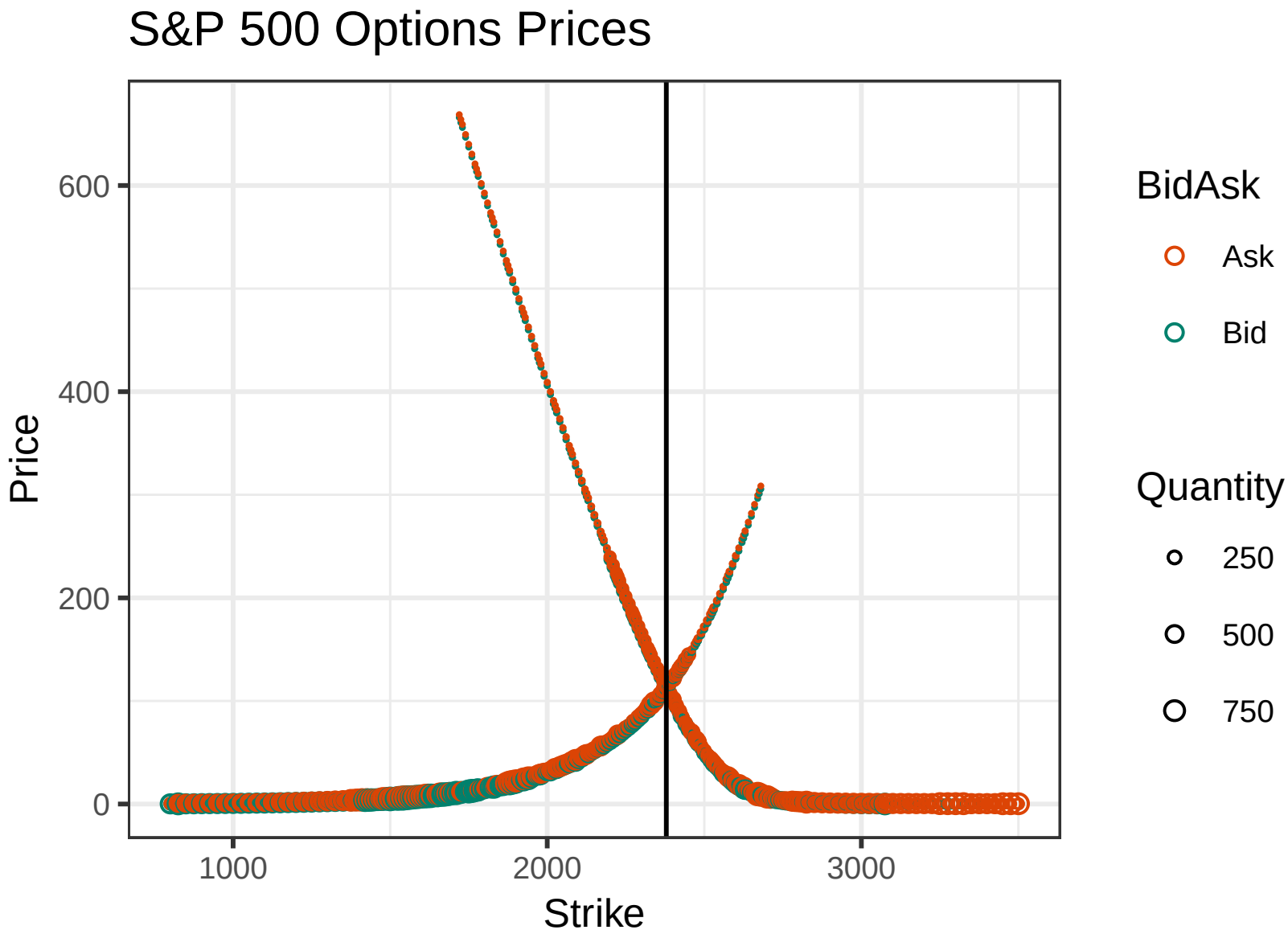
- Task: Model S&P 500® options prices
- The art of modeling is deciding what to discard so as to simplify the world…
- …and what to retain
- Where do we want to disagree with the market?
- Plot it! `ggplot2` is your friend!
- There's a package for that: `sn`

Read the text slides at home…

All of this is my personal opinion and none of this is investment advice

Starting Point: Transactable Prices

ES options Dec 2017 expiry on 15 March 2017



Prices as Implied Volatility: The Smirk

Forward price and rate from put/call parity

What Do We Care About?

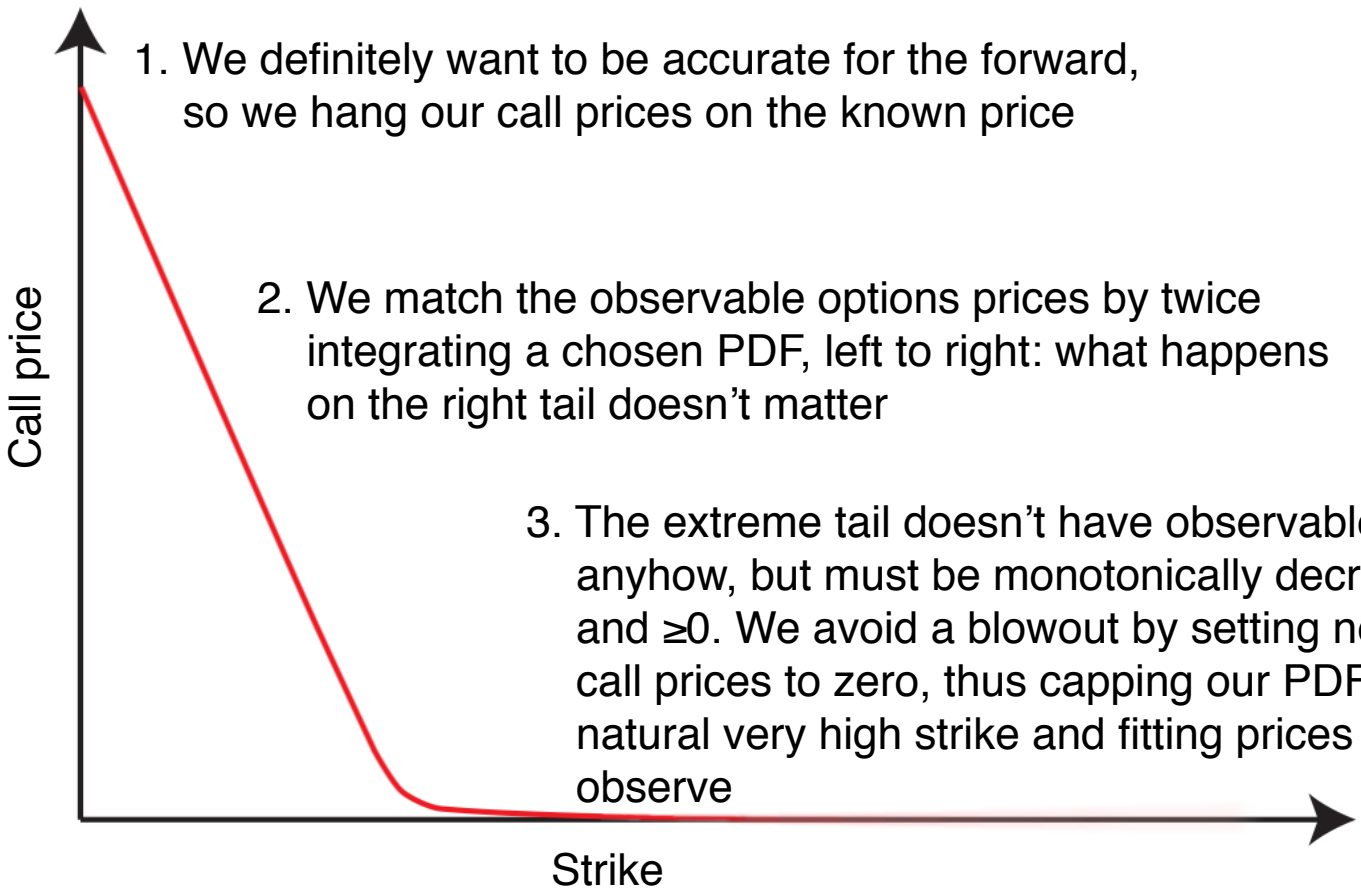
Four parameters seem about right

Forward	Location parameter of the distribution
Volatility	Stretch parameter of the distribution
Skewness	Shape parameter of distribution shifting median *vs.* mean
Tails	Power-law tails left and right

- This one should be the only controversial proposal of the four
- At least for *P* measure both tails tend to be heavy (Mandelbrot)
- For *Q* measure right tail should not be attenuated heavily at least for securities where representative investor dislikes price shocks
- At least ask prices do not decline to near 0 for any strike assuming writer could actually pay
- Smirk does not not flatten with tenor as Central Limit Theorem would imply
- But there is a problem…

How to Avoid the Ultraviolet Catastrophe

- Right power law tails blow up expected value to infinity, but we know the finite forward value
- Max left skew avoids this, but undervalues little calls
- Arbitrary truncation parameter is odd
- A bit of handwaving to the rescue! Use forward as an input



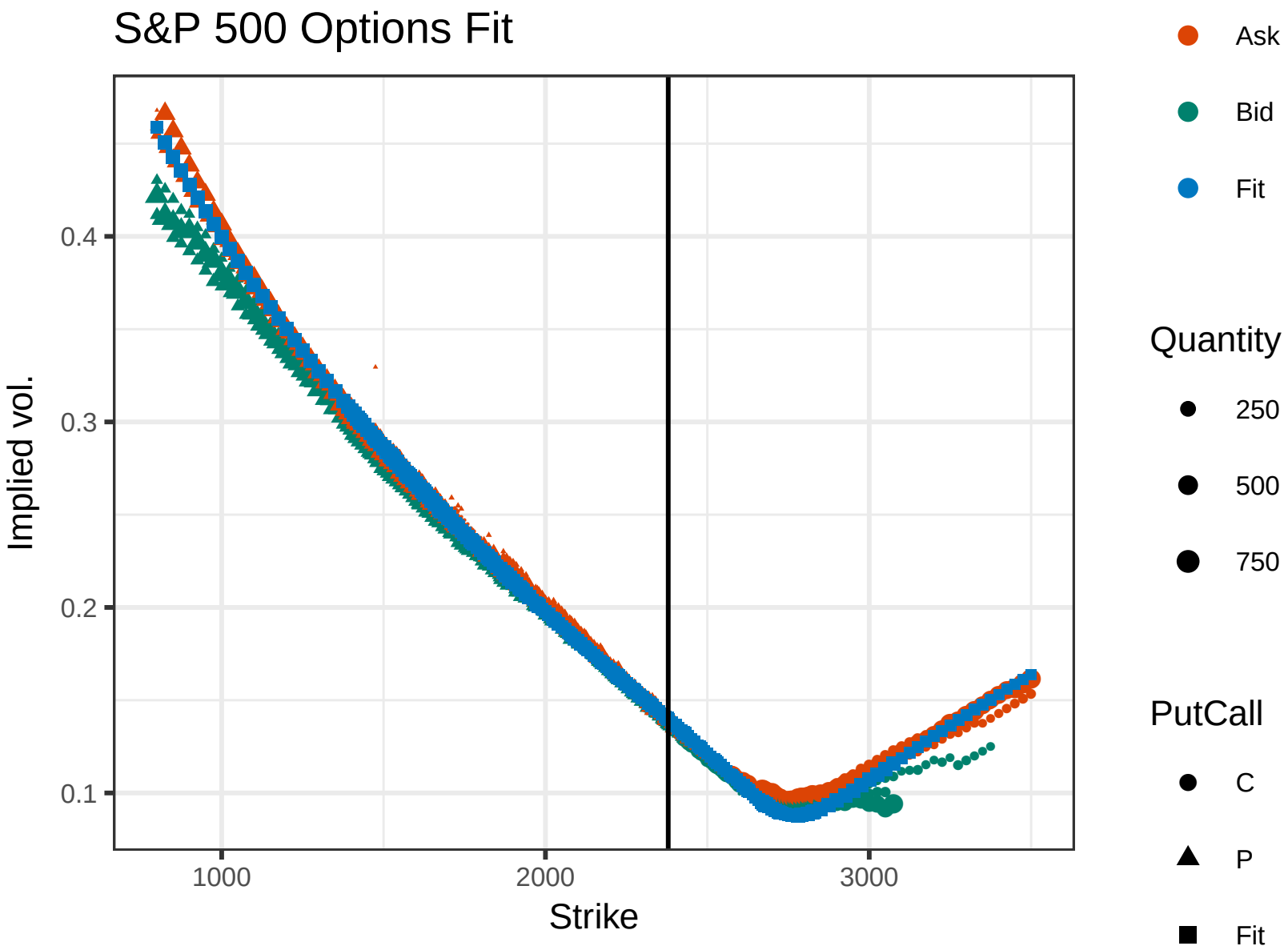
The skew-*t* distribution from the `sn` package

- `dst(x, xi=0, omega=1, alpha=0, nu=Inf, dp=NULL, method=0, ...)` and `pst`, `qst`, `rst`
- `xi` is location parameter
- `omega` is stretch parameter
- `alpha` is shape parameter
- `nu` is tail heaviness as in Student's *t*

- `alpha=0` gives Student's *t*
- `alpha=0` and `nu=Inf` gives Gaussian
- `alpha=0` and `nu=1` gives Cauchy (Here be dragons!)
- Can have `nu`>2, better fit than L-stable distributions (but beware the Central Limit Theorem)

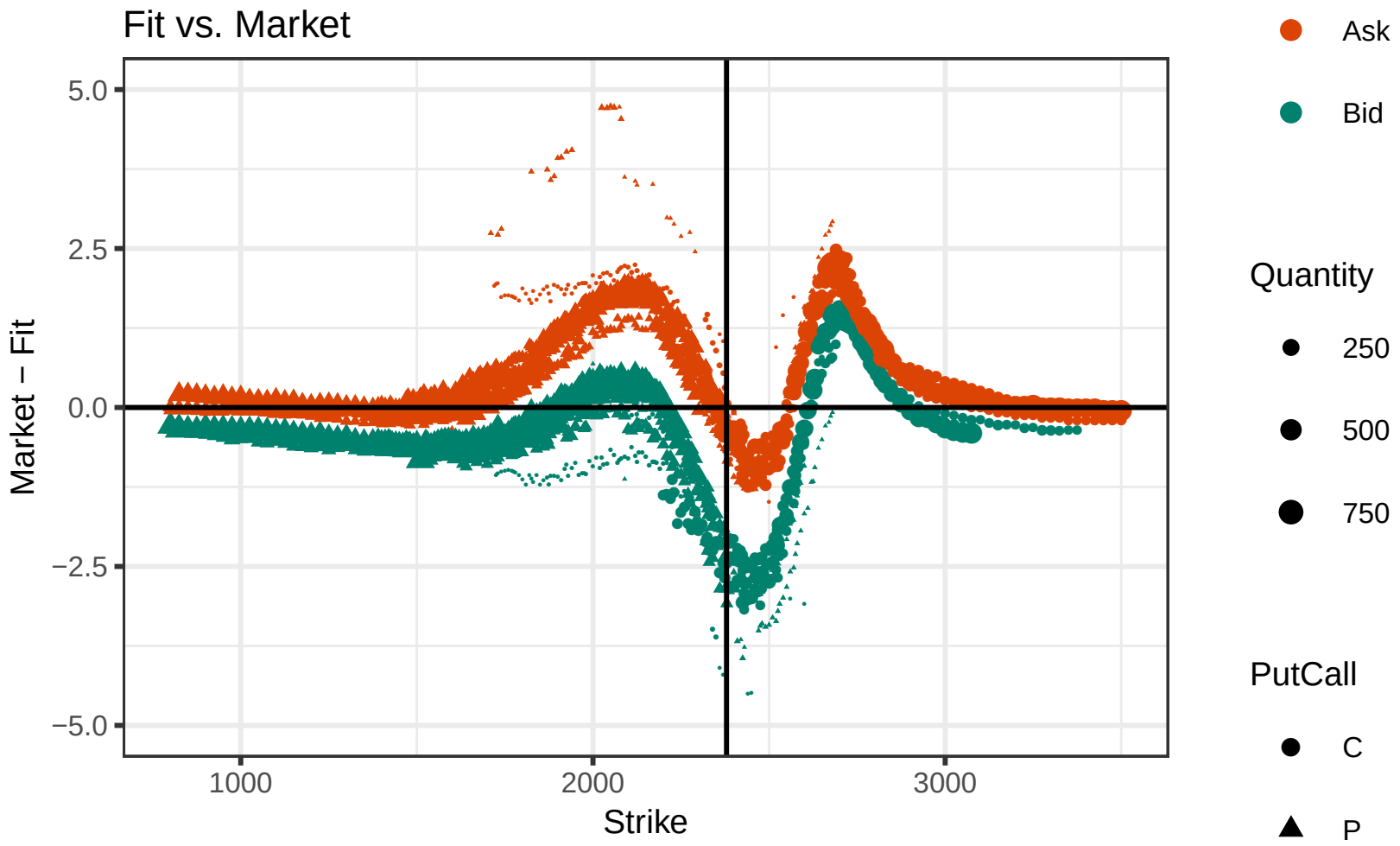
It Fits!

Fit to minimize bid/ask price violations



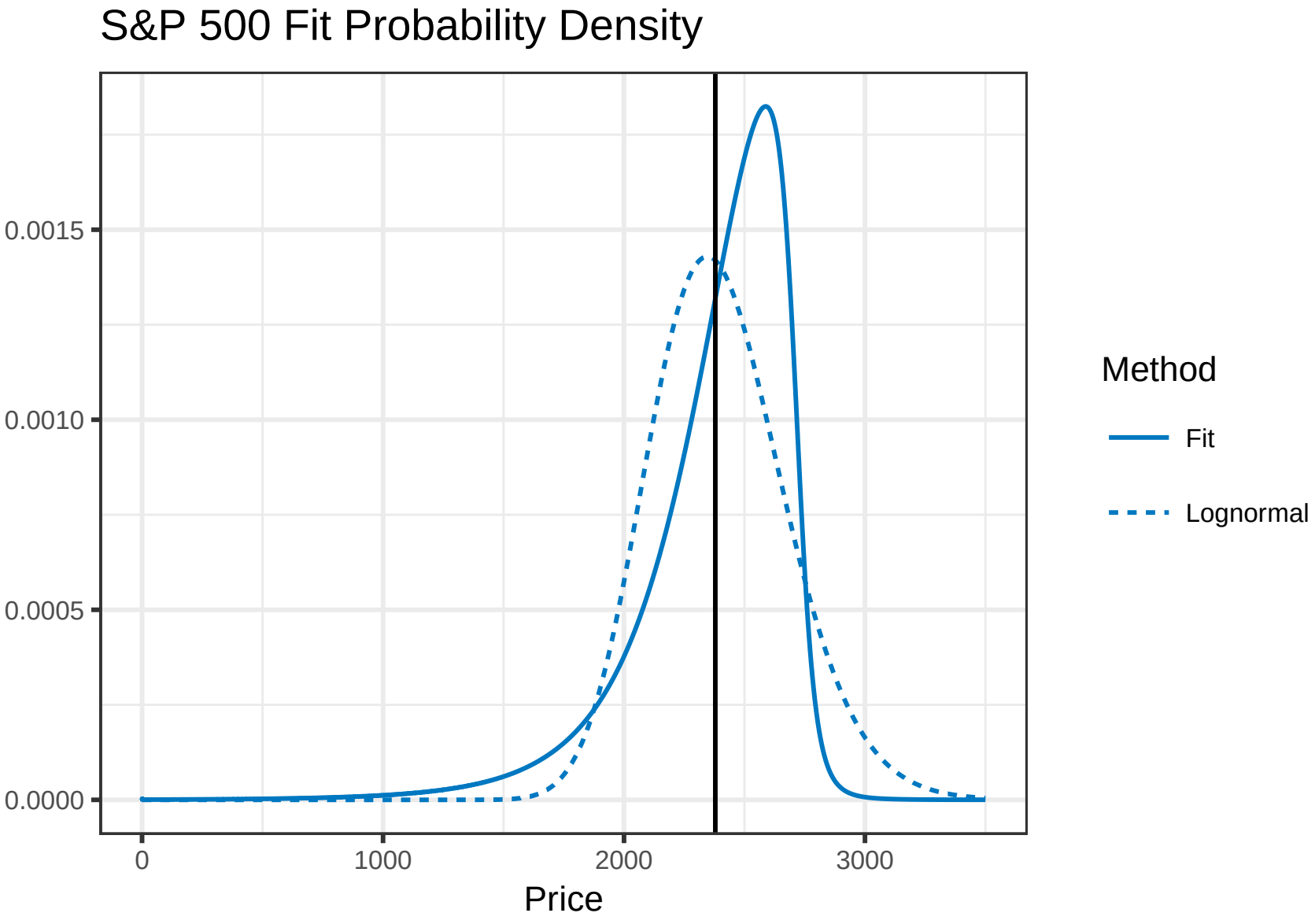
It Fits!

- Right tail a bit too heavy, left tail a bit too thin
- A bit too pointy at the mode—probably because right tail is too heavy



Implied Risk-neutral PDF

S&P at 1,500 has significant probability mass, 3,000 not so much



Applications

- Greeks against skew (shall we call it ‘škoda’?) and tail parameters (‘kappa’) can be useful additions to delta and vega
- You can Monte Carlo this for exotic options
  - Careful with the tails—they’re fat!
  - Use appropriate variance reduction techniques
  - Make sure the forward price checks out
  - After (many) iterations the Central Limit Theorem will kick in—don’t use smaller steps in time than needed

What's Next?

- The `xi` parameter is less embarrassing than an arbitrary truncation parameter, but still embarrassing
  - For `nu=Inf` there's not really a choice at all
  - For smaller `nu` there's little choice
- In *P*-space and in *Q*-space for many distributions power-law α should be larger for right than for left tail
- Some control of kurtosis around the center of the distribution would seem useful–Gaussian mixin?
- Can we come up with a distribution/call price that fits observed prices for a broad range of products? What about the parameters:
  - Forward price
  - Stretch parameter
  - Left tail power law exponent
  - Right tail power law exponent—this could give shape by itself
  - Maybe an additional shape modifier as needed