Practical Options Modeling with the \texttt{sn} Package, Fat Tails, and How to Avoid the Ultraviolet Catastrophe

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R/Finance 2017
Practical Options Modeling

- Task: Model S&P 500® options prices
- The art of modeling is deciding what to discard so as to simplify the world…
- …and what to retain
- Where do we want to disagree with the market?
- Plot it! ggplot2 is your friend!
- There's a package for that: sn

Read the text slides at home…

All of this is my personal opinion and none of this is investment advice
Starting Point: Transactable Prices
ES options Dec 2017 expiry on 15 March 2017

S&P 500 Options Prices

BidAsk
- Ask
- Bid

Quantity
- 250
- 500
- 750
Prices as Implied Volatility: The Smirk
Forward price and rate from put/call parity

S&P 500 Options Prices

BidAsk
- Ask
- Bid

Quantity
- 250
- 500
- 750

PutCall
- C
- P

PutCall

Forward price and rate from put/call parity
What Do We Care About?

Four parameters seem about right

**Forward**  Location parameter of the distribution

**Volatility**  Stretch parameter of the distribution

**Skewness**  Shape parameter of distribution shifting median vs.
mean

**Tails**  Power-law tails left and right

- This one should be the only controversial proposal of the four
- At least for $P$ measure both tails tend to be heavy (Mandelbrot)
- For $Q$ measure right tail should not be attenuated heavily at
  least for securities where representative investor dislikes price
  shocks
- At least ask prices do not decline to near 0 for any strike
  assuming writer could actually pay
- Smirk does not not flatten with tenor as Central Limit Theorem
  would imply
- But there is a problem…
How to Avoid the Ultraviolet Catastrophe

- Right power law tails blow up expected value to infinity, but we know the finite forward value
- Max left skew avoids this, but undervalues little calls
- Arbitrary truncation parameter is odd
- A bit of handwaving to the rescue! Use forward as an input

1. We definitely want to be accurate for the forward, so we hang our call prices on the known price
2. We match the observable options prices by twice integrating a chosen PDF, left to right: what happens on the right tail doesn’t matter
3. The extreme tail doesn’t have observable prices anyhow, but must be monotonically decreasing and $\geq 0$. We avoid a blowout by setting negative call prices to zero, thus capping our PDF at a natural very high strike and fitting prices we can observe
The skew-\(t\) distribution from the\texttt{sn} package

- \texttt{dst(x, xi=0, omega=1, alpha=0, nu=Inf, dp=NULL, method=0, ...)} and \texttt{pst, qst, rst}
- \(xi\) is location parameter
- \(omega\) is stretch parameter
- \(alpha\) is shape parameter
- \(nu\) is tail heaviness as in Student’s \(t\)

- \(alpha=0\) gives Student’s \(t\)
- \(alpha=0\) and \(nu=\text{Inf}\) gives Gaussian
- \(alpha=0\) and \(nu=1\) gives Cauchy (Here be dragons!)
- Can have \(nu>2\), better fit than L-stable distributions (but beware the Central Limit Theorem)
It Fits!
Fit to minimize bid/ask price violations

S&P 500 Options Fit

- Orange dots represent Ask prices
- Green triangles represent Bid prices
- Blue line represents the fit

Implied vol.

Strike

Quantity
- Black circle: 250
- Black square: 500
- Black triangle: 750

PutCall
- Black circle: C (Call)
- Black triangle: P (Put)
- Black square: Fit

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It Fits!

- Right tail a bit too heavy, left tail a bit too thin
- A bit too pointy at the mode—probably because right tail is too heavy

Fit vs. Market

Market − Fit

Fit vs. Market

Strike

Fit vs. Market

Market − Fit

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Strike
Implied Risk-neutral PDF

S&P at 1,500 has significant probability mass, 3,000 not so much

S&P 500 Fit Probability Density

Method
- Fit
- Lognormal
Applications

- Greeks against skew (shall we call it ‘škoda’?) and tail parameters (‘kappa’) can be useful additions to delta and vega
- You can Monte Carlo this for exotic options
  - Careful with the tails—they’re fat!
  - Use appropriate variance reduction techniques
  - Make sure the forward price checks out
  - After (many) iterations the Central Limit Theorem will kick in—don’t use smaller steps in time than needed
What’s Next?

- The $\xi$ parameter is less embarrassing than an arbitrary truncation parameter, but still embarrassing
  - For $\nu=\infty$ there’s not really a choice at all
  - For smaller $\nu$ there’s little choice
- For $\nu=\infty$ there’s not really a choice at all
- For smaller $\nu$ there’s little choice
- In $P$-space and in $Q$-space for many distributions power-law $\alpha$ should be larger for right than for left tail
- Some control of kurtosis around the center of the distribution would seem useful—Gaussian mixin?
- Can we come up with a distribution/call price that fits observed prices for a broad range of products? What about the parameters:
  - Forward price
  - Stretch parameter
  - Left tail power law exponent
  - Right tail power law exponent—this could give shape by itself
  - Maybe an additional shape modifier as needed