# Efficient, Consistent and Flexible Credit Default Simulation

## with TRNG and RcppParallel

Riccardo Porreca

Roland Schmid

Mirai Solutions GmbH
Tödistrasse 48
CH-8002 Zurich
Switzerland

info@mirai-solutions.com
www.mirai-solutions.com

# Integrated market and default risk model

- Portfolio of defaultable securities issued by a set of counterparties
- State of the credit environment driving the default of counterparty j in [1,J]

$$Y_j = \beta_j Z_j + \sigma_j \varepsilon_j \text{ , i.i.d. } \varepsilon_j \sim N(0,1);$$

systemic component — specific component (idiosyncratic return)

$$D_j = \begin{cases} 1, & Y_j < \Phi^{-1}\left(P_j^D\right) \\ 0, & Y_j \geq \Phi^{-1}\left(P_j^D\right) \end{cases}$$

default indicator

$$V_j = V_j^0 (1 + r_j);$$

market value

$$L_j = V_j^0 - \left[\left(1 - D_j\right) V_j + D_j R_j\right]$$

integrated model loss

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integrated model loss

- Assumption: M scenarios of the state of the world are available

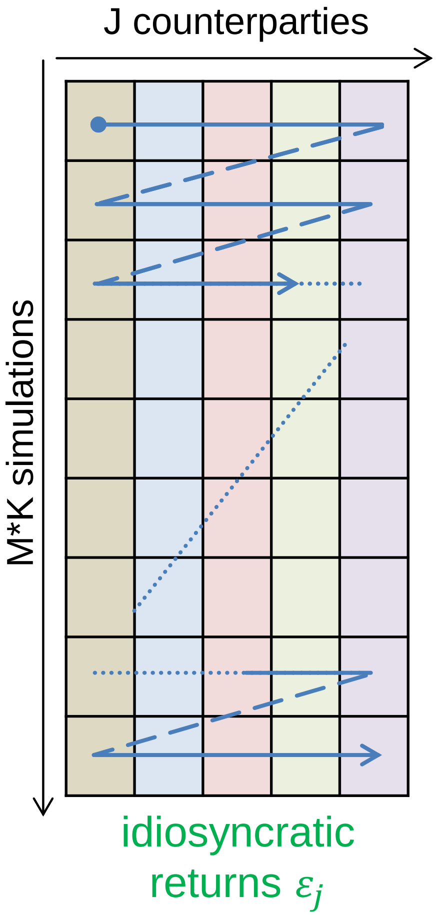
$$\left\{ Z_j^{(m)}, r_j^{(m)} \right\}_{m=1}^{M}$$

- Simulation problem: for each scenario m in [1,M], generate K samples of $\varepsilon_j$ to obtain M*K realizations of the credit environment return $Y_j$
  - combined simulation size M*K high enough to capture the rare nature of default events

# Efficient, consistent, flexible MC simulation

Exact reproducibility of full-simulation results (*fair playing*)

- Parallel execution (*block splitting*)
- Sub-portfolio simulation
- Insight for given scenarios of interest
- Limitation: (Pseudo)Random Number Generators (RNGs) are intrinsically sequential



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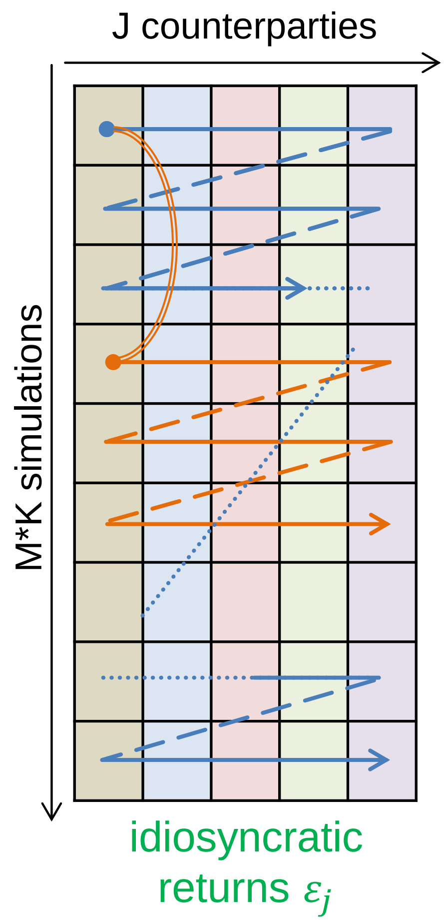
- TRNG: "*state of the art C++ pseudo-random number generator library for sequential and parallel Monte Carlo simulations*" [H. Bauke, http://numbercrunch.de/trng]
  - RNGs with powerful jump and split capabilities



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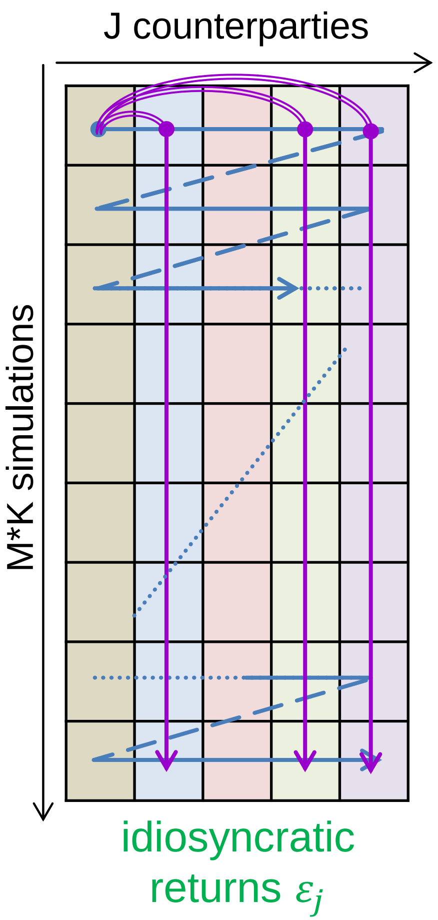

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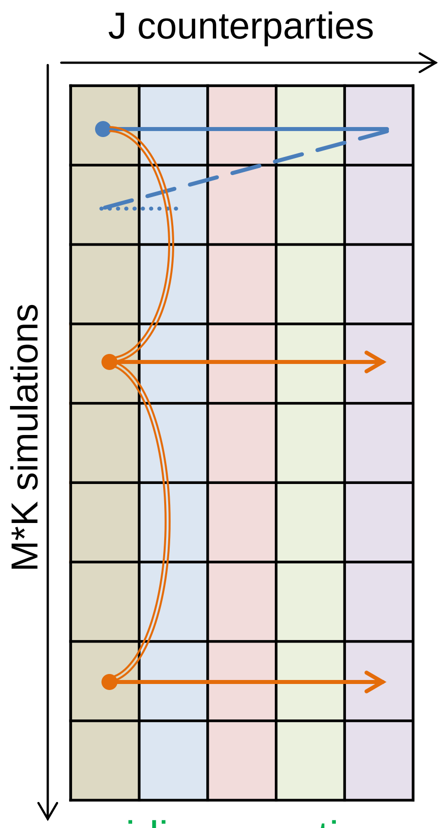


idiosyncratic returns $\varepsilon_j$

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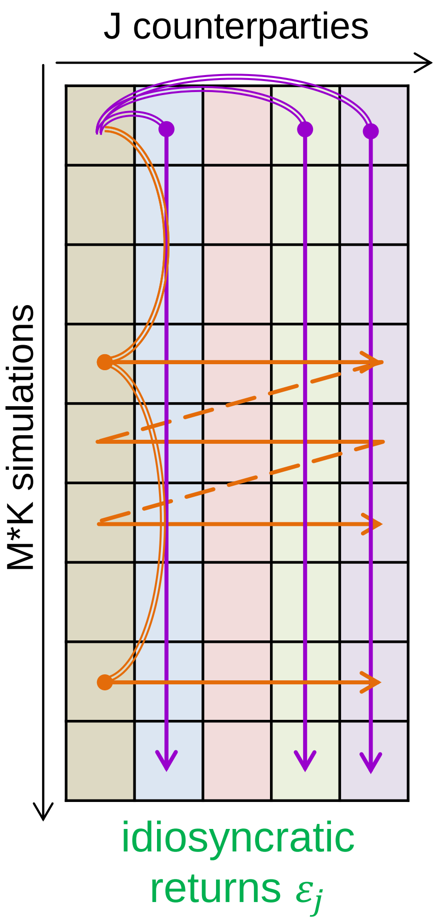
- TRNG: "*state of the art C++ pseudo-random number generator library for sequential and parallel Monte Carlo simulations*" [H. Bauke, http://numbercrunch.de/trng]
  - RNGs with powerful jump and split capabilities
- Available to the R community via rTRNG package
  - `install_github("miraisolutions/rTRNG")`



# R(cpp) simulation kernel and data

https://github.com/miraisolutions/PortfolioRiskMC

| j | V0 | R | beta | PD | rtng | ... |
|---|---|---|---|---|---|---|
| 2 | 32606970 | 910000 | 0.707 | 0.0025 | BBB | . |
| 3 | 8932752 | 92800 | 0.707 | 0.0010 | A | . |
| 6 | 564931 | 335675 | 0.707 | 0.1000 | CCC | . |
| 7 | 3494502 | 82000 | 0.707 | 0.0400 | B | . |
| 9 | 6679886 | 1000500 | 0.707 | 0.0100 | BB | . |

| m \ j | 1 | . | . | . | J |
|---|---|---|---|---|---|
| 1 | 0.5241 | -0.484 | -0.402 | -0.774 | -0.702 |
| . | -2.2608 | -0.666 | -1.003 | 0.423 | 0.683 |
| . | -0.0197 | -0.174 | -0.178 | -0.607 | -0.858 |
| . | 0.1831 | -1.011 | -0.488 | 0.209 | 0.368 |
| M | -0.3614 | 0.740 | 0.928 | -0.777 | -1.430 |

| m \ j | 1 | . | . | . | J |
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| 1 | -0.512 | -0.376 | -1.059 | -0.165 | 0.511 |
| . | -0.268 | -1.135 | -1.318 | 1.760 | -3.088 |
| . | -0.199 | 0.609 | -0.204 | 1.612 | 0.474 |
| . | 0.857 | 0.566 | -0.601 | 0.803 | 0.626 |
| M | -0.167 | -0.736 | 0.661 | 1.265 | 0.981 |

```
simulationKernel(pf, Z, r,
                 J,
                 K, mk = seq_len(K * nrow(Z)),
                 agg = factor(rep("PF", nrow(pf))),
                 seed)
```

$$Y_j = \beta_j Z_j + \sqrt{1-\beta_j^2}\,\varepsilon_j$$

$$V_j = V_j^0(1+r_j)$$

$$D_j = \begin{cases} 1, & Y_j < \Phi^{-1}\left(P_j^D\right) \\ 0, & Y_j \geq \Phi^{-1}\left(P_j^D\right) \end{cases}$$

$$L_j = V_j^0 - \left[\left(1-D_j\right)V_j + D_j\, R_j\right]$$

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(sub)portfolio

| m \ j | 1 | . | . | . | J |
|---|---|---|---|---|---|
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**Usage**

```
simulationKernel(pf, Z, r,
                 J,                                   
                 K, mk = seq_len(K * nrow(Z)),
                 agg = factor(rep("PF", nrow(pf))),
                 seed)
```

J: total nr. of counterparties

seed: initial RNG state

**Value**

aggregation criterion

| mk \ agg | 1 | . | . | A |
|---|---|---|---|---|
| 1 | 2942986 | 2144142 | 2551616 | 2128115 |
| . | 3269994 | 1886979 | 2881280 | 1447524 |
| . | 3874471 | 1711273 | 2544507 | 3696855 |
| . | 2809653 | 3757694 | 1816909 | 3071337 |
| . | 4100697 | 2123775 | 2544716 | 1878057 |
| . | 2106241 | 4758459 | 4014828 | 3573142 |
| M*K | 2045032 | 2990315 | 4636829 | 2588612 |

simulations of interest

# Examples and results

https://github.com/miraisolutions/PortfolioRiskMC

- Full simulation (multi-threaded): J = 6'000, M = 10'000, K = 100
  - Aggregation criterion: rating (credit quality)

```
L_rtng <- simulationKernel(pf, Z, r, J, K,
                           agg = pf$rtng, seed = s)
```

  - ES99: average loss in the 1% worst scenarios

```
ES99_rtng <- ES99(L_rtng)
##        BBB         AA        AAA          A         BB ...
## 9878920788 8620051762 7468245838 4796596354 1190520347 ...
```

- Consistent simulation for the sub-portfolio with BBB rating

```
L_BBB <- simulationKernel(pf %>% filter(rtng == "BBB"),
                          Z, r, J, K, seed = s)
all.equal(c(L_BBB), L_rtng[, "BBB"], check.attributes = FALSE)
## [1] TRUE
ES99_BBB <- ES99(L_BBB)
## 9878920788
```

# Examples and results

https://github.com/miraisolutions/PortfolioRiskMC

- Risk insight for BBB
  - Contribution of individual counterparties to the BBB total ES99

```
pfBBB <- pf %>% filter(rtng == "BBB")
L_jBBBtail <- simulationKernel(pfBBB, Z, r, J, K, agg = pfBBB$j,
                               mk = tail99(L_BBB), seed = s)
ContrES99_jBBB <- colMeans(L_jBBBtail)
all.equal(sum(ContrES99_jBBB), ES99_BBB, check.attributes = FALSE)
## [1] TRUE
```

  - Focus on the top 3 counterparties (highest contribution)

```
pftop3BBB <- pfBBB %>% filter(j %in% top3jBBB)
L_top3BBB <- simulationKernel(pftop3BBB, Z, r, J, K,
                              agg = pftop3BBB$j, seed = s)
ES99_top3BBB <- ES99(L_top3BBB)
##  j         V0         R       ES99  ContrES99       Div   Contr/V0
##  2 9444041000 278250000 4720889738 4277383374 0.9060545 0.4529188
##  8 3260697000  91000000 1672364832  999636420 0.5977382 0.3065714
## 70  298111300  13483307  963482756  436598485 0.4531461 1.4645486
```

# Examples and results

https://github.com/miraisolutions/PortfolioRiskMC

- What-if scenario
  - Top 3 BBB counterparties downgraded => PD from 0.0025 to 0.01

```
pfBBBwi <- pf %>% filter(rtng == "BBB") %>%
  mutate(PD = replace(PD, j %in% top3jBBB, 0.01))
pftop3BBBwi <- pfBBBwi %>% filter(j %in% top3jBBB)
L_top3BBBwi <- simulationKernel(pftop3BBBwi, Z, r, J, K,
                                agg = pftop3BBBwi$j, seed = s)
```

  - Effect on the BBB total

```
L_BBBwi <- L_BBB + (rowSums(L_top3BBBwi) - rowSums(L_top3BBB))
ES99_BBBwi <- ES99(L_BBBwi)
##  ES99_BBBwi    ES99_BBB
## 11943875892 9878920788
```

  - New contribution for the full BBB sub-portfolio

```
L_jBBBtailwi <- simulationKernel(pfBBBwi, Z, r, J, K, agg = pfBBBwi$j,
                                 mk = tail99(L_BBBwi), seed = s)
```

**All this achieved without re-simulating the whole BBB portfolio!**

# Summary

- Monte Carlo simulation of an integrated market and default risk model
  - Flexible, consistent, slim, multi-purpose simulation kernel
- TRNG state-of-the-art parallel random number generators
  - rTRNG for prototyping in R and broader usage in R/C++ projects
- Flexible and fast ad-hoc assessments on sub-portfolios, simulations of interest, what-if scenarios
- Incremental simulations and updates possible
- Can also be used for driver or change analysis, isolating away the MC variability

**=> Achieve fast re-simulation** instead of

- storing GBs or TBs of granular results
- using complex analytic approximation models that are hard to explain and understand

# Appendix
# Integrated market and default risk model

- Portfolio of defaultable securities issued by a set of counterparties

- Model market risk in correlation with credit default risk using an integrated approach
  - Market and default risk are intrinsically related
  - Dependency must be properly taken into account

- Simplifying assumptions
  - Default occurrence determined at counterparty level
  - Exactly one security per each counterparty in the portfolio
  - We ignore non-defaultable securities subject to market risk only

# Integrated market and default risk model

- State of the credit environment driving the default of counterparty j in [1,J]

$$Y_j = \beta_j Z_j + \sigma_j \varepsilon_j \text{ , i.i.d. } \varepsilon_j \sim N(0,1)$$

systemic component (reflects the state of the world)

specific component (idiosyncratic return)

- Return $r_j$ drives the market value at horizon (based on the state of the world)

$$V_j = V_j^0 (1 + r_j)$$

- Default indicator

$$D_j = \begin{cases} 1, & Y_j < \theta_j \\ 0, & Y_j \geq \theta_j \end{cases}, \quad \theta_j \ : \mathrm{P}\left(Y_j < \theta_j\right) = P_j^D$$

- Loss (including occurrence of defaults)

$$L_j = V_j^0 - \left[\left(1 - D_j\right) V_j + D_j R_j\right], \quad 0 \leq R_j \leq V_j^0$$

- Default events $D_j$ and losses $L_j$ inherit the correlation structure of $r_j$ and $Z_j$ with other counterparties

# Appendix
# Integrated market and default risk model

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- Assumption: M scenarios of the state of the world are available

$$\left\{Z_j^{(m)}, r_j^{(m)}\right\}_{m=1}^{M}$$

  - Monte Carlo realizations from a given market risk model, which we extend by the occurrence of defaults
  - we also assume (WLOG): $Z_j \sim N(0,1)$, $\sigma_j = \sqrt{1-\beta_j^2} \Rightarrow \theta_j = \Phi^{-1}\left(P_j^D\right)$

$$Y_j = \beta_j Z_j + \sqrt{1-\beta_j^2}\,\varepsilon_j$$

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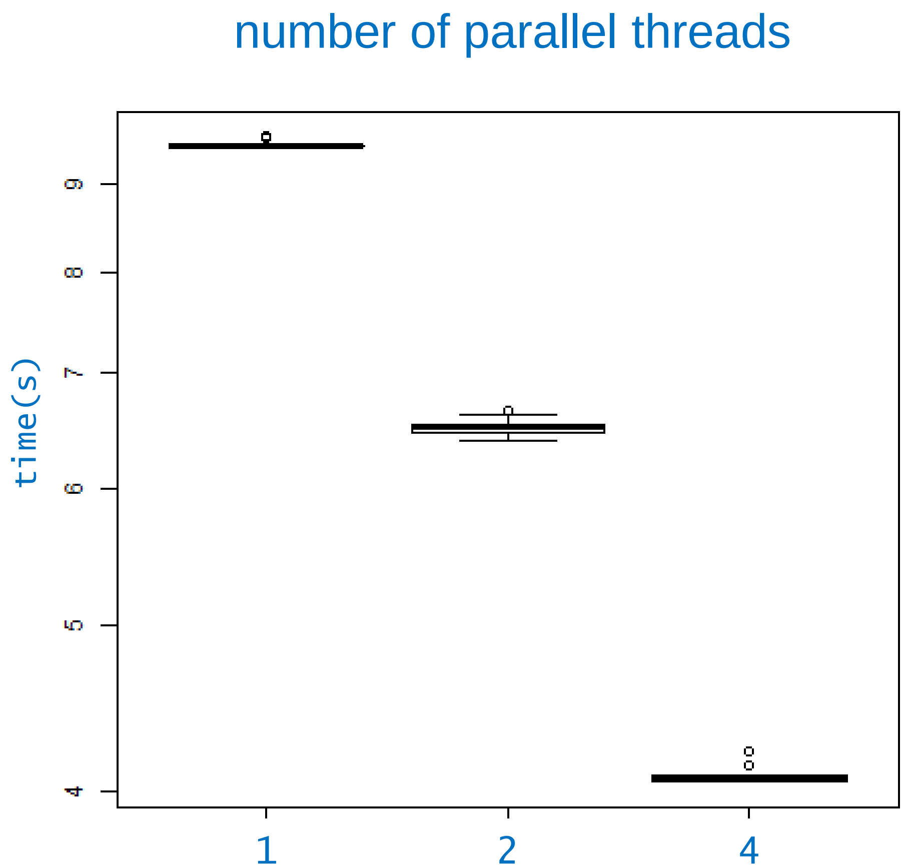
  - we also assume (WLOG): $Z_j \sim N(0,1),\ \ \sigma_j = \sqrt{1-\beta_j^2} \Rightarrow \theta_j = \Phi^{-1}\left(P_j^D\right)$
- Monte Carlo approach for simulating the integrated model:
  - combine $V_j$ and $Z_j$ for the available scenarios with independent realizations of the idiosyncratic returns $\varepsilon_j$
  - for each scenario m in [1,M], generate K samples of $\varepsilon_j$ to obtain M*K realizations of the credit environment return $Y_j$
  - combined simulation size M*K high enough to capture the rare nature of default events

`microbenchmark` results (M = 1000, K = 10)



# Appendix
# Performance benchmark

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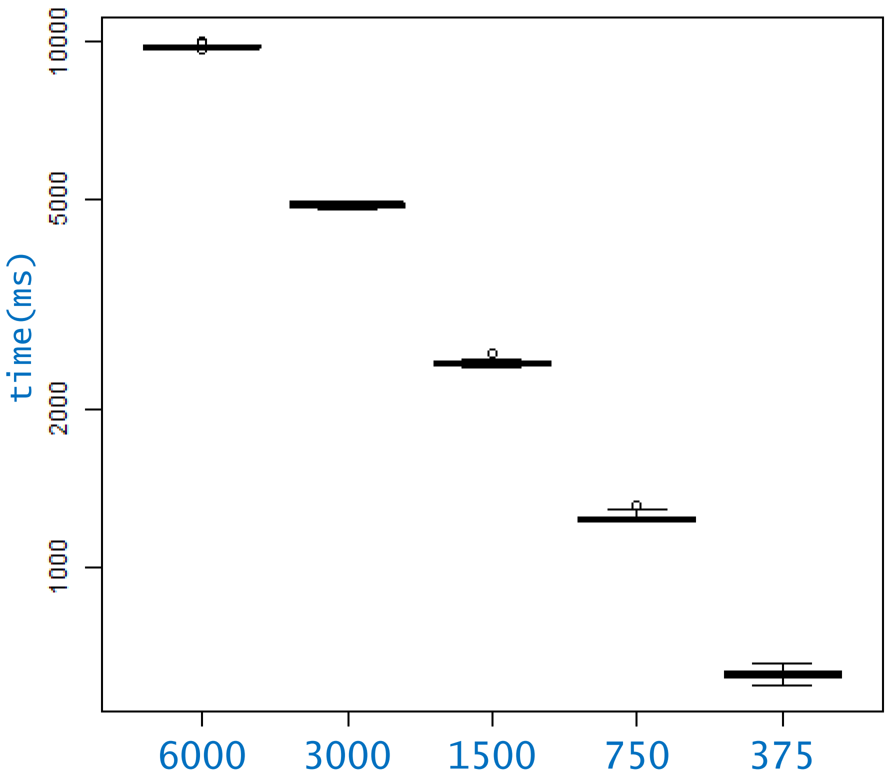
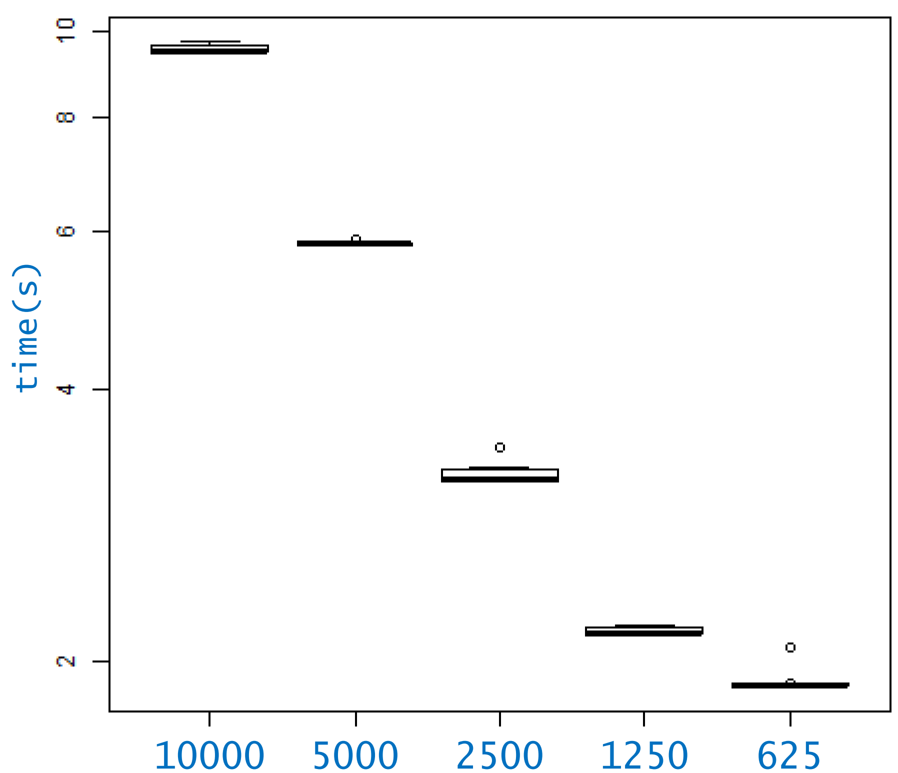
size of the sub-portfolio




number of sub-simulations



# Contact

Riccardo Porreca
Senior Solutions Consultant
E-Mail: riccardo.porreca@mirai-solutions.com
Mobile: +41 (0)76 786 10 28

Roland Schmid
Partner, Applications Lead
E-Mail: roland.schmid@mirai-solutions.com
Mobile: +41 (0)79 478 31 82

Mirai Solutions GmbH
Tödistrasse 48
CH-8002 Zurich
Switzerland

info@mirai-solutions.com
[www.mirai-solutions.com](http://www.mirai-solutions.com)