The PE Package
Modeling private equity in the 21st Century

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3. Risk Management Framework (I): Outline
4. Modeling PE Fund Dynamics
5. Risk Management Framework (II): Risk Measures
6. Fund Structure & Fees
7. Extensive List of To-Dos
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Private equity: Why bother?

- Private equity ("PE") investments continue to increase within institutional portfolios:

  - Investors are looking for diversification (relative to traditional investments)
  - Investors are looking for yield

Total assets under management in PE now exceed USD 3.0 trillion globally.

Despite the apparent importance of PE as an asset class, industry-wide understanding of how originators and investors alike can measure the risks of investing in PE remains limited, modeling is primitive by quantitative-finance standards, and investors have no way to gauge the cost of fees other than to use rules of thumb from historical data, if available.

Objectives:

1. Outline the first comprehensive risk-management framework for private equity fund investments.
2. Describe the underlying stochastic model for the dynamics of PE funds: We introduce a continuous-time model for cash-flow and value dynamics.
3. Describe the structure of fixed and variable fees within an equilibrium valuation framework and evaluate their impact on PE fund performance.
4. Make this model Open Source: We want this framework to become the standard model used by investors for their PE positions.

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What is the structure of private equity?

**Figure:** Partnership structure of private-equity funds: the General Partner ("GP") is the investment manager for the Limited Partners ("LP") who invest in the GP’s fund(s).

- **GP**
- **Fund I**
- **Fund II**
- **Fund VI**
- **LP 1**
- **LP 2**
- **LP 3**
- **LP N**
What is a PE fund’s life cycle? (I)

1. GP forms a new fund

2. GP raises capital from LPs

3. LP commits $C_0$ in capital for $T_L$

4. GP draws on each LP's $C_0$ for $T_I$, where $I \leq L$

5. GP invests in portfolio companies throughout $T_I$

6. GP harvests investments at any time $0 < t \leq T_L$

7. GP exacts fees from LPs' committed capital (some fixed, some variable)

8. GP distributes proceeds according to the fund's waterfall

9. GP fully liquidates the fund at some time $0 \leq t \leq T_L$
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![Graph of PE Capital Drawdowns](image)

- Capital distributions, or returns
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![Private Manager Screening Process](image-url)
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\begin{align*}
\Delta D_t &= \delta_t (C_0 - D_t) \\
\Delta R_t &= \nu_t V_t (1 + G_t) \\
\nu_t &= \max(Y_t, (t_L - B)) \\
\Delta V_t &= V_t G + \Delta D_t - \Delta R_t,
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The drawdown rate, \(\delta_t\), is provided by the user, as are \(G\) (the exogenous growth rate), \(Y\) (the exogenous yield), and \(B\) (a “bowing factor” to control the rate of distribution).

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Yale Endowment Model [25]

Illiquid Alternative Asset Fund Modeling

Dean Takahashi
Senior Director, Yale University Investments Office

Seth Alexander
Associate Director, Yale University Investments Office
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The above are the culmination of work described in Axel’s prior publications in the field of PE modeling [16, 20, 9, 10, 11, 23, 19, 7, 6, 8, 22, 1, 5, 17, 21, 4, 2, 18, 3, 15, 14, 13].
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The specifics of PE present a challenge

PE funds have (at least) two key features that make risk management challenging:

1. **PE investments are long-term and illiquid:**
   - Fund lifetimes: $10 \leq T_L \leq 14$ years
   - Secondary markets for PE positions are highly inefficient

2. **PE investments exhibit idiosyncratic dynamics:**
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The goal of the RM paper [12] is to develop the first comprehensive risk-management framework for PE fund investments that accounts for the idiosyncrasies of PE.
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A risk-management framework for PE fund investments must capture the three principal sources of risk to which PE positions are exposed:

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  The risk of losses in the market prices of the portfolio companies held by a fund exposes investors to market risk.
Main Sources of Risk (I)

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3 Risk Management Framework (I): Outline

4 Modeling PE Fund Dynamics

5 Risk Management Framework (II): Risk Measures

6 Fund Structure & Fees

7 Extensive List of To-Dos

8 References
Let $V_t$ denote the value of the fund at time $t$. 

Assumption

The dynamics of the fund value, $V_t$, under the real-world probability measure $P$, can be described by the stochastic process $\{V_t, 0 \leq t \leq T\}$:

$$dV_t = V_t(\mu_V dt + \beta_V \sigma_M dB_M, t) + \sigma_\epsilon dB_\epsilon, t)$$

where $\mu_V > 0$ is the mean rate of return of the fund, and $\beta_V$ is the market beta of the fund. 

Thomas P. Harte & Axel Buchner

† The PE Package
Fund value

- Let $V_t$ denote the value of the fund at time $t$
- Let $D_t$ denote the cumulative capital drawdowns from the LPs up to time $t$
Fund value

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$B_{M,t}$ is a standard Brownian motion driving aggregate stock market returns, such that $r_{M,t} = \mu_M + \sigma_M dB_{M,t}$, where $\mu_M$ is the mean rate of return of the aggregate stock market (“the market”), and $\sigma_M$ is the returns volatility of the market.
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$B_{\varepsilon,t}$ is a second Brownian motion, representing idiosyncratic shocks to the fund, where $dB_{M,t} \, dB_{\varepsilon,t} = 0$, the mean rate of return of the idiosyncratic shocks is zero, and $\sigma_\varepsilon$ is the volatility of the idiosyncratic shocks.
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Assumption

The dynamics of the fund value, $V_t$, under the real-world probability measure $\mathbb{P}$, can be described by the stochastic process $\{V_t, 0 \leq t \leq T_L\}$:

$$dV_t = V_t(\mu_\nu dt + \beta_\nu \sigma_M dB_{M,t} + \sigma_\epsilon dB_{\epsilon,t}) + dD_t - dR_t,$$

where $\mu_\nu > 0$ is the mean rate of return of the fund, and $\beta_\nu$ is the market beta of the fund
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\[dV_t = V_t(\mu_V dt + \beta_V \sigma_M dB_{M,t} + \sigma_{\epsilon} dB_{\epsilon,t}) + dD_t - dR_t,\]

(1)

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Let $I_0$ be the capital available for investment, \( i.e. \ C_0 \) less fees. For simplicity we can at first assume that $I_0 = C_0$
Capital drawdowns

- Let $I_0$ be the capital available for investment, *i.e.* $C_0$ less fees. For simplicity we can at first assume that $I_0 = C_0$

**Assumption**

The dynamics of the cumulative capital drawdowns, $D_t$, can be described by the ordinary differential equation:

$$\begin{align*}
\text{d}D_t &= \delta_t(I_0 - D_t) \mathbf{1}_{\{0 \leq t \leq T\}} \text{d}t,
\end{align*}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function. The fund’s drawdown rate $\delta_t$ is assumed to follow a stochastic process $\{\delta_t, 0 \leq t \leq T\}$ given by:

$$\begin{align*}
\delta_t &= \delta + \sigma_{\delta} B_{\delta,t},
\end{align*}$$

where $\delta > 0$ is the mean of the drawdown rate, $\sigma_{\delta} > 0$ is the volatility of the drawdown rate; $B_{\delta,t}$ is a third standard Brownian motion for which it is assumed that $\text{d}B_{\delta,t} \text{d}B_{\delta,t} = \rho_{\delta} \text{d}t$, where $\rho_{\delta}$ is the correlation between drawdown rate and stock market returns, and $\text{d}B_{\delta,t} \text{d}B_{\varepsilon,t} = 0$. In order to avoid negative drawdown rates, we use $\delta_t^+ = \max(\delta_t, 0)$ in the model implementation.
Let $I_0$ be the capital available for investment, \( i.e. \ C_0 \) less fees. For simplicity we can at first assume that $I_0 = C_0$

**Assumption**

The dynamics of the cumulative capital drawdowns, $D_t$, can be described by the ordinary differential equation:

$$dD_t = \delta_t(I_0 - D_t)1_{\{0 \leq t \leq T_I\}}dt,$$

where $1_{\{\cdot\}}$ is an indicator function. The fund’s drawdown rate $\delta_t$ is assumed to follow a stochastic process \{\(\delta_t, 0 \leq t \leq T_I\}\} given by:

$$\delta_t = \delta + \sigma_{\delta}B_{\delta,t},$$

where $\delta > 0$ is the mean of the drawdown rate, $\sigma_{\delta} > 0$ is the volatility of the drawdown rate; $B_{\delta,t}$ is a third standard Brownian motion for which it is assumed that $dB_{\delta,t}dB_{M,t} = \rho_{\delta}dt$, where $\rho_{\delta}$ is the correlation between drawdown rate and stock market returns, and $dB_{\delta,t}dB_{\varepsilon,t} = 0$. In order to avoid negative drawdown rates, we use $\delta_t^+ = \max(\delta_t, 0)$ in the model implementation.
Capital distributions

Assumption

The dynamics of the cumulative capital distributions, $R_t$, can be described by:

$$dR_t = \nu_t V_t dt, \quad \text{for } t < T_L,$$

and

$$R_t = V_t 1_{\{t=T_L\}} + \int_0^t \nu_u V_u du, \quad \text{for } t \leq T_L \quad (4)$$

The fund’s distribution rate $\nu_t$ is assumed to follow a stochastic process $\{\nu_t, \ 0 \leq t \leq T_L\}$ given by:

$$\nu_t = \nu t + \sigma_{\nu} B_{\nu,t}, \quad (5)$$

where $\nu$ is the mean distribution rate, and $\sigma_{\nu} > 0$ is the volatility of the distribution rate; $B_{\nu,t}$ is a fourth standard Brownian motion for which it is assumed that $dB_{\nu,t} dB_{M,t} = \rho_{\nu} dt$, where $\rho_{\nu}$ is the correlation between the drawdown rate and stock market returns, and $dB_{\nu,t} dB_{\varepsilon,t} = 0$. In order to avoid negative distributions rates, we use $\nu^+_t = \max(\nu_t, 0)$ in the model implementation.
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Market risk: Value at Risk (“VaR”)

- VaR is always forward-looking: VaR is a forecast of the uncertainty in the P&L of a portfolio at the end of the holding period. If we let \( d_{t,h} \) be the discount factor with term \( t \) and tenor \( h \) and let \( P_t \) be the PE investor’s position at time \( t \), then the discounted forecast of the P&L at time \( t + h \) in present-value terms is:

\[
P&L_{t+h} = d_{t,h}P_{t+h} - P_t
\]
Market risk: Value at Risk ("VaR")

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- The dynamics of the PE investor’s position, \( P_t \), at time \( t \) are given by:

\[
dP_t = dV_t + dC_t
\]

\[
= V_t(\mu_V dt + \beta_V \sigma_M dB_{M,t} + \sigma_e dB_{\epsilon,t}) + dD_t - dR_t + C_t r_c dt - dD_t + dR_t
\]

\[
= V_t(\mu_V dt + \beta_V \sigma_M dB_{M,t} + \sigma_e dB_{\epsilon,t}) + C_t r_c dt \tag{6}
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(6)

(We’ll give a definition of the dynamics of the investor’s cash position shortly when we define CFaR)
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$$= V_t(\mu_V dt + \beta_V \sigma_M dB_{M,t} + \sigma_\varepsilon dB_{\varepsilon,t}) + C_t r_c dt$$

(6)

- The portfolio VaR at any time $t$, which we will denote by $\text{VaR}_{t,h}^{\alpha,\$}$ when expressed in dollar terms for a significance level $\alpha \in [0, 1]$ and a holding period $h$, is defined as:

$$\text{Pr}(P&L_{t+h} < q_h^{\alpha,\$}) = \alpha \iff \text{VaR}_{t,h}^{\alpha,\$} = -q_h^{\alpha,\$}$$
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$$\Pr(\text{P&L}_{t+h} < q_h^{\alpha,\$}) = \alpha \iff \text{VaR}_{t,h}^{\alpha,\$} = -q_h^{\alpha,\$}$$
Liquidity risk: Liquidity Adjusted Value at Risk ("LVaR")

Figure: Median Discount (+) / Premium (-) to fund NAVs by fund type, 2004–2013.

Source: Preqin Secondary Market Monitor
Liquidity risk: Liquidity Adjusted Value at Risk ("LVaR")

- The point of LVaR is to incorporate the secondary-market discount rate as an exogenous liquidity risk in the calculation of VaR.
Liquidity risk: Liquidity Adjusted Value at Risk ("LVaR")

- The point of LVaR is to incorporate the secondary-market discount rate as an exogenous liquidity risk in the calculation of VaR

Assumption

The dynamics of the secondary-market discount rate $\pi_t$ are assumed to follow a stochastic process given by:

$$d\pi_t = \kappa_\pi (\theta_\pi - \pi_t)dt + \sigma_\pi dB_{\pi,t},$$

where $\theta_\pi > 0$ is the long-run mean of the discount rate, $\kappa_\pi > 0$ is the rate of reversion to this mean, and $\sigma_\pi > 0$ reflects the volatility of the discount rate. $B_{\pi,t}$ is a fifth standard Brownian motion for which it is assumed that $dB_{\pi,t}dB_{M,t} = \rho_\pi dt$, where $\rho_\pi$ is the correlation between drawdown rate and stock market returns, and $dB_{\pi,t}dB_{\varepsilon,t} = 0$. 

$$P&L(L_{t+h}) = (1 - \pi_{t+h})V_{t+h} + C_{t+h} - P_t,$$

with $\pi_{t+h}$ being the forecast of the secondary-market discount for the fund at time $t+h$. 

\[8\]
The point of LVaR is to incorporate the secondary-market discount rate as an exogenous liquidity risk in the calculation of VaR.

**Assumption**

The dynamics of the secondary-market discount rate \( \pi_t \) are assumed to follow a stochastic process given by:

\[
d\pi_t = \kappa_\pi (\theta_\pi - \pi_t) dt + \sigma_\pi dB_{\pi,t},
\]

where \( \theta_\pi > 0 \) is the long-run mean of the discount rate, \( \kappa_\pi > 0 \) is the rate of reversion to this mean, and \( \sigma_\pi > 0 \) reflects the volatility of the discount rate. \( B_{\pi,t} \) is a fifth standard Brownian motion for which it is assumed that \( dB_{\pi,t}dB_{M,t} = \rho_\pi dt \), where \( \rho_\pi \) is the correlation between drawdown rate and stock market returns, and \( dB_{\pi,t}dB_{\varepsilon,t} = 0 \).

The LVaR\(_{t,h}^{\alpha,\$} \) is defined by:

\[
\Pr(P&L^{(L)}_{t+h} < q^{(L),\alpha,\$}_h) = \alpha \iff LVaR_{t,h}^{\alpha,\$} = -q^{(L),\alpha,\$}_h
\]

where \( P&L^{(L)}_{t+h} \) is the liquidity-adjusted P&L forecast of the investor’s position in the fund for time \( t + h \):

\[
P&L^{(L)}_{t+h} = ((1 - \pi_{t+h})V_{t+h} + C_{t+h}) - P_t,
\]

with \( \pi_{t+h} \) being the forecast of the secondary-market discount for the fund at time \( t + h \).
Liquidity risk: Liquidity Adjusted Value at Risk ("LVaR")

- The point of LVaR is to incorporate the secondary-market discount rate as an exogenous liquidity risk in the calculation of VaR.

**Assumption**

The dynamics of the secondary-market discount rate $\pi_t$ are assumed to follow a stochastic process given by:

$$d\pi_t = \kappa_\pi (\theta_\pi - \pi_t)dt + \sigma_\pi dB_{\pi,t}, \quad (7)$$

where $\theta_\pi > 0$ is the long-run mean of the discount rate, $\kappa_\pi > 0$ is the rate of reversion to this mean, and $\sigma_\pi > 0$ reflects the volatility of the discount rate. $B_{\pi,t}$ is a fifth standard Brownian motion for which it is assumed that $dB_{\pi,t} dB_{M,t} = \rho_\pi dt$, where $\rho_\pi$ is the correlation between drawdown rate and stock market returns, and $dB_{\pi,t} dB_{\epsilon,t} = 0$.

- The LVaR$_{t,h}^{\alpha,\$}$ is defined by:

$$\Pr\left(P&L_{t+h}^{(L)} < q_{h}^{(L),\alpha,\$}\right) = \alpha \iff \text{LVaR}_{t,h}^{\alpha,\$} = -q_{h}^{(L),\alpha,\$} \quad (8)$$

where $P&L_{t+h}^{(L)}$ is the liquidity-adjusted P&L forecast of the investor’s position in the fund for time $t + h$:

$$P&L_{t+h}^{(L)} = ((1 - \pi_{t+h})V_{t+h} + C_{t+h}) - P_t, \quad (9)$$

with $\pi_{t+h}$ being the forecast of the secondary-market discount for the fund at time $t + h$. 
The risk measure CFaR is defined as the change (or possibly loss) in the investor’s cash position, $C_t$, which is exceeded with some given probability $\alpha$, over a given time horizon $h$. 

\[ CFaR_{\alpha, t, h} = -q(C_t, \alpha, h) \]
The risk measure CFaR is defined as the change (or possibly loss) in the investor’s cash position, $C_t$, which is exceeded with some given probability $\alpha$, over a given time horizon $h$.

**Assumption**

The dynamics of the investor’s cash position are given by:

$$dC_t = C_t r_c dt - dD_t + dR_t$$

**Note:** $r_c$ is the rate of return on cash.
Cash-flow risk: Cash Flow at Risk ("CFaR")

- The risk measure CFaR is defined as the change (or possibly loss) in the investor’s cash position, \( C_t \), which is exceeded with some given probability \( \alpha \), over a given time horizon \( h \).

Assumption

*The dynamics of the investor’s cash position are given by:*

\[
dC_t = C_t r_c dt - dD_t + dR_t
\]

*(10)*

*where \( r_c \) is rate of return on cash*

- The CFaR\(_{\alpha, t, h}^{\alpha, \$}\) is defined by:

\[
\Pr\left( C_{t+h} - C_t < q_{\alpha, t, h}^{(C, \alpha, \$)} \right) = \alpha \iff \text{CFaR}_{\alpha, t, h}^{\alpha, \$} = -q_{\alpha, t, h}^{(C, \alpha, \$)}
\]

*(11)*
The risk measure CFaR is defined as the change (or possibly loss) in the investor’s cash position, $C_t$, which is exceeded with some given probability $\alpha$, over a given time horizon $h$.

**Assumption**

*The dynamics of the investor’s cash position are given by:*

$$dC_t = C_tr_c dt - dD_t + dR_t$$  \hspace{1cm} (10)

*where $r_c$ is rate of return on cash*

The CFaR$^\alpha_\$ t,h is defined by:

$$\Pr(C_{t+h} - C_t < q^{(C),\alpha,\$}_h) = \alpha \iff \text{CFaR}_t^{\alpha,\$} = -q^{(C),\alpha,\$}_h$$  \hspace{1cm} (11)
Cash-flow risk: Cash Flow at Risk ("CFaR")

- The risk measure CFaR is defined as the change (or possibly loss) in the investor’s cash position, \( C_t \), which is exceeded with some given probability \( \alpha \), over a given time horizon \( h \).

**Assumption**

*The dynamics of the investor’s cash position are given by:*

\[
dC_t = C_t r_c dt - dD_t + dR_t
\]  

where \( r_c \) is rate of return on cash.

- The CFaR\(_{t,h}^{\alpha,\$}\) is defined by:

\[
Pr\left( C_{t+h} - C_t < q^{(C),\alpha,\$}_h \right) = \alpha \iff CFaR_{t,h}^{\alpha,\$} = -q^{(C),\alpha,\$}_h
\]  

\( \text{(11)} \)
**Calibrated model parameters**

**Table**: Summary of baseline parameters used in illustration of risk-management model

Note: All model parameters are stated as annualized units, except where indicated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life of the PE fund investment (years)</td>
<td>$T_L$</td>
<td>12</td>
</tr>
<tr>
<td>Simulation frequency (years)</td>
<td>$dt$</td>
<td>1/4</td>
</tr>
<tr>
<td>Committed capital (US dollars)</td>
<td>$C_0$</td>
<td>100</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r_f$</td>
<td>0.05</td>
</tr>
<tr>
<td>Return on cash positions</td>
<td>$r_c$</td>
<td>0</td>
</tr>
<tr>
<td>Expected return of stock market</td>
<td>$\mu_M$</td>
<td>0.11</td>
</tr>
<tr>
<td>Volatility of stock market returns</td>
<td>$\sigma_M$</td>
<td>0.15</td>
</tr>
<tr>
<td>Alpha of PE funds</td>
<td>$\alpha$</td>
<td>0.04</td>
</tr>
<tr>
<td>Market beta of PE funds</td>
<td>$\beta_M$</td>
<td>1.30</td>
</tr>
<tr>
<td>Idiosyncratic volatility of PE fund returns</td>
<td>$\sigma_\varepsilon$</td>
<td>0.35</td>
</tr>
<tr>
<td>Drawdown rate of PE funds</td>
<td>$\delta$</td>
<td>0.41</td>
</tr>
<tr>
<td>Volatility of the drawdown rate</td>
<td>$\sigma_\delta$</td>
<td>0.21</td>
</tr>
<tr>
<td>Correlation between drawdown rate and stock market returns</td>
<td>$\rho_\delta$</td>
<td>0.50</td>
</tr>
<tr>
<td>Average distribution rate</td>
<td>$\nu$</td>
<td>0.08</td>
</tr>
<tr>
<td>Volatility of the distribution rate</td>
<td>$\sigma_\nu$</td>
<td>0.11</td>
</tr>
<tr>
<td>Correlation between distribution rate and stock market returns</td>
<td>$\rho_\nu$</td>
<td>0.80</td>
</tr>
<tr>
<td>Long-run mean of secondary market discounts</td>
<td>$\theta_\pi$</td>
<td>0.16</td>
</tr>
<tr>
<td>Mean-reversion speed of secondary market discounts</td>
<td>$\kappa_\pi$</td>
<td>0.42</td>
</tr>
<tr>
<td>Volatility of secondary market discounts</td>
<td>$\sigma_\pi$</td>
<td>0.16</td>
</tr>
<tr>
<td>Initial secondary market discount</td>
<td>$\pi_0$</td>
<td>$\theta_\pi$</td>
</tr>
<tr>
<td>Correlation between discount rate and stock market returns</td>
<td>$\rho_\pi$</td>
<td>-0.60</td>
</tr>
</tbody>
</table>
It’s Monte Carlo time...
Figure: Cumulative capital drawdowns (left) and cumulative capital distributions (right). Solid lines represent Monte Carlo estimates of the average and dotted lines represent the 10th & 90th quantiles over 500,000 simulations.
**Figure:** Fund values (left) and cumulative net fund cash flows (right). Solid lines represent Monte Carlo estimates of the average and dotted lines represent the 10th & 90th quantiles over 500,000 simulations.
Figure: VaR dynamics over the fund lifecycle: (left) VaR at fund initiation, $\text{VaR}^{\alpha,0}_{0,h}$, plotted as a function of the time horizon $h$; (right) quarterly VaR, i.e., $\text{VaR}^{\alpha,0.25}_{t,0.25}$, plotted as a function of time $t$. The thickest line represents the Monte Carlo estimate of the 1% VaR over 500,000 simulations (also shown are the 5% VaR and the 10% VaR).
**Figure:** LVaR dynamics over the fund lifecycle: (left) LVaR at fund initiation, $\text{LVaR}^{\alpha,h}_{0}$, plotted as a function of time horizon $h$; (right) quarterly LVaR, *i.e.* $\text{LVaR}^{\alpha,0.25}_{t}$, plotted as a function of time $t$. The thickest line represents the Monte Carlo estimate of the 1% LVaR over 500,000 simulations (also shown are the 5% LVaR and the 10% LVaR)
Figure: CFaR dynamics over the fund lifecycle: (left) CFaR at fund initiation, $\text{CFaR}_{0,h}^{\alpha,\$}$, plotted as a function of time horizon $h$; (right) quarterly CFaR, i.e., $\text{CFaR}_{t,0.25}^{\alpha,\$}$, plotted as a function of time $t$. The thickest line represents the Monte Carlo estimate of the 1% CFaR over 500,000 simulations (also shown are the 5% CFaR and the 10% CFaR).
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Management fees

- The management fee is levied against a basis: this is usually either the committed capital, $C_0$, or the net invested capital (“NIC”), and it is one of four different types that is specified in the limited partnership agreement (“LPA”):

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1. flat fee
2. tapered fee: tapers after the investment period, $T_I < t \leq T_L$
3. change basis to NIC after investment period with flat fee
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Let $MF_t$ denote the cumulative management fees up to some time $t \in [0, T_L]$.

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**Fixed Management Fees:** If management fees are defined as a percentage $c_{MF}$ of the committed capital $C_0$ and are paid continuously, the dynamics are given by:

$$dMF_t = c_{MF}C_0dt$$ (12)

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**Management Fees with Change in Basis:** Latterly, tapered management fees appear to be gaining in popularity. The tapering typically begins after the investment period, i.e. for $T_I < t \leq T_L$, and reflects the fact that less time is required by the GP in managing the activities of the portfolio companies. Many funds change the fee basis from committed capital (during the commitment period) to NIC capital (after the commitment period).

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  3. change basis to NIC after investment period with flat fee\(^5\)
  4. change basis to NIC after investment period with tapered fee

- Let $MF_t$ denote the cumulative management fees up to some time $t \in [0, T_L]$.

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\(^4\) Invested capital minus the cost basis of exited investments, \textit{ibid.} p. 2315, \textit{et seq.}

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Management fees: basis change to NIC requires *ex ante* computation

- If *ab initio* the basis for management-fee calculation is agreed to change from committed capital, $C_0$, for $0 \leq t \leq T_I$, to NIC for $T_I < t \leq T_L$, then how do GPs determine $I_C$, the capital available for investment, for $t \leq T_I$? Is it specified in the LPA?

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- We use an iterative algorithm to arrive at the NIC (convergence is rapid):

\[ \begin{align*}
1 & \text{ Set the initial guess for NIC to } C_0 \\
2 & \text{ Subtract the fixed management fees applicable for } t \leq T_I, \text{ which we know at } t = 0 \text{ to follow } \\
3 & \text{ the value of NIC for } t = T_I \text{ is then initialized to } C_0 - MF_{T_I} \\
4 & \text{ The dynamics of management fees for } T_I < t \leq T_L \text{ are assumed to follow: } \\
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$$dMF_t = c_{MF} C_0 dt \; 1_{0 \leq t \leq T_I}$$

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the value of NIC$_t$ for $t = T_I$ is then initialized to $C_0 - MF_{T_I}$

3. The dynamics of management fees for $T_I < t \leq T_L$ are assumed to follow:

$$dMF_t = c_{MF}NIC_t dt 1_{T_I < t \leq T_L}$$

(14)

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   \[ \text{dMF}_t = c_{MF} C_0 \text{d}t \quad 0 \leq t \leq T_I \]  
   (13)
   
   the value of NIC$_t$ for $t = T_I$ is then initialized to $C_0 - \text{MF}_{T_I}$
3. The dynamics of management fees for $T_I < t \leq T_L$ are assumed to follow:
   \[ \text{dMF}_t = c_{MF} \text{NIC}_t \text{d}t \quad 1 \leq t \leq T_L \]  
   (14)

4. The fund’s distribution rate, $\nu_t$, is assumed to follow a stochastic process $\{\nu_t, \ 0 \leq t \leq T_L\}$ given by $\nu_t = \nu t + \sigma_\nu B_{\nu,t}$, as per Equation 5, and this rate is applied to the NIC to give its dynamics as:
   \[ \text{dNIC}_t = \nu_t \text{NIC}_t \text{d}t \]  
   (15)

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Management fees: basis change to NIC requires *ex ante* computation

- If *ab initio* the basis for management-fee calculation is agreed to change from committed capital, \( C_0 \), for \( 0 \leq t \leq T_I \), to NIC for \( T_I < t \leq T_L \), then how do GPs determine \( I_C \), the capital available for investment, for \( t \leq T_I \)? Is it specified in the LPA?

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     dMF_t = c_MF C_0 dt \quad 0 \leq t \leq T_I
     \] (13)
     the value of NIC\( _t \) for \( t = T_I \) is then initialized to \( C_0 - MF_{T_I} \)
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     \[
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- The fund’s distribution rate, \( \nu_t \), is assumed to follow a stochastic process \( \{\nu_t, 0 \leq t \leq T_L\} \) given by \( \nu_t = \nu t + \sigma_{\nu} B_{\nu,t} \), as per Equation 5, and this rate is applied to the NIC to give its dynamics as:
  \[
  dNIC_t = \nu_t NIC_t dt
  \] (15)

- Finally, we can solve for the invested capital \( I_C \), by noting\(^6\) that at \( t = 0 \) it must be the case that \( I_C = C_0 - NPV(MF_{T_I}) - NPV(MF_{T_L}) \), where the last term can be expressed as \( x \times I_C \) for some fraction \( x \)

\(^6\)As Metrick & Yasuda suggest, *ibid.* p. 2309, *et seq.*
Management fees: basis change to NIC requires *ex ante* computation

- If *ab initio* the basis for management-fee calculation is agreed to change from committed capital, $C_0$, for $0 \leq t \leq T_I$, to NIC for $T_I < t \leq T_L$, then how do GPs determine $I_C$, the capital available for investment, for $t \leq T_I$? Is it specified in the LPA?

- We use an iterative algorithm to arrive at the NIC (convergence is rapid):
  1. Set the initial guess for NIC to $C_0$
  2. Subtract the fixed management fees applicable for $t \leq T_I$, which we know at $t = 0$ to follow

$$dMF_t = c_{MF}C_0 dt\ 1_{0\leq t \leq T_I}$$

(13)

the value of NIC$_t$ for $t = T_I$ is then initialized to $C_0 - MF_{T_I}$

3. The dynamics of management fees for $T_I < t \leq T_L$ are assumed to follow:

$$dMF_t = c_{MF}NIC_t dt\ 1_{T_I < t \leq T_L}$$

(14)

4. The fund’s distribution rate, $\nu_t$, is assumed to follow a stochastic process $\{\nu_t,\ 0 \leq t \leq T_L\}$ given by $\nu_t = \nu t + \sigma_\nu B_{\nu,t}$, as per Equation 5, and this rate is applied to the NIC to give its dynamics as:

$$dNIC_t = \nu_t NIC_t dt$$

(15)

5. Finally, we can solve for the invested capital $I_C$, by noting\(^6\) that at $t = 0$ it must be the case that $I_C = C_0 - NPV(MF_{T_I}) - NPV(MF_{T_L})$, where the last term can be expressed as $x \times I_C$ for some fraction $x$

---

\(^6\) As Metrick & Yasuda suggest, *ibid.* p. 2309, *et seq.*
Let $CI_t$ denote the cumulative carried interest up to some time $t \in [0, T_L]$. 

Carried interest (I)

- Let $CI_t$ denote the cumulative carried interest up to some time $t \in [0, T_L]$. 

Catch-up provision: Most LPAs that contain a hurdle rate also include a provision that provides the GPs with a greater share of the profits once the hurdle rate has been paid and until the carry level has been reached.
Let $CI_t$ denote the cumulative carried interest up to some time $t \in [0, T_L]$.

**Carried Interest:** Let the carried interest level be given by $c_{CI}$ and let $h$ denote the hurdle rate. The dynamics of carried interest are given by:

$$dCI_t = c_{CI} \max\left\{dR_t - dD_t - dMF_t, \ 0\right\} \mathbf{1}_{\{\text{IRR}_t > h\}}$$

where $\mathbf{1}_{\{\text{IRR}_t > h\}}$ indicates that carried interest is only payable at time $t$ if the internal rate of return of the fund at $t$, $\text{IRR}_t$, exceeds the hurdle rate $h$. 

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$$dCI_t = c_{CI} \max \left( \underbrace{dR_t - dD_t - dMF_t}_{\text{net cash flow}} , \ 0 \right) 1_{\{\text{IRR}_t > h\}}$$

where $1_{\{\text{IRR}_t > h\}}$ indicates that carried interest is only payable at time $t$ if the internal rate of return of the fund at $t$, $\text{IRR}_t$, exceeds the hurdle rate $h$

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where $1_{\{\text{IRR}_t > h\}}$ indicates that carried interest is only payable at time $t$ if the internal rate of return of the fund at $t$, $\text{IRR}_t$, exceeds the hurdle rate $h$

Catch-up provision: Most LPAs that contain a hurdle rate also include a provision that provides the GPs with a greater share of the profits once the hurdle rate has been paid and until the carry level has been reached
Carried Interest (II)

- **Carried interest with catch-up**: If the carried interest is paid with a 100% catch-up provision, then its dynamics are given by:

\[
\begin{align*}
\text{dCI}_t &= \begin{cases} 
  c_{\text{CI}} \max \{d\text{NCF}_t, 0\} 1_{\{\text{IRR}_t > h\}}, & \text{if } \frac{\text{CI}_t}{R_t - C_0} = c_{\text{CI}} \\
  \min \{c_{\text{CI}}(R_t - C_0) - \text{CI}_t, d\text{NCF}_t\} 1_{\{\text{IRR}_t > h\}}, & \text{if } \frac{\text{CI}_t}{R_t - C_0} < c_{\text{CI}}
\end{cases}
\end{align*}
\]

where \(d\text{NCF}_t = dR_t - dD_t - d\text{MF}_t\)

\[c_{\text{CI}} \geq \frac{1}{R_t - C_0}\]
Table: Carried Interest Calculation

This table illustrates the carried interest calculation for a $100M fund with a carried interest level of 20 percent, a hurdle rate of 8 percent, and a lifetime of ten years. The calculation is shown for a fund with no catch-up clause and fund with a catch-up clause of 100 percent.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>40</td>
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<td>150</td>
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<tr>
<td>IRR (in % p.a.)</td>
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<td>-100</td>
<td>-100</td>
<td>-100</td>
<td>-33</td>
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<td>Carried Interest (No Catch-Up)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>8</td>
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<td>22</td>
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<tr>
<td>Carried Interest (With Catch-Up)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>8</td>
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Additional compensation may come from the GP charging transaction fees and monitoring fees, most commonly in Leveraged Buyout strategies.
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Let $TF_t$ denote the cumulative transaction fees paid up to time $t \in [0, T_l]$ and assume that transaction fees are fully paid at entry (purchase) as a fraction $c_{TF}$ of the deal size.
Additional compensation may come from the GP charging **transaction fees** and **monitoring fees**, most commonly in Leveraged Buyout strategies.

Let $TF_t$ denote the cumulative transaction fees paid up to time $t \in [0, T]$ and assume that transaction fees are fully paid at entry (purchase) as a fraction $c_{TF}$ of the deal size.

If $l$ denotes the average leverage ratio applied, the **dynamics of the transaction fees** can be represented by:

$$dTF_t = c_{TF}dD_t \times (1 + l)$$  (17)
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$$d\text{TF}_t = c_{\text{TF}}dD_t \times (1 + l)$$ \hspace{1cm} (17)

The typical profit-sharing rule between the GP and LPs for transaction fees is that they split the proceeds 50/50, *i.e.* $d\text{TF}_{t}^{(LP)} = d\text{TF}_{t}^{(GP)} = 0.5 \times d\text{TF}_t$.
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\[
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The typical profit-sharing rule between the GP and LPs for transaction fees is that they split the proceeds 50/50, i.e. \(dTF_t^{(LP)} = dTF_t^{(GP)} = 0.5 \times dTF_t\).
Let MoF$_t$ denote the cumulative monitoring fees paid up to time $t \in [0, T_L]$ and assume that monitoring fees are paid at exit as a fraction $c_{MoF}$ of the total firm value.
Let \( \text{MoF}_t \) denote the cumulative monitoring fees paid up to time \( t \in [0, T_L] \) and assume that monitoring fees are paid at exit as a fraction \( c_{\text{MoF}} \) of the total firm value.

If \( s_F \) denotes the (average) share the fund holds in its portfolio companies, the dynamics of the monitoring fees can be modeled by:

\[
d\text{MoF}_t = c_{\text{MoF}} dR_t \times \left( \frac{1 + l}{s_F} \right)
\]  

(18)
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\]  

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We use the typical sharing rule and allocate 20% of the monitoring fees to the GP and 80% to the LPs, *i.e.* \( d\text{MoF}^{(LP)}_t = 0.8 \times d\text{MoF}_t \) and \( d\text{MoF}^{(GP)}_t = 0.2 \times d\text{MoF}_t \).
Let $\text{MoF}_t$ denote the cumulative monitoring fees paid up to time $t \in [0, T_L]$ and assume that monitoring fees are paid at exit as a fraction $c_{\text{MoF}}$ of the total firm value.

If $s_F$ denotes the (average) share the fund holds in its portfolio companies, the dynamics of the monitoring fees can be modeled by:

$$d\text{MoF}_t = c_{\text{MoF}}dR_t \times \left(1 + \frac{l}{s_F}\right)$$

(18)

We use the typical sharing rule and allocate 20% of the monitoring fees to the GP and 80% to the LPs, i.e. $d\text{MoF}_t^{(LP)} = 0.8 \times d\text{MoF}_t$ and $d\text{MoF}_t^{(GP)} = 0.2 \times d\text{MoF}_t$. 
We assume an equilibrium framework in which LPs’ expected excess returns (net of fees) equal zero, such that GPs capture all rents from managing the funds:

\[
E_Q \left[ \int_0^{T_1} e^{-rf_u} \left( dR_u - dD_u - dMF_u - dCI_u + dPF_{uLP} \right) \right] = 0
\]
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$$E_Q \left[ \int_0^{T_1} e^{-rf} \left( dR_u - dD_u - dMF_u - dCI_u + dPF^{LP}_u \right) \right] = 0$$

We solve the equilibrium condition for the (ex ante) expected rate of return $\mu^*_V$ by using Monte Carlo simulations.
We assume an equilibrium framework in which LPs’ expected excess returns (net of fees) equal zero, such that GPs capture all rents from managing the funds:

\[ E_Q \left[ \int_0^{T_f} e^{-rfu} (dR_u - dD_u - dMF_u - dCI_u + dPF_{LP}^u) \right] = 0 \]

We solve the equilibrium condition for the \((ex \ ante)\) expected rate of return \(\mu^*_V\) by using Monte Carlo simulations.

Using this result, we compute the gross-of-fees abnormal rate of return \(\alpha\) (the break-even alpha) that the GPs have to generate by:

\[ \alpha = \mu^*_V - \mu_V = \mu^*_V - r_f - \beta_V (\mu_M - r_f) \]
We assume an equilibrium framework in which LPs’ expected excess returns (net of fees) equal zero, such that GPs capture all rents from managing the funds:

\[ \mathbb{E}_Q \left[ \int_0^{T_1} e^{-r_f t} \left( dR_u - dD_u - dMF_u - dCI_u + dPF_{LP} \right) \right] = 0 \]

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- We also extend the framework by allowing LPs to earn a positive out-performance after fees.
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\[
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\]

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\]

We also extend the framework by allowing LPs to earn a positive out-performance after fees.
Theorem (Fee Value): Applying a risk-neutral valuation approach, the arbitrage-free present value of the fund fees $V_0^{(GP)}$ is given by:

$$V_0^{(GP)} = E_Q \left[ \int_0^{T_L} e^{-r_f u} dMF_u \right] + E_Q \left[ \int_0^{T_L} e^{-r_f u} dCI_u \right] + E_Q \left[ \int_0^{T_L} e^{-r_f u} dPF_u^{(GP)} \right],$$

(19)

where $V_0^{(MF)}$ is the present value of management fees, $V_0^{(CI)}$ is the present value of carried interest payments, and $V_0^{(PF)}$ is the present value of lifetime portfolio company fees received by the GPs.
Fee valuation: single fund

- **Theorem (Fee Value):** Applying a risk-neutral valuation approach, the arbitrage-free present value of the fund fees $V^{(GP)}_0$ is given by:

$$V^{(GP)}_0 = E_Q \left[ \int_0^{T_L} e^{-r_f u} dMF_u \right] + E_Q \left[ \int_0^{T_L} e^{-r_f u} dCI_u \right] + E_Q \left[ \int_0^{T_L} e^{-r_f u} dPF^{(GP)}_u \right],$$

where $V^{(MF)}_0$ is the present value of management fees, $V^{(CI)}_0$ is the present value of carried interest payments, and $V^{(PF)}_0$ is the present value of lifetime portfolio company fees received by the GPs.

- We use Monte Carlo simulations to solve for the present values.
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where $V_{0}^{(MF)}$ is the present value of management fees, $V_{0}^{(CI)}$ is the present value of carried interest payments, and $V_{0}^{(PF)}$ is the present value of lifetime portfolio company fees received by the GPs.

We use Monte Carlo simulations to solve for the present values.
Death & taxes?

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<td>$15.86</td>
<td>$15.86</td>
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<td>(MF)</td>
<td></td>
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</tr>
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<td>(MoF 95%)</td>
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**Figure:** Net present value of a fund’s fees. Management fee is denoted by “MF”, carried interest by “CI”, transaction fees by “TF” and monitoring fees by “MoF” (the latter pair being most common in Leveraged Buyout strategies). The means are shown in large font, while the values in parenthesis are either the standard deviations of the means, or the quantiles of the Monte Carlo distributions, as indicated by the quantile and the % sign.
### Figure: Net present value of management fee (denoted by “MF”)
### Figure: Net present value of carried interest (denoted by “CI”)
**Figure:** Net present value of transaction fees (denoted by “TF”) for Leveraged Buyout funds

| MF   |
|------|---------------------------------------------------------------------|
|      | No deal fees                                                        |
|      | Constant fee basis                                                  |
|      | Fee basis change                                                    |
|      | No catch-up             | With catch-up             | No catch-up             | With catch-up             |
| MF   | $15.86                  | $15.86                    | $11.43                  | $11.43                    |
| (MF) |                        |                          | $0.40                   | $0.39                     |
| CI   | $3.88                   | $4.23                    | $4.44                   | $4.82                     |
| (CI) |                        |                          | $0.49                   | $0.49                     |
| (CI 5%) |                     |                          |                        |                           |
| (CI 10%) |                   |                          |                        |                           |
| (CI 20%) |                   |                          |                        |                           |
| (CI 30%) |                   |                          |                        |                           |
| (CI 50%) |                   |                          |                        |                           |
| (CI 60%) |                   |                          |                        |                           |
| (CI 70%) |                   |                          |                        |                           |
| (CI 75%) |                   |                          |                        |                           |
| (CI 80%) |                   |                          |                        |                           |
| (CI 85%) |                   |                          |                        |                           |
| (CI 90%) |                   |                          |                        |                           |
| (CI 95%) |                   |                          |                        |                           |

| TF   |
|------|---------------------------------------------------------------------|
|      | With deal fees                                                      |
|      | Constant fee basis                                                  |
|      | Fee basis change                                                    |
|      | No catch-up             | With catch-up             | No catch-up             | With catch-up             |
| TF   | $15.86                  | $15.86                    | $11.43                  | $11.43                    |
| (TF) |                        |                          | $0.39                   | $0.39                     |
| MoF  | $7.30                   | $7.30                    | $7.71                   | $7.70                     |
| (MoF)|                        |                          | $3.96                   | $3.98                     |
| (MoF 5%) |                   |                          |                        |                           |
| (MoF 25%) |                   |                          |                        |                           |
| (MoF 75%) |                   |                          |                        |                           |
| (MoF 95%) |                   |                          |                        |                           |
Death & taxes?

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</tr>
<tr>
<td>(CI50%)</td>
<td></td>
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<tr>
<td>(CI60%)</td>
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<tr>
<td>(CI 65%)</td>
<td></td>
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</tr>
<tr>
<td>(CI 70%)</td>
<td>$2.05</td>
<td>$4.52</td>
<td>$3.09</td>
<td>$5.35</td>
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<tr>
<td>(CI 75%)</td>
<td>$3.88</td>
<td>$5.96</td>
<td>$5.06</td>
<td>$6.87</td>
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<tr>
<td>(CI 80%)</td>
<td>$6.12</td>
<td>$7.73</td>
<td>$7.41</td>
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<tr>
<td>(CI 85%)</td>
<td>$9.01</td>
<td>$10.15</td>
<td>$10.44</td>
<td>$11.36</td>
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<tr>
<td>(CI 90%)</td>
<td>$13.15</td>
<td>$13.70</td>
<td>$14.82</td>
<td>$15.10</td>
</tr>
<tr>
<td>(CI 95%)</td>
<td>$20.68</td>
<td>$20.30</td>
<td>$22.59</td>
<td>$21.97</td>
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<td>TF (TF)</td>
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<tr>
<td>MoF (MoF)</td>
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<tr>
<td>(MoF5%)</td>
<td></td>
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<tr>
<td>(MoF25%)</td>
<td></td>
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</tr>
<tr>
<td>(MoF75%)</td>
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<tr>
<td>(MoF95%)</td>
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</tbody>
</table>

**Figure:** Net present value of monitoring fees (denoted by “MoF”) for Leveraged Buyout funds
To-do list

- Currently working on parameter estimation from the Preqin data set

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  - Modeling the underlying portfolio companies and aggregating to the fund level

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Watch this space: [https://github.com/tharte/PE](https://github.com/tharte/PE)

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References


