# The PE Package Modeling private equity in the 21<sup>st</sup> Century

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• Equity invested in non-quoted companies

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• Investments structured as convertible debt

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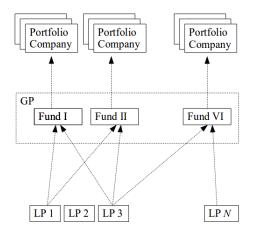


Figure: Partnership structure of private-equity funds: the General Partner ("GP") is the investment manager for the Limited Partners ("LP") who invest in the GP's fund(s).

O GP forms a new fund

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#### OP forms a new fund

② GP raises capital from LPs

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#### OP raises capital from LPs

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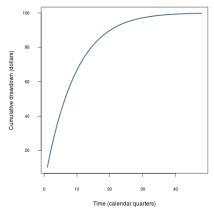
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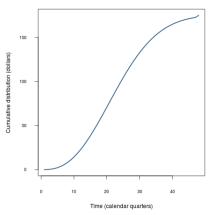
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PE Capital Drawdowns

• Capital distributions, or returns

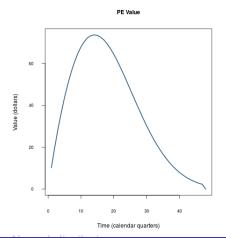
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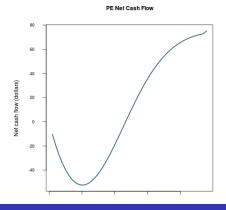
Fund value

PE Capital Distributions

- Capital drawdowns, or calls
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- Fund value



- Capital drawdowns, or calls
- Capital distributions, or returns
- Fund value
- Net cash distribution



• Thomson ONE (formerly "Venture Economics" / "TVE")

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### • Cepres

# What is the state of the art in PE data?

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Market Benchmarks	Co-Investment Multi-Manager	Direct Lending     Direct Lending			
Custom Benchmarks Quarterly Index	Distressed Debt	Fund of Funds     Infrastructure Fund of Funds	Growth		
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#### MODEL INSIGHT - THE BARRA PRIVATE EQUITY MODEL (PEQ2) - AUGUST 2014

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In this Model Insight we preset the latest latest Point Point (style Model VECI), which experiments a major attraction cardinal information of the drivers of mixed materia in global private receiption (style style). The style segment is in the point of the style segment in the Model Compared in the Start Insight and the style segment is in the point of the style segment in the style segment is in the point of the style segment is interested in the many segment deviation point of the style segment is the style segment point is point of the style segment is in the point of the style segment is interested in the many segment deviation point of the style segment point is point of the style segment point is point of the style segment point of the style segment point is point of the style segment point is point of the style segment point of the style segment point is point of the style segment point of the style segment point is point of the style segment point point of the style segment point of the style segment point point of the style segment point p

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### • Yale Endowment Model [25]

### Illiquid Alternative Asset Fund Modeling

Dean Takahashi Senior Director, Yale University Investments Office

Seth Alexander Associate Director, Yale University Investments Office

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$$\Delta D_t = \delta_t (C_0 - D_t)$$
  

$$\Delta R_t = v_t V_t (1 + G)$$
  

$$v_t = \max \left( Y, \left( \frac{t}{L} \right)^B \right)$$
  

$$\Delta V_t = V_t G + \Delta D_t - \Delta R_t$$

The drawdown rate,  $\delta_t$ , is provided by the user, as are *G* ( the exogenous growth rate), *Y* (the exogenous yield), and *B* (a "bowing factor" to control the rate of distribution)

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The dynamics of the fund value,  $V_t$ , under the real-world probability measure  $\mathbb{P}$ , can be described by the stochastic process  $\{V_t, 0 \le t \le T_L\}$ :

$$dV_t = V_t (\mu_V dt + \beta_V \sigma_M dB_{M,t} + \sigma_\varepsilon dB_{\varepsilon,t}) + dD_t - dR_t,$$
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$$dD_t = \delta_t (I_0 - D_t) \mathbf{1}_{\{0 \le t \le T_I\}} dt,$$
(2)

where  $\mathbf{1}_{\{\cdot\}}$  is an indicator function. The fund's drawdown rate  $\delta_t$  is assumed to follow a stochastic process  $\{\delta_t, 0 \le t \le T_I\}$  given by:

$$\delta_t = \delta + \sigma_\delta B_{\delta,t},\tag{3}$$

where  $\delta > 0$  is the mean of the drawdown rate,  $\sigma_{\delta} > 0$  is the volatility of the drawdown rate;  $B_{\delta,t}$ is a third standard Brownian motion for which it is assumed that  $dB_{\delta,t}dB_{M,t} = \rho_{\delta}dt$ , where  $\rho_{\delta}$  is the correlation between drawdown rate and stock market returns, and  $dB_{\delta,t}dB_{\varepsilon,t} = 0$ . In order to avoid negative drawdown rates, we use  $\delta_t^+ = \max(\delta_t, 0)$  in the model implementation.

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#### Assumption

The dynamics of the cumulative capital distributions,  $R_t$ , can be described by:

$$dR_t = v_t V_t dt, \text{ for } t < T_L, \quad and \quad R_t = V_t \mathbf{1}_{\{t=T_L\}} + \int_0^t v_u V_u du, \text{ for } t \le T_L$$
(4)

The fund's distribution rate  $v_t$  is assumed to follow a stochastic process  $\{v_t, 0 \le t \le T_L\}$  given by:

$$v_t = vt + \sigma_v B_{v,t},\tag{5}$$

where v is the mean distribution rate, and  $\sigma_v > 0$  is the volatility of the distribution rate;  $B_{v,t}$  is a fourth standard Brownian motion for which it is assumed that  $dB_{v,t}dB_{M,t} = \rho_v dt$ , where  $\rho_v$  is the correlation between the drawdown rate and stock market returns, and  $dB_{v,t}dB_{\varepsilon,t} = 0$ . In order to avoid negative distributions rates, we use  $v_t^+ = \max(v_t, 0)$  in the model implementation.

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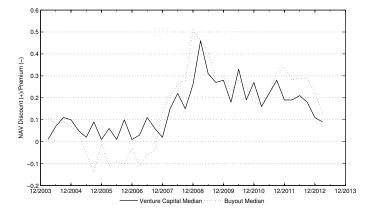


Figure: Median Discount (+) / Premium (-) to fund NAVs by fund type, 2004–2013. Source: Pregin Secondary Market Monitor

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#### Assumption

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$$d\pi_t = \kappa_\pi (\theta_\pi - \pi_t) dt + \sigma_\pi dB_{\pi,t}, \tag{7}$$

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where  $\theta_{\pi} > 0$  is the long-run mean of the discount rate,  $\kappa_{\pi} > 0$  is the rate of reversion to this mean, and  $\sigma_{\pi} > 0$  reflects the volatility of the discount rate.  $B_{\pi,t}$  is a fifth standard Brownian motion for which it is assumed that  $dB_{\pi,t}dB_{M,t} = \rho_{\pi}dt$ , where  $\rho_{\pi}$  is the correlation between drawdown rate and stock market returns, and  $dB_{\pi,t}dB_{E,t} = 0$ .

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• The LVaR $_{t,h}^{\alpha,\$}$  is defined by:

$$\Pr\left(\mathsf{P\&L}_{t+h}^{(L)} < q_h^{(L),\alpha,\$}\right) = \alpha \quad \Leftrightarrow \quad \mathsf{LVaR}_{t,h}^{\alpha,\$} = -q_h^{(L),\alpha,\$} \tag{8}$$

where  $P\&L_{t+h}^{(L)}$  is the liquidity-adjusted P&L forecast of the investor's position in the fund for time t + h:

$$P\&L_{t+h}^{(L)} = ((1 - \pi_{t+h})V_{t+h} + C_{t+h}) - P_t,$$
(9)

with  $\pi_{t+h}$  being the forecast of the secondary-market discount for the fund at time t + h

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The dynamics of the investor's cash position are given by:

$$\mathrm{d}C_t = C_t r_c \mathrm{d}t - \mathrm{d}D_t + \mathrm{d}R_t \tag{10}$$

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$$\Pr(C_{t+h} - C_t < q_h^{(C),\alpha,\$}) = \alpha \quad \Leftrightarrow \quad \operatorname{CFaR}_{t,h}^{\alpha,\$} = -q_h^{(C),\alpha,\$} \tag{11}$$

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## Calibrated model parameters

#### Table: Summary of baseline parameters used in illustration of risk-management model

Note: All model parameters are stated as annualized units, except where indicated

Parameter	Notation	Value
Life of the PE fund investment (years)	$T_L$	12
Simulation frequency (years)	dt	1/4
Committed capital (US dollars)	$C_0$	100
Risk-free rate	$r_{f}$	0.05
Return on cash positions	r <sub>c</sub>	0
Expected return of stock market	$\mu_M$	0.11
Volatility of stock market returns	$\sigma_M$	0.15
Alpha of PE funds	α	0.04
Market beta of PE funds	$\beta_M$	1.30
Idiosyncratic volatility of PE fund returns	$\sigma_{\varepsilon}$	0.35
Drawdown rate of PE funds	δ	0.41
Volatility of the drawdown rate	$\sigma_{\delta}$	0.21
Correlation between drawdown rate and stock market returns	$\rho_{\delta}$	0.50
Average distribution rate	ν	0.08
Volatility of the distribution rate	$\sigma_{v}$	0.11
Correlation between distribution rate and stock market returns	$\rho_{v}$	0.80
Long-run mean of secondary market discounts	$\theta_{\pi}$	0.16
Mean-reversion speed of secondary market discounts	$K_{\pi}$	0.42
Volatility of secondary market discounts	$\sigma_{\pi}$	0.16
Initial secondary market discount	$\pi_0$	$\theta_{\pi}$
Correlation between discount rate and stock market returns	$\rho_{\pi}$	-0.60

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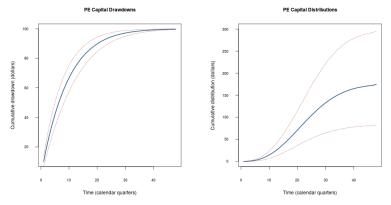


Figure: Cumulative capital drawdowns (left) and cumulative capital distributions (right). Solid lines represent Monte Carlo estimates of the average and dotted lines represent the 10<sup>th</sup> & 90<sup>th</sup> quantiles over 500,000 simulations

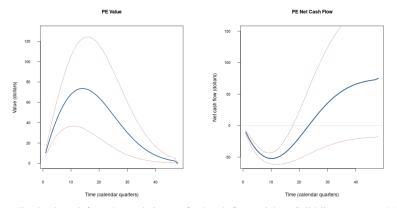
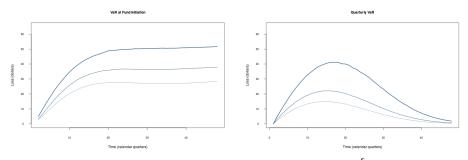
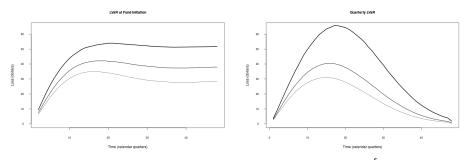


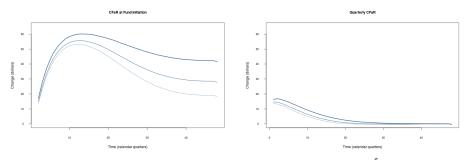
Figure: Fund values (left) and cumulative net fund cash flows (right). Solid lines represent Monte Carlo estimates of the average and dotted lines represent the 10<sup>th</sup> & 90<sup>th</sup> quantiles over 500,000 simulations



**Figure:** VaR dynamics over the fund lifecycle: (left) VaR at fund initiation,  $VaR_{0,h}^{\alpha,\$}$ , plotted as a function of the time horizon *h*; (right) quarterly VaR, *i.e.*  $VaR_{t,0.25}^{\alpha,\$}$ , plotted as a function of time *t*. The thickest line represents the Monte Carlo estimate of the 1% VaR over 500,000 simulations (also shown are the 5% VaR and the 10% VaR)



**Figure:** LVaR dynamics over the fund lifecycle: (left) LVaR at fund initiation,  $\text{LVaR}_{0,h}^{\alpha,\$}$ , plotted as a function of time horizon *h*; (right) quarterly LVaR, *i.e.*  $\text{LVaR}_{t,0.25}^{\alpha,\$}$ , plotted as a function of time *t*. The thickest line represents the Monte Carlo estimate of the 1% LVaR over 500,000 simulations (also shown are the 5% LVaR and the 10% LVaR)



**Figure:** CFaR dynamics over the fund lifecycle: (left) CFaR at fund initiation,  $\text{CFaR}_{0,h}^{\alpha,\$}$ , plotted as a function of time horizon *h*; (right) quarterly CFaR, *i.e.*  $\text{CFaR}_{t,0.25}^{\alpha,\$}$ , plotted as a function of time *t*. The thickest line represents the Monte Carlo estimate of the 1% CFaR over 500,000 simulations (also shown are the 5% CFaR and the 10% CFaR)

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2 Introduction

3 Risk Management Framework (I): Outline

Modeling PE Fund Dynamics

5 Risk Management Framework (II): Risk Measures

6 Fund Structure & Fees

7 Extensive List of To-Dos

8 References

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# Management fees

• The management fee is levied against a basis: this is usually either the committed capital,  $C_0$ , or the net invested capital ("NIC"),<sup>4</sup> and it is one of four different types that is specified in the limited partnership agreement ("LPA"):

<sup>&</sup>lt;sup>4</sup>Invested capital minus the cost basis of exited investments, *ibid.* p. 2315, *et seq.* 

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  - flat fee
  - 2 tapered fee: tapers after the investment period,  $T_I < t \leq T_L$
  - (a) change basis to NIC after investment period with flat fee<sup>5</sup>
  - Change basis to NIC after investment period with tapered fee

<sup>&</sup>lt;sup>4</sup>Invested capital minus the cost basis of exited investments, *ibid.* p. 2315, *et seq.* <sup>5</sup>*ibid.* p. 2315, *et seq.* 

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• Let MF<sub>t</sub> denote the cumulative management fees up to some time  $t \in [0, T_L]$ .

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## • Let $MF_t$ denote the cumulative management fees up to some time $t \in [0, T_L]$ .

• Fixed Management Fees: If management fees are defined as a percentage  $c_{MF}$  of the committed capital  $C_0$  and are paid continuously, the dynamics are given by:

$$\mathrm{dMF}_t = c_{\mathrm{MF}} C_0 \mathrm{d}t \tag{12}$$

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• Management Fees with Change in Basis: Latterly, tapered management fees appear to be gaining in popularity. The tapering typically begins after the investment period, *i.e.* for  $T_I < t \le T_L$ , and reflects the fact that less time is required by the GP in managing the activities of the portfolio companies. Many funds change the fee basis from committed capital (during the commitment period) to NIC capital (after the commitment period).

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<sup>&</sup>lt;sup>4</sup>Invested capital minus the cost basis of exited investments, *ibid.* p. 2315, *et seq.* 

<sup>&</sup>lt;sup>5</sup> ibid. p. 2315, et seq.

• If *ab initio* the basis for management-fee calculation is agreed to change from committed capital,  $C_0$ , for  $0 \le t \le T_I$ , to NIC for  $T_I < t \le T_L$ , then how do GPs determine  $I_C$ , the capital available for investment, for  $t \le T_I$ ? Is it specified in the LPA?

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$$\mathrm{dMF}_t = c_{\mathrm{MF}} C_0 \mathrm{d}t \, \mathbf{1}_{0 \le t \le T_I} \tag{13}$$

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the value of NIC<sub>t</sub> for  $t = T_I$  is then initialized to  $C_0 - MF_{T_I}$ 

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**(a)** The fund's distribution rate,  $v_t$ , is assumed to follow a stochastic process { $v_t$ ,  $0 \le t \le T_L$ } given by  $v_t = vt + \sigma_v B_{v,t}$ , as per Equation 5, and this rate is applied to the NIC to give its dynamics as:

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• **Carried Interest**: Let the carried interest level be given by *c*<sub>CI</sub> and let *h* denote the hurdle rate. The **dynamics of carried interest** are given by:

$$\mathrm{dCI}_t = c_{\mathrm{CI}} \max\{\underline{\mathrm{d}R_t - \mathrm{d}D_t - \mathrm{d}MF_t}, 0\} \mathbb{1}_{\{\mathrm{IRR}_t > h\}}$$

net cash flow =  $dNCF_{t}$ 

where  $\mathbf{1}_{\{IRR_t > h\}}$  indicates that carried interest is only payable at time *t* if the internal rate of return of the fund at *t*, IRR<sub>t</sub>, exceeds the hurdle rate *h* 

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• **Catch-up provision**: Most LPAs that contain a hurdle rate also include a provision that provides the GPs with a greater share of the profits once the hurdle rate has been paid and until the carry level has been reached

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• **Carried interest with catch-up**: If the carried interest is paid with a 100% catch-up provision, then its dynamics are given by:

$$d\mathbf{CI}_{t} = \begin{cases} c_{\mathrm{CI}} \max\{d\mathrm{NCF}_{t}, 0\} \mathbf{1}_{\{\mathrm{IRR}_{t} > h\}}, & \text{if } \frac{\mathrm{CI}_{t}}{R_{t} - C_{0}} = c_{\mathrm{CI}} \\ \\ \min\{c_{\mathrm{CI}}(R_{t} - C_{0}) - \mathrm{CI}_{t}, d\mathrm{NCF}_{t}\} \mathbf{1}_{\{\mathrm{IRR}_{t} > h\}}, & \text{if } \frac{\mathrm{CI}_{t}}{R_{t} - C_{0}} < c_{\mathrm{CI}} \end{cases}$$
(16)

where  $dNCF_t = dR_t - dD_t - dMF_t$ 

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#### Table: Carried Interest Calculation

This table illustrates the carried interest calculation for a \$100M fund with a carried interest level of 20 percent, a hurdle rate of 8 percent, and a lifetime of ten years. The calculation is shown for a fund with no catch-up clause and fund with a catch-up clause of 100 percent.

Year	1	2	3	4	5	6	7	8	9	10	Total
Cash Flows	-50	-30	-10	-10	30	50	60	50	40	20	150
IRR (in % p.a.)	-100	-100	-100	-100	-33	-6	8	14	17	18	18
Carried Interest (No Catch-Up)	0	0	0	0	0	0	0	10	8	4	22
Carried Interest (With Catch-Up)	0	0	0	0	0	0	0	18	8	4	30

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- If *l* denotes the average leverage ratio applied, the **dynamics of the transaction fees** can be represented by:

$$dTF_t = c_{TF} dD_t \times (1+l)$$
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• **Theorem (Fee Value)**: Applying a risk-neutral valuation approach, the arbitrage-free present value of the fund fees  $V_0^{(GP)}$  is given by:

$$V_0^{(\text{GP})} = \underbrace{\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_L} e^{-r_f u} d\mathbf{M} \mathbf{F}_u\right]}_{=V_0^{(\text{MF})}} + \underbrace{\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_L} e^{-r_f u} d\mathbf{C} \mathbf{I}_u\right]}_{=V_0^{(\text{CD})}} + \underbrace{\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_L} e^{-r_f u} d\mathbf{P} \mathbf{F}_u^{(\text{GP})}\right]}_{=V_0^{(\text{PF})}}, \quad (19)$$

where  $V_0^{(MF)}$  is the present value of management fees,  $V_0^{(CI)}$  is the present value of carried interest payments, and  $V_0^{(PF)}$  is the present value of lifetime portfolio company fees received by the GPs

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		NO DEAL	FEES		WITH DEAL FEES					
	CONSTANT	FEE BASIS	FEE BASIS	CHANGE	CONSTANT	FEE BASIS	FEE BASIS	CHANGE		
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up		
MF	\$15.86	\$15.86	\$11.43	\$11.43	\$15.86	\$15.86	\$11.43	\$11.43		
(MF)			\$0.40	\$0.39			\$0.39	\$0.39		
CI	\$3.88	\$4.23	\$4.44	\$4.82	\$3.42	\$3.81	\$4.02	\$4.41		
(CI)	\$8.78	\$8.40	\$9.49	\$9.04	\$8.08	\$7.99	\$8.94	\$8.72		
(CI 5%)										
(CI 10%)										
(CI 20%)										
(CI 30%)										
(CI 50%)										
(CI 60%)										
(CI 65%)			\$1.51	\$4.06			\$0.80	\$3.04		
(CI 70%)	\$2.05	\$4.52	\$3.09	\$5.35	\$1.20	\$3.73	\$2.32	\$4.73		
(CI 75%)	\$3.88	\$5.96	\$5.06	\$6.87	\$2.92	\$5.23	\$4.17	\$6.20		
(CI 80%)	\$6.12	\$7.73	\$7.41	\$8.79	\$5.09	\$6.93	\$6.43	\$8.01		
(CI 85%)	\$9.01	\$10.15	\$10.44	\$11.36	\$7.89	\$9.27	\$9.41	\$10.45		
(CI 90%)	\$13.15	\$13.70	\$14.82	\$15.10	\$11.86	\$12.63	\$13.60	\$14.09		
(CI 95%)	\$20.68	\$20.30	\$22.59	\$21.97	\$18.92	\$18.71	\$21.10	\$20.80		
TF					\$4.19	\$4.19	\$4.42	\$4.42		
(TF)					\$0.09	\$0.09	\$0.09	\$0.09		
MoF					\$7.30	\$7.30	\$7.71	\$7.70		
(MoF)					\$3.96	\$3.98	\$4.22	\$4.23		
(MoF 5%)					\$3.28	\$3.28	\$3.45	\$3.45		
(MoF 25%)					\$4.73	\$4.73	\$4.99	\$4.99		
(MoF 75%)					\$8.68	\$8.69	\$9.16	\$9.16		
(MoF 95%)					\$14.56	\$14.53	\$15.39	\$15.43		

Figure: Net present value of a fund's fees. Management fee is denoted by "MF", carried interest by "CI", transaction fees by "TF" and monitoring fees by "MoF" (the latter pair being most common in Leveraged Buyout strategies). The means are shown in large font, while the values in parenthesis are either the standard deviations of the means, or the quantiles of the Monte Carlo distributions, as indicated by the quantile and the % sign + (=)

		NO DEAL	FEES		WITH DEAL FEES					
	CONSTANT	FEE BASIS	FEE BASIS	CHANGE	CONSTANTI	FEE BASIS	FEE BASIS	CHANGE		
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up		
MF	\$15.86	\$15.86	\$11.43	\$11.43	\$15.86	\$15.86	\$11.43	\$11.43		
(MF)			\$0.40	\$0.39			\$0.39	\$0.39		
CI	\$3.88	\$4.23	\$4.44	\$4.82	\$3.42	\$3.81	\$4.02	\$4.41		
(CI)	\$8.78	\$8.40	\$9.49	\$9.04	\$8.08	\$7.99	\$8.94	\$8.72		
(CI 5%)										
(CI 10%)										
(CI 20%)										
(CI 30%)										
(CI 50%)										
(CI 60%)										
(CI 65%)			\$1.51	\$4.06			\$0.80	\$3.04		
(CI 70%)	\$2.05	\$4.52	\$3.09	\$5.35	\$1.20	\$3.73	\$2.32	\$4.73		
(CI 75%)	\$3.88	\$5.96	\$5.06	\$6.87	\$2.92	\$5.23	\$4.17	\$6.20		
(CI 80%)	\$6.12	\$7.73	\$7.41	\$8.79	\$5.09	\$6.93	\$6.43	\$8.01		
(CI 85%)	\$9.01	\$10.15	\$10.44	\$11.36	\$7.89	\$9.27	\$9.41	\$10.45		
(CI 90%)	\$13.15	\$13.70	\$14.82	\$15.10	\$11.86	\$12.63	\$13.60	\$14.09		
(CI 95%)	\$20.68	\$20.30	\$22.59	\$21.97	\$18.92	\$18.71	\$21.10	\$20.80		
TF					\$4.19	\$4.19	\$4.42	\$4.42		
(TF)					\$0.09	\$0.09	\$0.09	\$0.09		
MoF					\$7.30	\$7.30	\$7.71	\$7.70		
(MoF)					\$3.96	\$3.98	\$4.22	\$4.23		
(MoF 5%)					\$3.28	\$3.28	\$3.45	\$3.45		
(MoF 25%)					\$4.73	\$4.73	\$4.99	\$4.99		
(MoF 75%)					\$8.68	\$8.69	\$9.16	\$9.16		
(MoF 95%)					\$14.56	\$14.53	\$15.39	\$15.43		

Figure: Net present value of management fee (denoted by "MF")

		NO DEAL	FEES		WITH DEAL FEES					
	CONSTANT	FEE BASIS	FEE BASIS CHANGE		CONSTANT	EE BASIS	FEE BASIS CHANGE			
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up		
MF	\$15.86	\$15.86	\$11.43	\$11.43	\$15.86	\$15.86	\$11.43	\$11.43		
(MF)			\$0.40	\$0.39			\$0.39	\$0.39		
CI	\$3.88	\$4.23	\$4.44	\$4.82	\$3.42	\$3.81	\$4.02	\$4.41		
(CI)	\$8.78	\$8.40	\$9.49	\$9.04	\$8.08	\$7.99	\$8.94	\$8.72		
(CI 5%)										
(CI 10%)										
(CI 20%)										
(CI 30%)										
(CI 50%)										
(CI 60%)										
(CI 65%)			\$1.51	\$4.06			\$0.80	\$3.04		
(CI 70%)	\$2.05	\$4.52	\$3.09	\$5.35	\$1.20	\$3.73	\$2.32	\$4.73		
(CI 75%)	\$3.88	\$5.96	\$5.06	\$6.87	\$2.92	\$5.23	\$4.17	\$6.20		
(CI 80%)	\$6.12	\$7.73	\$7.41	\$8.79	\$5.09	\$6.93	\$6.43	\$8.01		
(CI 85%)	\$9.01	\$10.15	\$10.44	\$11.36	\$7.89	\$9.27	\$9.41	\$10.45		
(CI 90%)	\$13.15	\$13.70	\$14.82	\$15.10	\$11.86	\$12.63	\$13.60	\$14.09		
(CI 95%)	\$20.68	\$20.30	\$22.59	\$21.97	\$18.92	\$18.71	\$21.10	\$20.80		
TF					\$4.19	\$4.19	\$4.42	\$4.42		
(TF)					\$0.09	\$0.09	\$0.09	\$0.09		
MoF					\$7.30	\$7.30	\$7.71	\$7.70		
(MoF)					\$3.96	\$3.98	\$4.22	\$4.23		
(MoF 5%)					\$3.28	\$3.28	\$3.45	\$3.45		
(MoF 25%)					\$4.73	\$4.73	\$4.99	\$4.99		
(MoF 75%)					\$8.68	\$8.69	\$9.16	\$9.16		
(MoF 95%)					\$14.56	\$14.53	\$15.39	\$15.43		

Figure: Net present value of carried interest (denoted by "CI")

MF         \$15.86         \$11.43         \$11.43         \$15.86         \$11.43         \$           (MF)         \$3.88         \$4.23         \$4.44         \$4.82         \$3.42         \$3.81         \$4.02         \$           (C1)         \$8.78         \$8.40         \$9.49         \$9.04         \$8.08         \$7.99         \$8.94         \$           (C1)         \$8.78         \$8.40         \$9.49         \$9.04         \$8.08         \$7.99         \$8.94         \$           (C1 0%)         \$         \$1.51         \$4.06         \$			NO DEAL	FEES		WITH DEAL FEES				
MF         \$15.86         \$11.43         \$11.43         \$11.43         \$15.86         \$11.43         \$           (MF)         \$0.40         \$0.39         \$0.29		CONSTANT	FEE BASIS	FEE BASIS	CHANGE	CONSTANT	FEE BASIS	FEE BASIS	CHANGE	
(MF)         50.40         50.39           CI         \$3.88         \$4.23         \$4.44         \$4.82         \$3.42         \$3.81         \$4.02         \$3.60           (CI 5%)         \$8.78         \$8.40         \$9.49         \$9.04         \$8.08         \$7.99         \$8.94           (CI 5%)         (CI 10%)         (CI 10%)         \$3.60         \$7.99         \$8.94         \$4.02         \$3.60         \$7.99         \$8.94         \$4.02         \$3.60         \$5.99         \$8.94         \$4.02         \$3.60         \$5.99         \$8.94         \$4.02         \$3.60         \$5.99         \$5.99         \$5.99         \$6.93         \$6.91         \$5.05         \$5.05         \$5.05         \$5.09         \$5.09         \$5.03         \$5.04         \$5.09         \$5.03         \$5.643         \$5.09         \$5.09         \$5.03         \$5.643         \$5.09         \$5.09         \$5.03         \$5.643         \$5.09         \$5.09         \$5.03         \$5.643         \$5.09         \$5.03         \$5.643         \$5.09         \$5.03         \$5.643         \$5.09         \$5.09         \$5.03         \$5.643         \$5.09         \$5.09         \$5.09         \$5.03         \$5.643         \$5.09         \$5.09         \$5.09         <		No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	
CI         \$3.88         \$4.23         \$4.44         \$4.82           (CD         \$8.78         \$8.40         \$9.49         \$9.04         \$8.08         \$7.99         \$8.34           (C1 5%)         (C1 10%)         \$8.78         \$8.40         \$9.49         \$9.04         \$8.08         \$7.99         \$8.34           (C1 10%)         (C1 20%)         \$1.51         \$4.06         \$5.06         \$5.06         \$5.06         \$5.09         \$5.23         \$4.17         \$5.09         \$5.23         \$4.17         \$5.09         \$5.23         \$4.17         \$5.09         \$5.23         \$4.17         \$5.09         \$5.23         \$4.17         \$5.09         \$5.23         \$4.17         \$5.09         \$5.33         \$5.29         \$5.23         \$4.17         \$5.09         \$5.63         \$5.64         \$5.29         \$5.23         \$4.17         \$5.09         \$5.63         \$5.64         \$5.29         \$5.23         \$4.17         \$5.09         \$5.63         \$5.64         \$5.29         \$5.23         \$4.17         \$5.09         \$5.63         \$5.64         \$5.73         \$5.23         \$4.17         \$5.09         \$5.63         \$5.64         \$5.20         \$5.23         \$4.17         \$5.09         \$5.73         \$5.23         \$5.41	MF	\$15.86	\$15.86	\$11.43	\$11.43	\$15.86	\$15.86	\$11.43	\$11.43	
(C1)         S8.78         S8.40         S9.49         S9.04         S8.08         S7.99         S8.94           (C1 5%)         (C1 5%)         (C1 0%)         -	(MF)			\$0.40	\$0.39			\$0.39	\$0.39	
(C1 5%)       (C1 0%)         (C1 20%)       (C1 20%)         (C1 20%)       (C1 20%)         (C1 50%)       (C1 50%)         (C1 60%)       (C1 60%)         (C1 65%)       \$1.51       \$4.06         (C1 70%)       \$2.05       \$4.52       \$3.09       \$5.35         (C1 75%)       \$3.88       \$5.56       \$5.06       \$6.87       \$2.92       \$5.23         (C1 65%)       \$6.12       \$7.73       \$7.41       \$8.79       \$5.09       \$6.93       \$6.43         (C1 65%)       \$5.15       \$10.44       \$11.36       \$7.89       \$9.97       \$9.41       \$2.06         (C1 65%)       \$5.06       \$5.06       \$11.66       \$12.63       \$13.60       \$2.06         (C1 65%)       \$9.01       \$10.15       \$10.44       \$11.36       \$7.89       \$9.97       \$9.41       \$2.06         (C1 65%)       \$52.068       \$20.30       \$22.59       \$21.97       \$18.82       \$18.71       \$21.10       \$2.10         TF       \$50.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$6.99       \$	CI	\$3.88	\$4.23	\$4.44	\$4.82	\$3.42	\$3.81	\$4.02	\$4.41	
(C1 10%)       (C1 20%)         (C1 20%)       (C1 50%)         (C1 50%)       \$1.51       \$4.06         (C1 50%)       \$1.51       \$4.06         (C1 50%)       \$1.51       \$4.06         (C1 70%)       \$2.05       \$4.52       \$3.09         (C1 70%)       \$3.88       \$5.56       \$5.66         (C1 70%)       \$3.88       \$5.56       \$5.09       \$6.93       \$6.43         (C1 80%)       \$6.12       \$7.73       \$7.41       \$8.79       \$5.09       \$6.93       \$6.43         (C1 80%)       \$6.12       \$7.73       \$7.41       \$8.79       \$5.09       \$6.93       \$6.43         (C1 90%)       \$3.15       \$1.3.70       \$1.42       \$1.1.36       \$7.89       \$9.27       \$9.41       \$9.69         (C1 90%)       \$3.15       \$1.3.70       \$1.42       \$1.1.36       \$7.89       \$9.27       \$9.41       \$9.69         (C1 90%)       \$3.15       \$1.3.70       \$1.42       \$1.1.36       \$7.89       \$9.27       \$9.41       \$9.69       \$9.69       \$0.69       \$1.10       \$9.7       \$1.892       \$18.71       \$21.10       \$2.10       \$2.10       \$2.10       \$2.10       \$2.10       \$2.1	(CI)	\$8.78	\$8.40	\$9.49	\$9.04	\$8.08	\$7.99	\$8.94	\$8.72	
(C1 20%)         (C1 30%)           (C1 50%)         \$1.51         \$4.06           (C1 60%)         \$1.51         \$4.06           (C1 60%)         \$1.51         \$4.06           (C1 70%)         \$2.05         \$4.52         \$3.09         \$5.35           (C1 70%)         \$2.05         \$4.52         \$3.09         \$5.35           (C1 70%)         \$2.05         \$4.52         \$3.09         \$5.25           (C1 70%)         \$2.05         \$4.52         \$3.09         \$5.35           (C1 70%)         \$2.05         \$4.52         \$3.09         \$5.25         \$5.23         \$4.17           (C1 80%)         \$6.12         \$7.73         \$7.41         \$8.79         \$5.92         \$5.23         \$4.17           (C1 80%)         \$50.01         \$10.15         \$10.44         \$11.36         \$7.89         \$9.27         \$9.41         \$9           (C1 55%)         \$20.08         \$20.30         \$22.59         \$21.17         \$18.82         \$18.71         \$21.10         \$1           C1 55%)         \$20.08         \$20.30         \$22.59         \$21.97         \$18.82         \$18.71         \$21.10         \$2           C1 55%         \$20.08	(CI 5%)									
(CI 30%)       (CI 50%)         (CI 60%)       \$1.51       \$4.06         (CI 55%)       \$2.05       \$4.52       \$3.09       \$5.35         (CI 75%)       \$3.88       \$5.56       \$5.06       \$6.87       \$2.92       \$5.23         (CI 85%)       \$6.12       \$7.73       \$7.41       \$8.79       \$5.09       \$6.93       \$6.43         (CI 85%)       \$9.01       \$10.15       \$10.44       \$11.36       \$7.89       \$9.97       \$9.41       \$2.06       \$13.70       \$1.422       \$1.50       \$13.80       \$5.06       \$5.06       \$13.60       \$2.06       \$13.80       \$2.030       \$22.59       \$21.97       \$18.92       \$18.71       \$21.10       \$21.00 <t< td=""><td>(CI 10%)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	(CI 10%)									
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(CI 20%)									
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(CI 30%)									
(C1 65%)         S1.51         S4.06         S0.89           (C1 75%)         S2.05         S4.52         S3.09         S5.35         S1.20         S3.73         S2.32           (C1 75%)         S2.88         S5.96         S5.06         S6.87         S2.92         S5.33         S2.32           (C1 60%)         S6.12         S7.73         S7.41         S8.79         S5.09         S6.93         S6.43           (C1 80%)         S9.01         S10.15         S10.04         S11.36         S7.89         S9.27         S9.41         S9.07           (C1 90%)         S13.15         S1.70         S14.82         S11.10         S18.68         S12.63         S13.60         S9           (C1 90%)         S20.68         S20.30         S22.59         S21.97         S18.92         S18.71         S21.10         S1           (C1 95%)         S20.68         S20.30         S22.59         S21.97         S18.92         S18.71         S21.10         S1           (C1 95%)         S20.68         S20.30         S22.59         S21.97         S18.92         S18.71         S21.10         S1           (C1 95%)         S20.68         S20.30         S22.59         S21.67         S1.8	(CI 50%)									
S10         S2.05         S4.52         S3.09         S5.35         S1.20         S3.73         S2.32           (C1 75%)         S3.88         S5.56         S5.06         S6.87         S2.92         S5.23         S4.17           (C1 75%)         S3.88         S5.56         S5.06         S6.87         S2.92         S5.23         S4.17           (C1 80%)         S6.12         S7.73         S7.41         S8.79         S5.99         S6.93         S6.43           (C1 80%)         S9.01         S10.15         S10.44         S11.36         S7.89         S9.27         S9.41         S9.07         S9.41         S9	(CI 60%)									
S13.88         S5.96         S5.06         S6.87         S2.92         S5.23         S4.17           (C1 75%)         S3.88         S5.96         S5.06         S6.87         S2.92         S5.23         S4.17           (C1 80%)         S6.12         S7.73         S7.41         S8.79         S5.09         S6.93         S6.43           (C1 80%)         S0.15         S10.44         S11.36         S7.89         S9.27         S9.41         S9           (C1 90%)         S13.15         S13.70         S14.82         S15.10         S11.86         S12.63         S13.60         S9           (C1 95%)         S20.68         S20.30         S22.59         S21.97         S18.92         S18.71         S21.10         S9           TF          S4.19         S4.19         S4.42         S9         S9         S0.09	(CI 65%)			\$1.51	\$4.06			\$0.80	\$3.04	
S6.12         \$7.73         \$7.41         \$8.79         \$5.09         \$6.93         \$6.43           (C1 80%)         \$9.01         \$10.15         \$10.44         \$11.36         \$7.89         \$8.27         \$9.41         \$9.01           (C1 80%)         \$9.01         \$10.15         \$10.44         \$11.36         \$7.89         \$8.27         \$9.41         \$9.01           (C1 90%)         \$13.15         \$13.70         \$14.42         \$15.10         \$11.86         \$12.63         \$13.60         \$9.07           (C1 95%)         \$20.68         \$20.30         \$22.59         \$21.97         \$18.92         \$18.71         \$21.10         \$9.07           TF          \$20.68         \$20.30         \$22.59         \$21.97         \$18.92         \$18.71         \$21.10         \$9.09           MoF          \$7.30         \$7.30         \$5.73         \$7.71         \$1.30         \$1.92 <td< td=""><td>(CI 70%)</td><td>\$2.05</td><td>\$4.52</td><td>\$3.09</td><td>\$5.35</td><td>\$1.20</td><td>\$3.73</td><td>\$2.32</td><td>\$4.73</td></td<>	(CI 70%)	\$2.05	\$4.52	\$3.09	\$5.35	\$1.20	\$3.73	\$2.32	\$4.73	
Space         Space <th< td=""><td>(CI 75%)</td><td>\$3.88</td><td>\$5.96</td><td>\$5.06</td><td>\$6.87</td><td>\$2.92</td><td>\$5.23</td><td>\$4.17</td><td>\$6.20</td></th<>	(CI 75%)	\$3.88	\$5.96	\$5.06	\$6.87	\$2.92	\$5.23	\$4.17	\$6.20	
S13.15         S13.70         S14.82         S15.10         S11.86         S12.63         S13.60         S12.63           (C1 90%)         S20.68         S20.30         S22.59         S21.97         S18.92         S18.71         S21.10         S1           TF	(CI 80%)	\$6.12	\$7.73	\$7.41	\$8.79	\$5.09	\$6.93	\$6.43	\$8.01	
S20.68         S20.30         S22.59         S21.37         S18.32         S18.71         S21.10         S2           TF         S4.19         S4.19         S4.42         S0.09         S0	(CI 85%)	\$9.01	\$10.15	\$10.44	\$11.36	\$7.89	\$9.27	\$9.41	\$10.45	
S20.68         S20.30         S22.59         S21.37         S18.32         S18.71         S21.10         S2           TF         S4.19         S4.19         S4.42         S0.09         S0	(CI 90%)	\$13.15	\$13.70	\$14.82	\$15.10	\$11.86	\$12.63	\$13.60	\$14.09	
S0.09         S0.09         S0.09           MoF         \$7.30         \$7.30         \$7.71         \$7.30           (MoF)         \$3.36         \$3.38         \$4.22           (MoF 5%)         \$3.28         \$3.28         \$3.45           (MoF 7%)         \$8.68         \$8.69         \$9.16		\$20.68	\$20.30	\$22.59	\$21.97	\$18.92	\$18.71	\$21.10	\$20.80	
S7.30         \$7.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$7.71         57.30         \$8.422         \$0.33         \$6.33         \$6.33         \$6.422         \$6.33         \$6.69         <	TF					\$4.19	\$4.19	\$4.42	\$4.42	
(MoF)         \$3.96         \$3.98         \$4.22           (MoF 5%)         \$3.28         \$3.28         \$3.45           (MoF 25%)         \$4.73         \$4.93         \$4.93           (MoF 75%)         \$8.68         \$8.69         \$9.16	(TF)					\$0.09	\$0.09	\$0.09	\$0.09	
Mor 5%)         \$3.28         \$3.28         \$3.45           (Mor 25%)         \$4.73         \$4.73         \$4.99           (Mor 75%)         \$8.68         \$8.69         \$9.16	MoF					\$7.30	\$7.30	\$7.71	\$7.70	
MoF 25%)         \$4,73         \$4,73         \$4.99           (MoF 75%)         \$8,68         \$8,69         \$9,16	(MoF)					\$3.96	\$3.98	\$4.22	\$4.23	
(MoF 75%) \$8.68 \$8.69 \$9.16	(MoF 5%)					\$3.28	\$3.28	\$3.45	\$3.45	
	(MoF 25%)					\$4.73	\$4.73	\$4.99	\$4.99	
(MoF 95%) \$14.56 \$14.53 \$15.39	(MoF 75%)					\$8.68	\$8.69	\$9.16	\$9.16	
	(MoF 95%)					\$14.56	\$14.53	\$15.39	\$15.43	

Figure: Net present value of transaction fees (denoted by "TF") for Leveraged Buyout funds

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		NO DEAL	FEES		WITH DEAL FEES					
	CONSTANT	FEE BASIS	FEE BASIS	CHANGE	CONSTANT	FEE BASIS	FEE BASIS	CHANGE		
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up		
MF	\$15.86	\$15.86	\$11.43	\$11.43	\$15.86	\$15.86	\$11.43	\$11.43		
(MF)			\$0.40	\$0.39			\$0.39	\$0.39		
CI	\$3.88	\$4.23	\$4.44	\$4.82	\$3.42	\$3.81	\$4.02	\$4.41		
(CI)	\$8.78	\$8.40	\$9.49	\$9.04	\$8.08	\$7.99	\$8.94	\$8.72		
(CI 5%)										
(CI 10%)										
(CI 20%)										
(CI 30%)										
(CI 50%)										
(CI 60%)										
(CI 65%)			\$1.51	\$4.06			\$0.80	\$3.04		
(CI 70%)	\$2.05	\$4.52	\$3.09	\$5.35	\$1.20	\$3.73	\$2.32	\$4.73		
(CI 75%)	\$3.88	\$5.96	\$5.06	\$6.87	\$2.92	\$5.23	\$4.17	\$6.20		
(CI 80%)	\$6.12	\$7.73	\$7.41	\$8,79	\$5.09	\$6.93	\$6,43	\$8.01		
(CI 85%)	\$9.01	\$10.15	\$10.44	\$11.36	\$7.89	\$9.27	\$9.41	\$10.45		
(CI 90%)	\$13.15	\$13.70	\$14.82	\$15.10	\$11.86	\$12.63	\$13.60	\$14.09		
(CI 95%)	\$20.68	\$20.30	\$22.59	\$21.97	\$18.92	\$18.71	\$21.10	\$20.80		
TF					\$4.19	\$4.19	\$4.42	\$4.42		
(TF)					\$0.09	\$0.09	\$0.09	\$0.09		
MoF					\$7.30	\$7.30	\$7.71	\$7.70		
(MoF)					\$3.96	\$3.98	\$4.22	\$4.23		
(MoF 5%)					\$3.28	\$3.28	\$3.45	\$3.45		
(MoF 25%)					\$4.73	\$4.73	\$4.99	\$4.99		
(MoF 75%)					\$8.68	\$8.69	\$9.16	\$9.16		
(MoF 95%)					\$14.56	\$14.53	\$15.39	\$15.43		

Figure: Net present value of monitoring fees (denoted by "MoF") for Leveraged Buyout funds

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