The PE Package

Modeling private equity in the 21st Century

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What is the structure of private equity?

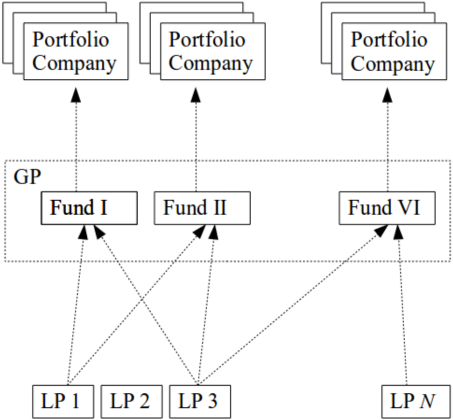

Figure: Partnership structure of private-equity funds: the General Partner ("GP") is the investment manager for the Limited Partners ("LP") who invest in the GP's fund(s).

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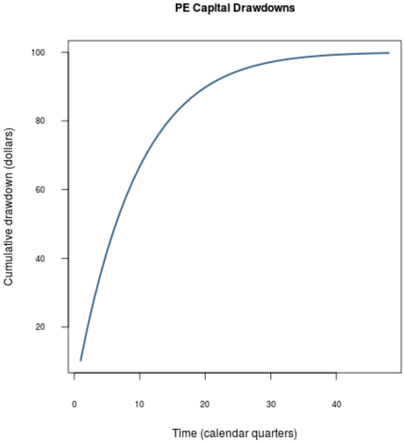

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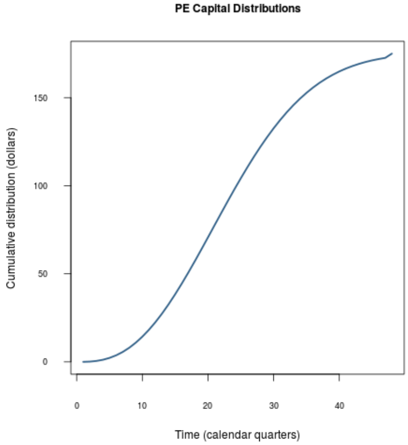

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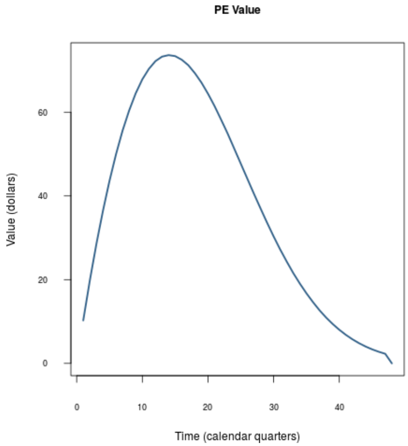

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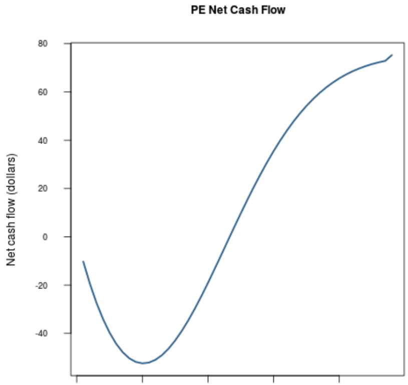

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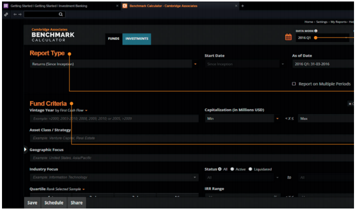

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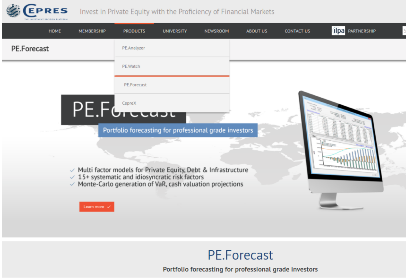

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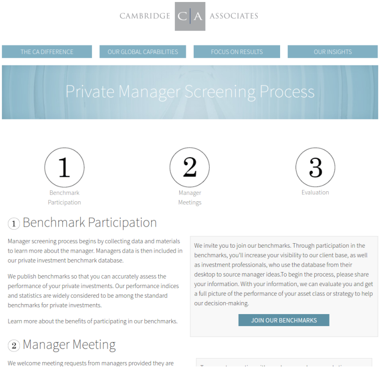

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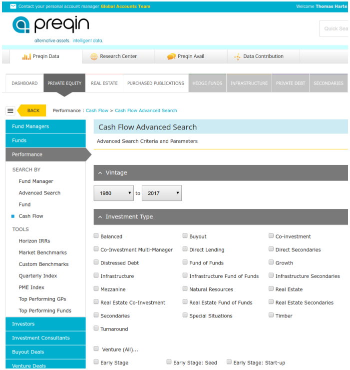

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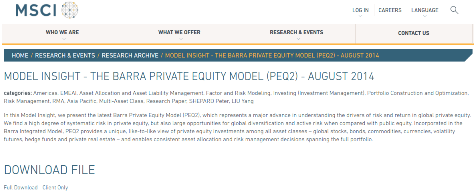
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categories: Americas, EMEAI, Asset Allocation and Asset Liability Management, Factor and Risk Modeling, Investing (Investment Management), Portfolio Construction and Optimization, Risk Management, RMA, Asia Pacific, Multi-Asset Class, Research Paper, SHEPARD Peter, LIU Yang

In this Model Insight, we present the latest Barra Private Equity Model (PEQ2), which represents a major advance in understanding the drivers of risk and return in global private equity. We find a high degree of systematic risk in private equity, but also large opportunities for global diversification and active risk when compared with public equity. Incorporated in the Barra Integrated Model, PEQ2 provides a unique, like-to-like view of private equity investments among all asset classes – global stocks, bonds, commodities, currencies, volatility futures, hedge funds and private real estate – and enables consistent asset allocation and risk management decisions spanning the full portfolio.

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Illiquid Alternative Asset Fund Modeling

Dean Takahashi
Senior Director, Yale University Investments Office

Seth Alexander
Associate Director, Yale University Investments Office

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$$\begin{aligned}
\Delta D_t &= \delta_t (C_0 - D_t) \\
\Delta R_t &= \nu_t V_t (1 + G) \\
\nu_t &= \max\left(Y, \left(\frac{t}{L}\right)^B\right) \\
\Delta V_t &= V_t G + \Delta D_t - \Delta R_t,
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The above are the culmination of work described in Axel's prior publications in the field of PE modeling [16, 20, 9, 10, 11, 23, 19, 7, 6, 8, 22, 1, 5, 17, 21, 4, 2, 18, 3, 15, 14, 13]

The PE package currently implements the models proposed in two of Axel's forthcoming papers:

1. **Risk Management Framework:**
   Buchner, A. [12] "Risk management for private equity funds", *Journal of Risk*, August 2017

2. **PE Fund Structure and Fees:**
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The dynamics of the fund value, V_t, under the real-world probability measure $\mathbb{P}$, can be described by the stochastic process $\{V_t,\ 0 \leq t \leq T_L\}$:

$$\mathrm{d}V_t = V_t(\mu_V \mathrm{d}t + \beta_V \sigma_M \mathrm{d}B_{M,t} + \sigma_\varepsilon \mathrm{d}B_{\varepsilon,t}) + \mathrm{d}D_t - \mathrm{d}R_t, \tag{1}$$

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The dynamics of the cumulative capital drawdowns, D_t, can be described by the ordinary differential equation:

$$\mathrm{d}D_t = \delta_t(I_0 - D_t)\mathbf{1}_{\{0 \le t \le T_I\}}\mathrm{d}t, \tag{2}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function. The fund's drawdown rate δ_t is assumed to follow a stochastic process $\{\delta_t, 0 \le t \le T_I\}$ given by:

$$\delta_t = \delta + \sigma_\delta B_{\delta,t}, \tag{3}$$

where $\delta > 0$ is the mean of the drawdown rate, $\sigma_\delta > 0$ is the volatility of the drawdown rate; $B_{\delta,t}$ is a third standard Brownian motion for which it is assumed that $\mathrm{d}B_{\delta,t}\mathrm{d}B_{M,t} = \rho_\delta \mathrm{d}t$, where ρ_δ is the correlation between drawdown rate and stock market returns, and $\mathrm{d}B_{\delta,t}\mathrm{d}B_{\varepsilon,t} = 0$. In order to avoid negative drawdown rates, we use $\delta_t^+ = \max(\delta_t, 0)$ in the model implementation.

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Assumption

The dynamics of the cumulative capital distributions, R_t, can be described by:

$$dR_t = \nu_t V_t dt, \ \textit{for } t < T_L, \qquad \textit{and} \qquad R_t = V_t \mathbf{1}_{\{t=T_L\}} + \int_0^t \nu_u V_u \mathrm{d}u, \ \textit{for } t \leq T_L \tag{4}$$

The fund's distribution rate ν_t is assumed to follow a stochastic process $\{\nu_t, \ 0 \leq t \leq T_L\}$ given by:

$$\nu_t = \nu t + \sigma_\nu B_{\nu,t}, \tag{5}$$

where ν is the mean distribution rate, and $\sigma_\nu > 0$ is the volatility of the distribution rate; $B_{\nu,t}$ is a fourth standard Brownian motion for which it is assumed that $\mathrm{d}B_{\nu,t}\mathrm{d}B_{M,t} = \rho_\nu \mathrm{d}t$, where ρ_ν is the correlation between the drawdown rate and stock market returns, and $\mathrm{d}B_{\nu,t}\mathrm{d}B_{\varepsilon,t} = 0$. In order to avoid negative distributions rates, we use $\nu_t^+ = \max(\nu_t, 0)$ in the model implementation.

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Market risk: Value at Risk ("VaR")

- VaR is always forward-looking: VaR is a forecast of the uncertainty in the P&L of a portfolio at the end of the holding period. If we let $d_{t,h}$ be the discount factor with term t and tenor h and let P_t be the PE investor's position at time t, then the discounted forecast of the P&L at time $t + h$ in present-value terms is:

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- The portfolio VaR at any time t, which we will denote by $\text{VaR}_{t,h}^{\alpha,\$}$ when expressed in dollar terms for a significance level $\alpha \in [0, 1]$ and a holding period h, is defined as:

$$\Pr\left(\text{P\&L}_{t+h} < q_h^{\alpha,\$}\right) = \alpha \quad \Leftrightarrow \quad \text{VaR}_{t,h}^{\alpha,\$} = -q_h^{\alpha,\$}$$

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Liquidity risk: Liquidity Adjusted Value at Risk (“LVaR”)

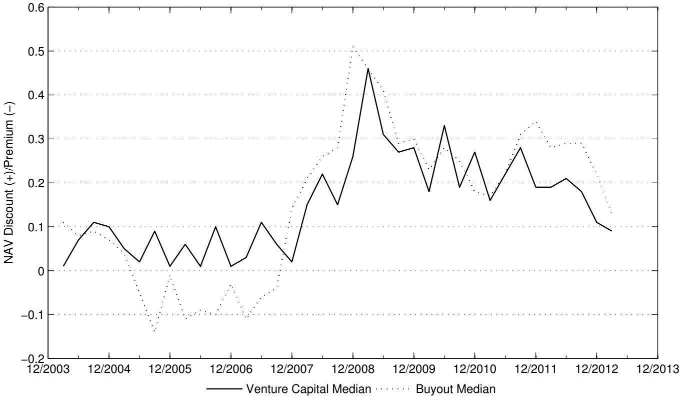

Figure: Median Discount (+) / Premium (-) to fund NAVs by fund type, 2004–2013.

Source: Preqin Secondary Market Monitor

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Assumption

The dynamics of the secondary-market discount rate π_t are assumed to follow a stochastic process given by:

$$d\pi_t = \kappa_\pi(\theta_\pi - \pi_t)dt + \sigma_\pi dB_{\pi,t}, \tag{7}$$

where $\theta_\pi > 0$ is the long-run mean of the discount rate, $\kappa_\pi > 0$ is the rate of reversion to this mean, and $\sigma_\pi > 0$ reflects the volatility of the discount rate. $B_{\pi,t}$ is a fifth standard Brownian motion for which it is assumed that $dB_{\pi,t}dB_{M,t} = \rho_\pi dt$, where ρ_π is the correlation between drawdown rate and stock market returns, and $dB_{\pi,t}dB_{\varepsilon,t} = 0$.

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- The $\mathrm{LVaR}_{t,h}^{\alpha,\$}$ is defined by:

$$\Pr\left(\mathrm{P\&L}_{t+h}^{(L)} < q_h^{(L),\alpha,\$}\right) = \alpha \quad \Leftrightarrow \quad \mathrm{LVaR}_{t,h}^{\alpha,\$} = -q_h^{(L),\alpha,\$} \tag{8}$$

where $\mathrm{P\&L}_{t+h}^{(L)}$ is the liquidity-adjusted P&L forecast of the investor's position in the fund for time $t+h$:

$$\mathrm{P\&L}_{t+h}^{(L)} = ((1-\pi_{t+h})V_{t+h} + C_{t+h}) - P_t, \tag{9}$$

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- The risk measure CFaR is defined as the change (or possibly loss) in the investor’s cash position, C_t, which is exceeded with some given probability α, over a given time horizon h

Cash-flow risk: Cash Flow at Risk ("CFaR")

- The risk measure CFaR is defined as the change (or possibly loss) in the investor's cash position, C_t, which is exceeded with some given probability α, over a given time horizon h

Assumption

The dynamics of the investor's cash position are given by:

$$dC_t = C_t r_c dt - dD_t + dR_t \tag{10}$$

where r_c is rate of return on cash

- The risk measure CFaR is defined as the change (or possibly loss) in the investor's cash position, C_t, which is exceeded with some given probability α, over a given time horizon h

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- The $\text{CFaR}_{t,h}^{\alpha,\$}$ is defined by:

$$\Pr\left(C_{t+h} - C_t < q_h^{(C),\alpha,\$}\right) = \alpha \quad \Leftrightarrow \quad \text{CFaR}_{t,h}^{\alpha,\$} = -q_h^{(C),\alpha,\$} \tag{11}$$

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Cash-flow risk: Cash Flow at Risk (“CFaR”)

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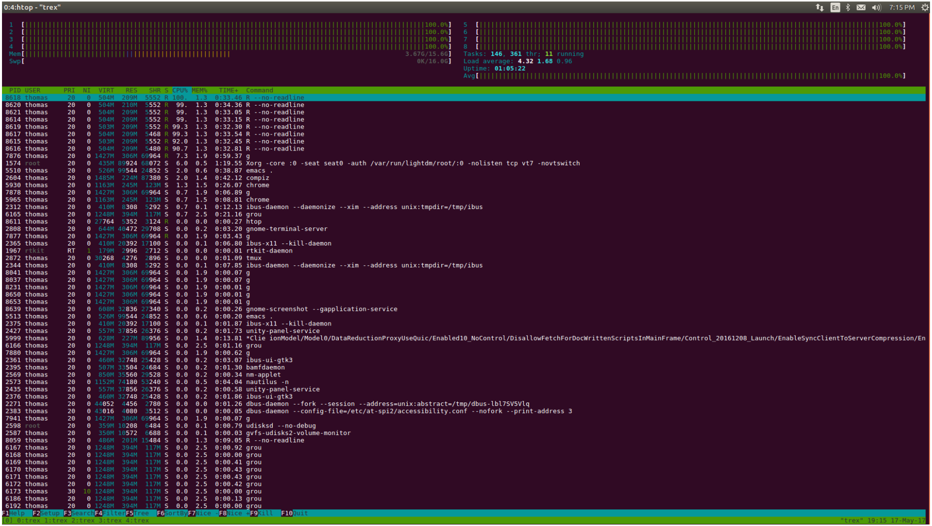
Calibrated model parameters

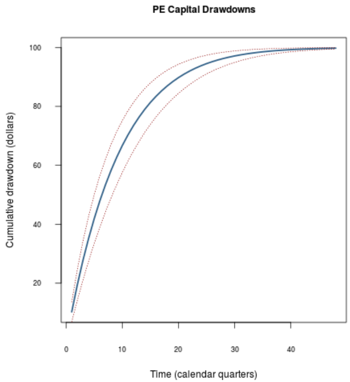
Table: Summary of baseline parameters used in illustration of risk-management model

Note: All model parameters are stated as annualized units, except where indicated

Parameter	Notation	Value
Life of the PE fund investment (years)	T_L	12
Simulation frequency (years)	$\mathrm{d}t$	1/4
Committed capital (US dollars)	C_0	100
Risk-free rate	r_f	0.05
Return on cash positions	r_c	0
Expected return of stock market	μ_M	0.11
Volatility of stock market returns	σ_M	0.15
Alpha of PE funds	α	0.04
Market beta of PE funds	β_M	1.30
Idiosyncratic volatility of PE fund returns	σ_ε	0.35
Drawdown rate of PE funds	δ	0.41
Volatility of the drawdown rate	σ_δ	0.21
Correlation between drawdown rate and stock market returns	ρ_δ	0.50
Average distribution rate	ν	0.08
Volatility of the distribution rate	σ_ν	0.11
Correlation between distribution rate and stock market returns	ρ_ν	0.80
Long-run mean of secondary market discounts	θ_π	0.16
Mean-reversion speed of secondary market discounts	κ_π	0.42
Volatility of secondary market discounts	σ_π	0.16
Initial secondary market discount	π_0	θ_π
Correlation between discount rate and stock market returns	ρ_π	-0.60

It's Monte Carlo time. . .





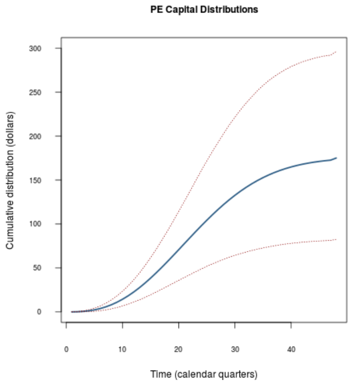


Figure: Cumulative capital drawdowns (left) and cumulative capital distributions (right). Solid lines represent Monte Carlo estimates of the average and dotted lines represent the 10^{th} & 90^{th} quantiles over 500,000 simulations

Fund dynamics (II)

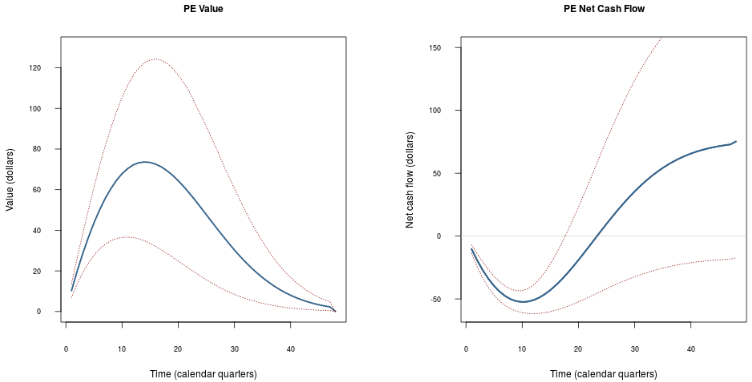


Figure: Fund values (left) and cumulative net fund cash flows (right). Solid lines represent Monte Carlo estimates of the average and dotted lines represent the 10^{th} & 90^{th} quantiles over 500,000 simulations

VaR dynamics

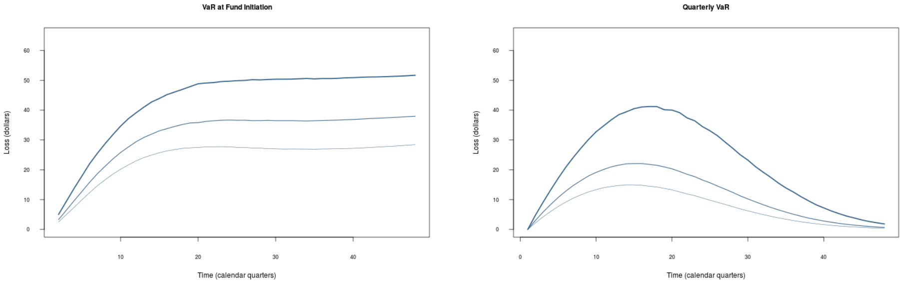
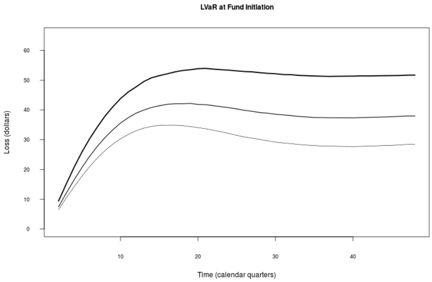


Figure: VaR dynamics over the fund lifecycle: (left) VaR at fund initiation, $\text{VaR}_{0,h}^{\alpha,\$}$, plotted as a function of the time horizon h; (right) quarterly VaR, *i.e.* $\text{VaR}_{t,0.25}^{\alpha,\$}$, plotted as a function of time t. The thickest line represents the Monte Carlo estimate of the 1% VaR over 500,000 simulations (also shown are the 5% VaR and the 10% VaR)

LVaR Dynamics



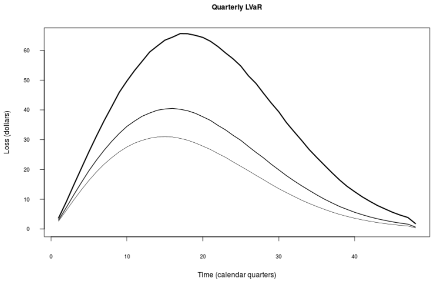
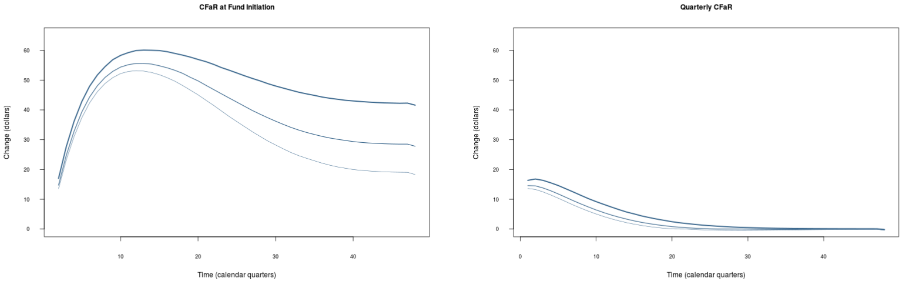


Figure: LVaR dynamics over the fund lifecycle: (left) LVaR at fund initiation, $\text{LVaR}_{0,h}^{\alpha,\$}$, plotted as a function of time horizon h; (right) quarterly LVaR, *i.e.* $\text{LVaR}_{t,0.25}^{\alpha,\$}$, plotted as a function of time t. The thickest line represents the Monte Carlo estimate of the 1% LVaR over 500,000 simulations (also shown are the 5% LVaR and the 10% LVaR)



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- If *ab initio* the basis for management-fee calculation is agreed to change from committed capital, C_0, for $0 \leq t \leq T_I$, to NIC for $T_I < t \leq T_L$, then how do GPs determine I_C, the capital available for investment, for $t \leq T_I$? Is it specified in the LPA?

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  1. Set the initial guess for NIC to C_0
  2. Subtract the fixed management fees applicable for $t \leq T_I$, which we know at $t = 0$ to follow

$$\mathrm{dMF}_t = c_{\mathrm{MF}} C_0 \mathrm{d}t \, \mathbf{1}_{0 \leq t \leq T_I} \tag{13}$$

  the value of NIC_t for $t = T_I$ is then initialized to $C_0 - \mathrm{MF}_{T_I}$

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Management fees: basis change to NIC requires *ex ante* computation

- If *ab initio* the basis for management-fee calculation is agreed to change from committed capital, C_0, for $0 \leq t \leq T_I$, to NIC for $T_I < t \leq T_L$, then how do GPs determine I_C, the capital available for investment, for $t \leq T_I$? Is it specified in the LPA?
- We use an iterative algorithm to arrive at the NIC (convergence is rapid):
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- **Carried interest with catch-up**: If the carried interest is paid with a 100% catch-up provision, then its dynamics are given by:

$$\mathrm{dCI}_t = \begin{cases} c_{\mathrm{CI}} \max\left\{\mathrm{dNCF}_t,\, 0\right\} \mathbf{1}_{\{\mathrm{IRR}_t > h\}}, & \text{if } \frac{\mathrm{CI}_t}{R_t - C_0} = c_{\mathrm{CI}} \\ \min\left\{c_{\mathrm{CI}}(R_t - C_0) - \mathrm{CI}_t,\, \mathrm{dNCF}_t\right\} \mathbf{1}_{\{\mathrm{IRR}_t > h\}}, & \text{if } \frac{\mathrm{CI}_t}{R_t - C_0} < c_{\mathrm{CI}} \end{cases} \quad (16)$$

where $\mathrm{dNCF}_t = \mathrm{d}R_t - \mathrm{d}D_t - \mathrm{dMF}_t$

Table: Carried Interest Calculation

This table illustrates the carried interest calculation for a $100M fund with a carried interest level of 20 percent, a hurdle rate of 8 percent, and a lifetime of ten years. The calculation is shown for a fund with no catch-up clause and fund with a catch-up clause of 100 percent.

Year	1	2	3	4	5	6	7	8	9	10	Total
Cash Flows	-50	-30	-10	-10	30	50	60	50	40	20	150
IRR (in % p.a.)	-100	-100	-100	-100	-33	-6	8	14	17	18	18
Carried Interest (No Catch-Up)	0	0	0	0	0	0	0	10	8	4	22
Carried Interest (With Catch-Up)	0	0	0	0	0	0	0	18	8	4	30

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Fee valuation: equilibrium framework

- We assume an equilibrium framework in which LPs' expected excess returns (net of fees) equal zero, such that GPs capture all rents from managing the funds:

$$\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_l} e^{-r_f u}\Big(dR_u - dD_u - dMF_u - dCI_u + dPF_u^{LP}\Big)\right] = 0$$

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$$V_0^{(GP)} = \underbrace{\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_L} e^{-r_f u} \mathrm{dMF}_u\right]}_{=V_0^{(MF)}} + \underbrace{\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_L} e^{-r_f u} \mathrm{dCI}_u\right]}_{=V_0^{(CI)}} + \underbrace{\mathscr{E}_{\mathbb{Q}}\left[\int_0^{T_L} e^{-r_f u} \mathrm{dPF}_u^{(GP)}\right]}_{=V_0^{(PF)}}, \qquad (19)$$

where $V_0^{(MF)}$ is the present value of management fees, $V_0^{(CI)}$ is the present value of carried interest payments, and $V_0^{(PF)}$ is the present value of lifetime portfolio company fees received by the GPs

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Death & taxes?

	NO DEAL FEES				WITH DEAL FEES			
	CONSTANT FEE BASIS		FEE BASIS CHANGE		CONSTANT FEE BASIS		FEE BASIS CHANGE	
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up
MF	$15.86	$15.86	$11.43	$11.43	$15.86	$15.86	$11.43	$11.43
(MF)			$0.40	$0.39			$0.39	$0.39
CI	$3.88	$4.23	$4.44	$4.82	$3.42	$3.81	$4.02	$4.41
(CI)	$8.78	$8.40	$9.49	$9.04	$8.08	$7.99	$8.94	$8.72
(CI 5%)								
(CI 10%)								
(CI 20%)								
(CI 30%)								
(CI 50%)								
(CI 60%)								
(CI 65%)			$1.51	$4.06			$0.80	$3.04
(CI 70%)	$2.05	$4.52	$3.09	$5.35	$1.20	$3.73	$2.32	$4.73
(CI 75%)	$3.88	$5.96	$5.06	$6.87	$2.92	$5.23	$4.17	$6.20
(CI 80%)	$6.12	$7.73	$7.41	$8.79	$5.09	$6.93	$6.43	$8.01
(CI 85%)	$9.01	$10.15	$10.44	$11.36	$7.89	$9.27	$9.41	$10.45
(CI 90%)	$13.15	$13.70	$14.82	$15.10	$11.86	$12.63	$13.60	$14.09
(CI 95%)	$20.68	$20.30	$22.59	$21.97	$18.92	$18.71	$21.10	$20.80
TF					$4.19	$4.19	$4.42	$4.42
(TF)					$0.09	$0.09	$0.09	$0.09
MoF					$7.30	$7.30	$7.71	$7.70
(MoF)					$3.96	$3.98	$4.22	$4.23
(MoF 5%)					$3.28	$3.28	$3.45	$3.45
(MoF 25%)					$4.73	$4.73	$4.99	$4.99
(MoF 75%)					$8.68	$8.69	$9.16	$9.16
(MoF 95%)					$14.56	$14.53	$15.39	$15.43

Figure: Net present value of a fund's fees. Management fee is denoted by "MF", carried interest by "CI", transaction fees by "TF" and monitoring fees by "MoF" (the latter pair being most common in Leveraged Buyout strategies). The means are shown in large font, while the values in parenthesis are either the standard deviations of the means, or the quantiles of the Monte Carlo distributions, as indicated by the quantile and the % sign

Death & taxes?

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	CONSTANT FEE BASIS		FEE BASIS CHANGE		CONSTANT FEE BASIS		FEE BASIS CHANGE	
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(MoF 25%)					$4.73	$4.73	$4.99	$4.99
(MoF 75%)					$8.68	$8.69	$9.16	$9.16
(MoF 95%)					$14.56	$14.53	$15.39	$15.43

Figure: Net present value of management fee (denoted by "MF")

	NO DEAL FEES				WITH DEAL FEES			
	CONSTANT FEE BASIS		FEE BASIS CHANGE		CONSTANT FEE BASIS		FEE BASIS CHANGE	
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up
MF	$15.86	$15.86	$11.43	$11.43	$15.86	$15.86	$11.43	$11.43
(MF)			$0.40	$0.39			$0.39	$0.39
CI	**$3.88**	**$4.23**	**$4.44**	**$4.82**	**$3.42**	**$3.81**	**$4.02**	**$4.41**
(CI)	**$8.78**	**$8.40**	**$9.49**	**$9.04**	**$8.08**	**$7.99**	**$8.94**	**$8.72**
(CI 5%)								
(CI 10%)								
(CI 20%)								
(CI 30%)								
(CI 50%)								
(CI 60%)								
(CI 65%)			**$1.51**	**$4.06**			**$0.80**	**$3.04**
(CI 70%)	**$2.05**	**$4.52**	**$3.09**	**$5.35**	**$1.20**	**$3.73**	**$2.32**	**$4.73**
(CI 75%)	**$3.88**	**$5.96**	**$5.06**	**$6.87**	**$2.92**	**$5.23**	**$4.17**	**$6.20**
(CI 80%)	**$6.12**	**$7.73**	**$7.41**	**$8.79**	**$5.09**	**$6.93**	**$6.43**	**$8.01**
(CI 85%)	**$9.01**	**$10.15**	**$10.44**	**$11.36**	**$7.89**	**$9.27**	**$9.41**	**$10.45**
(CI 90%)	**$13.15**	**$13.70**	**$14.82**	**$15.10**	**$11.86**	**$12.63**	**$13.60**	**$14.09**
(CI 95%)	**$20.68**	**$20.30**	**$22.59**	**$21.97**	**$18.92**	**$18.71**	**$21.10**	**$20.80**
TF					$4.19	$4.19	$4.42	$4.42
(TF)					$0.09	$0.09	$0.09	$0.09
MoF					$7.30	$7.30	$7.71	$7.70
(MoF)					$3.96	$3.98	$4.22	$4.23
(MoF 5%)					$3.28	$3.28	$3.45	$3.45
(MoF 25%)					$4.73	$4.73	$4.99	$4.99
(MoF 75%)					$8.68	$8.69	$9.16	$9.16
(MoF 95%)					$14.56	$14.53	$15.39	$15.43

Figure: Net present value of carried interest (denoted by "CI")

Death & taxes?

	NO DEAL FEES				WITH DEAL FEES			
	CONSTANT FEE BASIS		FEE BASIS CHANGE		CONSTANT FEE BASIS		FEE BASIS CHANGE	
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up
MF	$15.86	$15.86	$11.43	$11.43	$15.86	$15.86	$11.43	$11.43
(MF)			$0.40	$0.39			$0.39	$0.39
CI	$3.88	$4.23	$4.44	$4.82	$3.42	$3.81	$4.02	$4.41
(CI)	$8.78	$8.40	$9.49	$9.04	$8.08	$7.99	$8.94	$8.72
(CI 5%)								
(CI 10%)								
(CI 20%)								
(CI 30%)								
(CI 50%)								
(CI 60%)								
(CI 65%)			$1.51	$4.06			$0.80	$3.04
(CI 70%)	$2.05	$4.52	$3.09	$5.35	$1.20	$3.73	$2.32	$4.73
(CI 75%)	$3.88	$5.96	$5.06	$6.87	$2.92	$5.23	$4.17	$6.20
(CI 80%)	$6.12	$7.73	$7.41	$8.79	$5.09	$6.93	$6.43	$8.01
(CI 85%)	$9.01	$10.15	$10.44	$11.36	$7.89	$9.27	$9.41	$10.45
(CI 90%)	$13.15	$13.70	$14.82	$15.10	$11.86	$12.63	$13.60	$14.09
(CI 95%)	$20.68	$20.30	$22.59	$21.97	$18.92	$18.71	$21.10	$20.80
TF					**$4.19**	**$4.19**	**$4.42**	**$4.42**
(TF)					**$0.09**	**$0.09**	**$0.09**	**$0.09**
MoF					$7.30	$7.30	$7.71	$7.70
(MoF)					$3.96	$3.98	$4.22	$4.23
(MoF 5%)					$3.28	$3.28	$3.45	$3.45
(MoF 25%)					$4.73	$4.73	$4.99	$4.99
(MoF 75%)					$8.68	$8.69	$9.16	$9.16
(MoF 95%)					$14.56	$14.53	$15.39	$15.43

Figure: Net present value of transaction fees (denoted by “TF”) for Leveraged Buyout funds

Death & taxes?

	NO DEAL FEES				WITH DEAL FEES			
	CONSTANT FEE BASIS		FEE BASIS CHANGE		CONSTANT FEE BASIS		FEE BASIS CHANGE	
	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up	No catch-up	With catch-up
MF	$15.86	$15.86	$11.43	$11.43	$15.86	$15.86	$11.43	$11.43
(MF)			$0.40	$0.39			$0.39	$0.39
CI	$3.88	$4.23	$4.44	$4.82	$3.42	$3.81	$4.02	$4.41
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(CI 70%)	$2.05	$4.52	$3.09	$5.35	$1.20	$3.73	$2.32	$4.73
(CI 75%)	$3.88	$5.96	$5.06	$6.87	$2.92	$5.23	$4.17	$6.20
(CI 80%)	$6.12	$7.73	$7.41	$8.79	$5.09	$6.93	$6.43	$8.01
(CI 85%)	$9.01	$10.15	$10.44	$11.36	$7.89	$9.27	$9.41	$10.45
(CI 90%)	$13.15	$13.70	$14.82	$15.10	$11.86	$12.63	$13.60	$14.09
(CI 95%)	$20.68	$20.30	$22.59	$21.97	$18.92	$18.71	$21.10	$20.80
TF					$4.19	$4.19	$4.42	$4.42
(TF)					$0.09	$0.09	$0.09	$0.09
MoF					**$7.30**	**$7.30**	**$7.71**	**$7.70**
(MoF)					**$3.96**	**$3.98**	**$4.22**	**$4.23**
(MoF 5%)					**$3.28**	**$3.28**	**$3.45**	**$3.45**
(MoF 25%)					**$4.73**	**$4.73**	**$4.99**	**$4.99**
(MoF 75%)					**$8.68**	**$8.69**	**$9.16**	**$9.16**
(MoF 95%)					**$14.56**	**$14.53**	**$15.39**	**$15.43**

Figure: Net present value of monitoring fees (denoted by “MoF”) for Leveraged Buyout funds

Contents

To-do list

- Currently working on parameter estimation from the Preqin data set[7]

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- Release the package!

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Watch this space: https://github.com/tharte/PE

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