A multi-GPU implementation of the stochastic volatility and interest rate model in R

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Stochastic Volatility and Interest Rate Problem

The stochastic system with stochastic interest rate and stochastic volatility[1] takes the form of

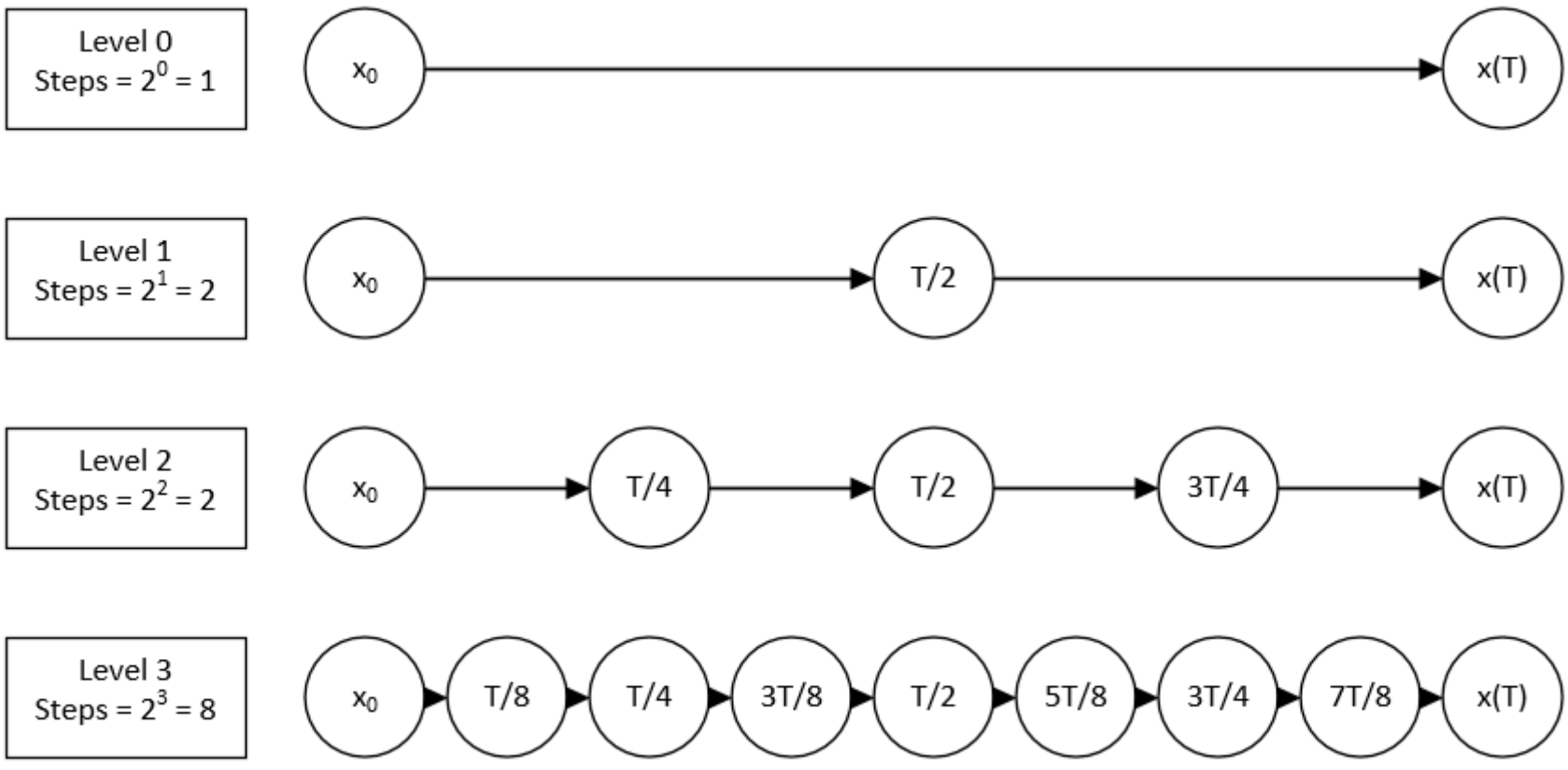
$$dS_t = r_t S_t dt + \sqrt{v_t} S_t dW_t^S$$

$$dv_t = \kappa_v(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$

$$dr_t = \kappa_r(\bar{r} - r_t)dt + \sigma_r \sqrt{r_t} dW_t^r$$

[1] Medvedev, Alexy, and Olivier Scaillet. 2010. "Pricing American Options Under Stochastic Volatility and Stochastic Interest Rates." *Journal of Financial Economics* 98 (1): 145–59.

Multilevel Monte Carlo



Multilevel Monte Carlo

- Multilevel Monte Carlo[2] performs Monte Carlo simulations using varying step sizes dt to reduce variance and thus computational complexity.
- If the MLMC has P levels, then for level N $= 0$ of the multilevel Monte Carlo,

$$E\Big[\text{MLMC}_{l_0}(T,\omega)\Big] = \frac{1}{M_0}\sum_{i=1}^{M_0} X_{l_0}(T,\omega)$$

- For levels $1 \leq N \leq P-1$

$$E\Big[\text{MLMC}_{l_N}(T,\omega)\Big] = \frac{1}{M_N}\sum_{i=1}^{M_N}\Big(X_{l_N}(T,\omega) - X_{l_{N-1}}(T,\omega)\Big)$$

[2]Giles, Michael B. 2008. "Multilevel Monte Carlo Path Simulation." *Operations Research* 56 (3): 607–17.

Multilevel Monte Carlo

The number of samples needed per level is estimated by[3]

$$N_l = \lceil 2\epsilon^{-2}\sqrt{V_l \delta t_l} \sum_{l=L_0}^{L_0+L-1} \sqrt{V_l/\delta t_l} \rceil$$

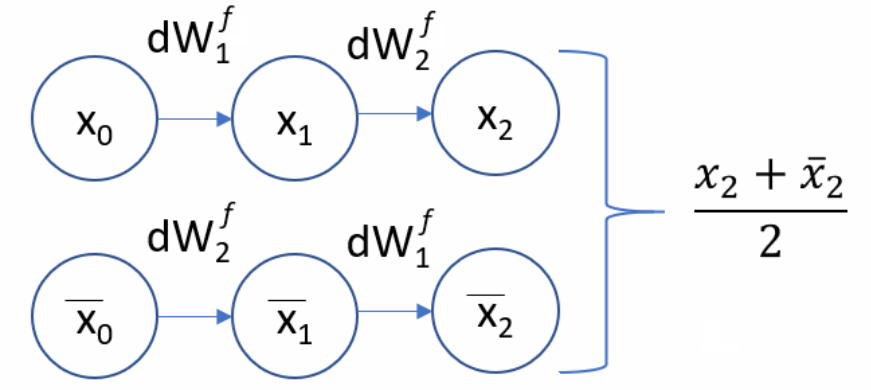
where

- N_l is the number of samples needed at level l
- ϵ is the error
- V_l is the variance at level l
- δt_l is the time step

[3] Giles, Michael B. 2008. "Multilevel Monte Carlo Path Simulation." *Operations Research* 56 (3): 607–17.

Antithetic Multilevel Monte Carlo

- Antithetic MLMC[4] further reduces variance in the system.
- Antithetic operation is performed on the *fine* step of the MLMC



[4] Giles, Michael B, and Lukasz Szpruch. 2014. "Antithetic Multilevel Monte Carlo Estimation for Multi-Dimensional Sdes Without Lévy Area Simulation." *The Annals of Applied Probability* 24 (4): 1585–1620.

MLMC GPU Implementation

Algorithm[5]

1. Read parameters for simulation
2. Pass parameters and initial sample size to C++ code via Rcpp.
3. C++ launches CUDA kernel on each GPU with an equal number of simulations
4. CUDA kernel performs simulations and returns mean and variance
5. R estimates samples by MLMC level
6. C++ code is called again with new sample sizes
7. Final results are returned from CUDA kernel

[5] **Lay, H.**, Colgin, Z., Reshniak, V., & Khaliq, A. (2018). "On the implementation of multilevel Monte Carlo simulation of the stochastic volatility and interest rate model using multi-GPU clusters." Under review.

MLMC GPU Implementation

- Each thread estimates the mean and variance of its group of samples
- Results are aggregated

$$\bar{X} = \frac{1}{\sum_{i \in \{G \times T\}} n_i} \sum_{i \in \{G \times T\}} n_i \bar{x}_i$$

$$\bar{s}^2 = \frac{1}{\{G \times T\}} \sum_{i \in \{G \times T\}} s_i^2$$

where $\bar{X}$ is the final mean and $\bar{s}^2$ is the final variance. For the threads T on GPU's G, $\bar{x}_i$ is the mean from that thread, n_i is the number of samples, and s_i^2 is the variance.

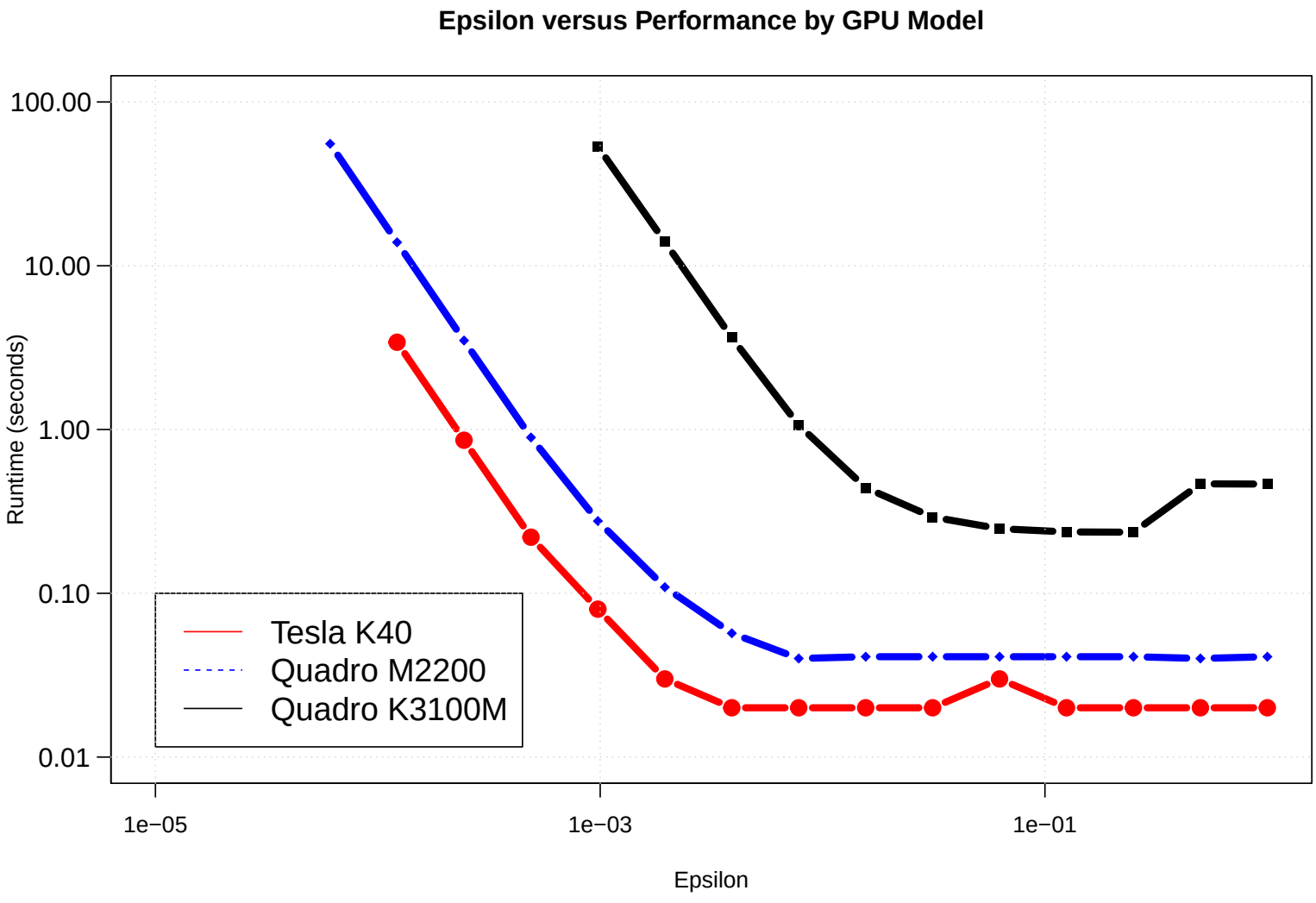
Results - Accuracy

- Accuracy of simulation for European put[6].
- Parameters used in all simulations: $\theta = 0.02$, $r_0 = 0.04$, $\bar{r} = 0.04$, $S_0 = 100$, $\rho_{13} = 0$, $\rho_{23} = 0$, $K_r = 0.3$, $\sigma_r = 0.1$

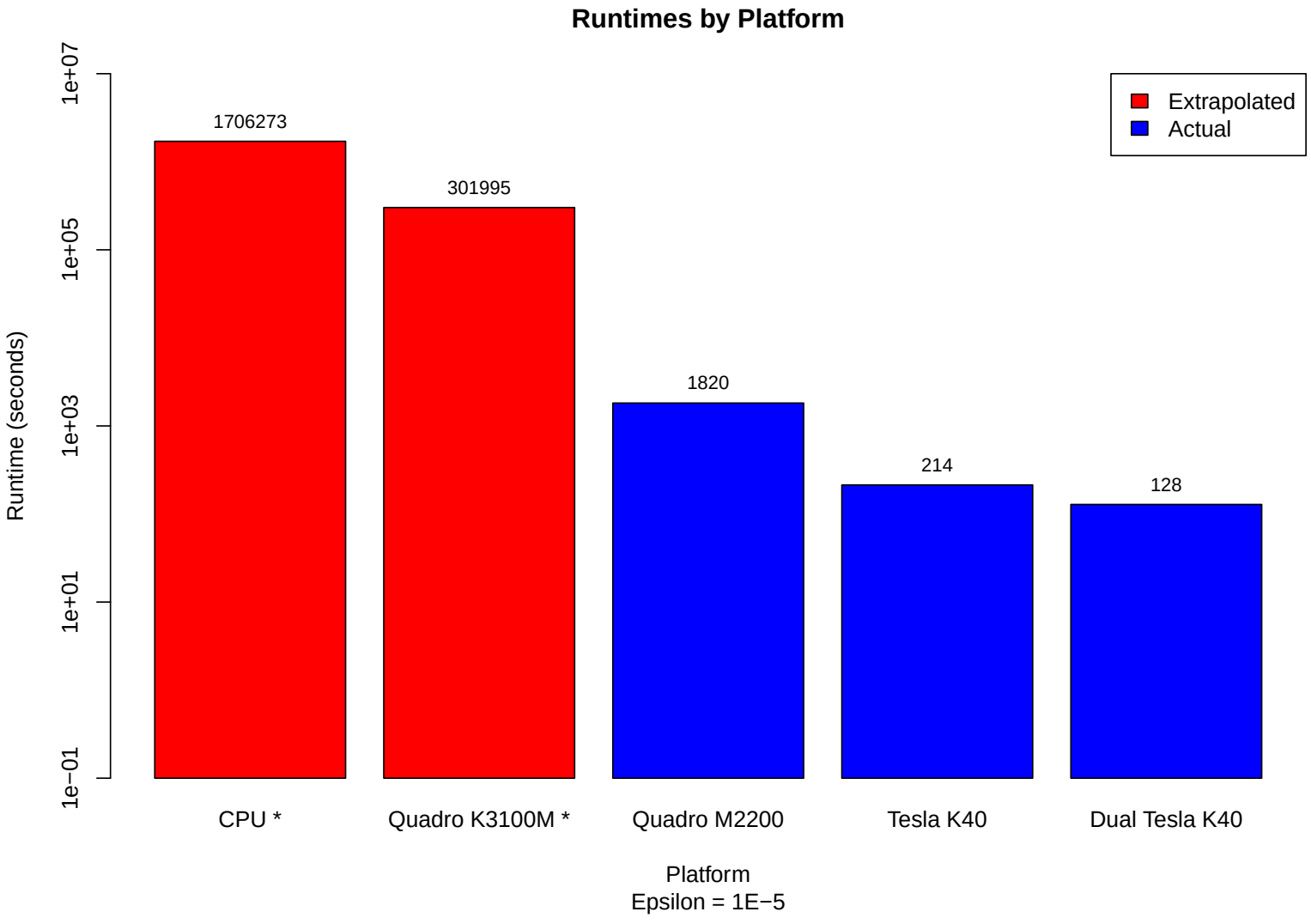
Parameters						Closed Form	Result	Error %
K	T	v	K_v	σ_v	ρ_{12}			
100	0.50	0.01	1.50	0.15	0.1	2.2254	2.2243	0.05%
110	0.25	0.04	0.75	0.30	0.1	9.0701	9.0699	0.00%
110	0.50	0.04	0.75	0.30	0.1	8.6410	8.6423	0.02%
100	0.25	0.04	1.50	0.30	0.1	3.3156	3.3151	0.02%
110	0.25	0.04	1.50	0.15	-0.5	10.0156	10.0160	0.00%
110	0.50	0.04	1.50	0.15	-0.5	10.3200	10.3190	0.01%

[6] Medvedev, Alexy, and Olivier Scaillet. 2010. "Pricing American Options Under Stochastic Volatility and Stochastic Interest Rates." *Journal of Financial Economics* 98 (1): 145–59.

Results - Performance



Results - Performance



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