Rational Explanation for Rule-of-Thumb Practices in Asset Allocation

Majeed Simaan[1,2] Yusif Simaan[3]

[1]Lally School of Management
Rensselaer Polytechnic Institute

[2]School of Business
Stevens Institute of Technology

[3]Gabelli School of Business
Fordham University

June 1st, 2018
R/Finance 2018 Annual Meeting
Chicago, IL

"*Markowitz came along, and there was light*"

-William F. Sharpe in Bernstein, 2011

In/Out Sample MV Efficient Frontier (MVEF)

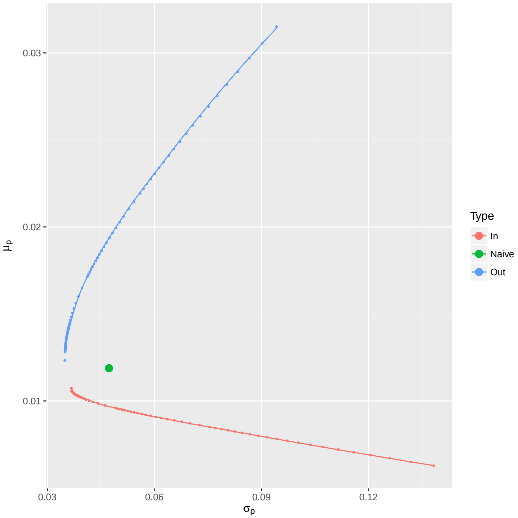


Figure: Fama-French 17 Industry Portfolios

In/Out Sample MV Efficient Frontier (MVEF)

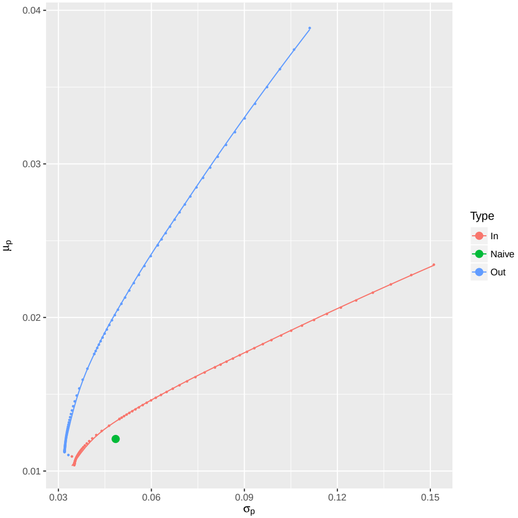


Figure: Fama-French 30 Industry Portfolios

In/Out Sample MV Efficient Frontier (MVEF)

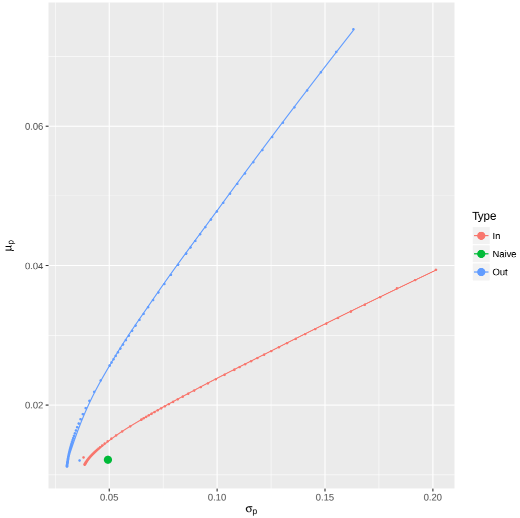


Figure: Fama-French 48 Industry Portfolios

In/Out Sample MV Efficient Frontier (MVEF)

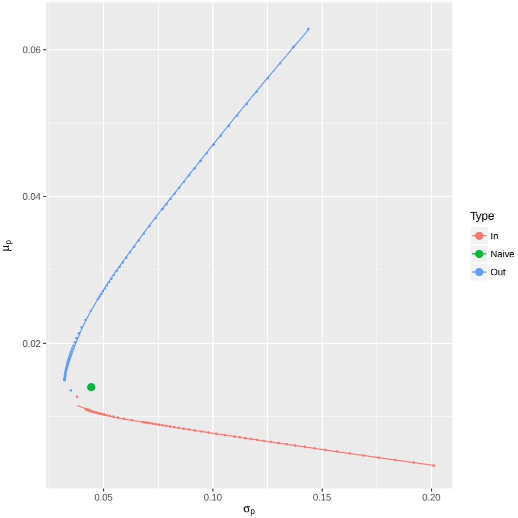


Figure: Largest 50 Market Cap S&P 500 Stocks

In/Out Sample MV Efficient Frontier (MVEF)

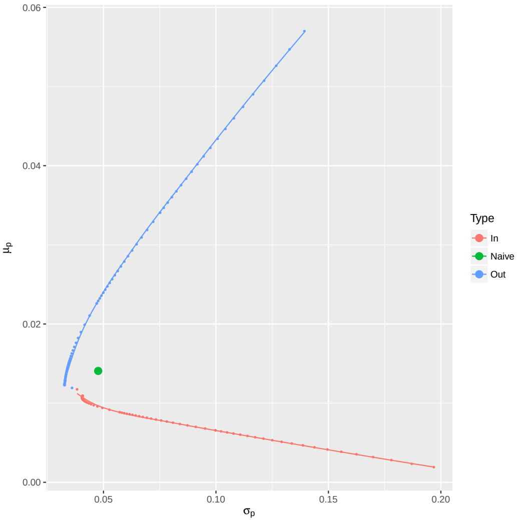


Figure: Second Largest 50 Market Cap S&P 500 Stocks

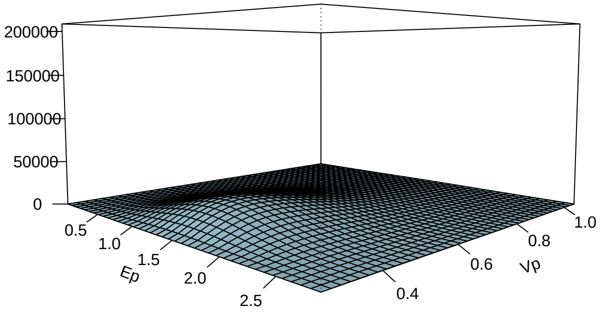
MVEF under Estimation Error

- "*Though Markowitz derived the [MVEF] more than 60 years ago, we still have no settled way to compute that frontier in real-world situations*." - Cochrane, 2014

MVEF under Estimation Error

- "*Though Markowitz derived the [MVEF] more than 60 years ago, we still have no settled way to compute that frontier in real-world situations.*" - Cochrane, 2014
- Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead

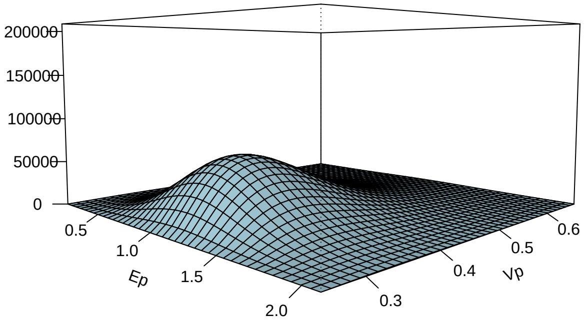
Figure: The sampling distribution of the MVEF using 10 years of data



MVEF under Estimation Error

- "*Though Markowitz derived the [MVEF] more than 60 years ago, we still have no settled way to compute that frontier in real-world situations.*" - Cochrane, 2014
- Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead

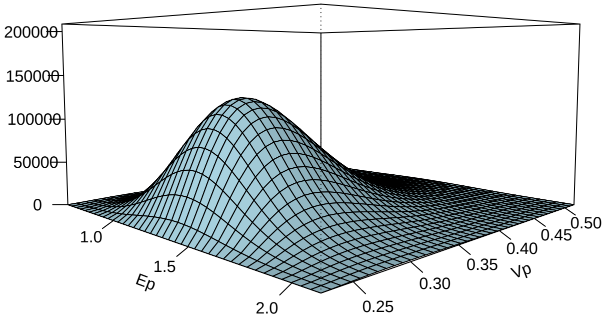
Figure: The sampling distribution of the MVEF using 20 years of data



MVEF under Estimation Error

- "*Though Markowitz derived the [MVEF] more than 60 years ago, we still have no settled way to compute that frontier in real-world situations*." - Cochrane, 2014
- Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead

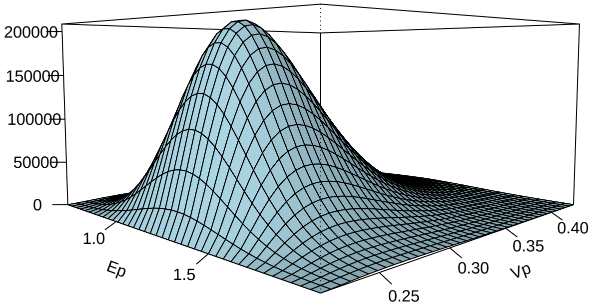
Figure: The sampling distribution of the MVEF using 30 years of data



MVEF under Estimation Error

- "*Though Markowitz derived the [MVEF] more than 60 years ago, we still have no settled way to compute that frontier in real-world situations.*" - Cochrane, 2014
- Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead

Figure: The sampling distribution of the MVEF using 40 years of data



Estimation Error

- Estimation error $\rightarrow$ poor out-of-sample performance, (see e.g. Michaud, 1989)
- Out-of-Sample Expected Utility (see e.g., Kan & Zhou, 2007)
- Shrinkage approaches?
  - Short-sale constraints (Jagannathan & Ma, 2003)
  - "Markowitz Meets Goldilocks" (Ledoit & Wolf, 2017)
- Should investors optimize?
  - 1/N naive portfolio by DeMiguel, Garlappi, & Uppal, 2009
  - "Markowitz meets Talmud" (Tu & Zhou, 2011)

This Research...

How to choose a portfolio under estimation error?

1. Bother estimating mean returns? (e.g., DeMiguel, Nogales, & Uppal, 2014)
2. Focus on variance alone? (e.g., Ledoit & Wolf, 2003)
3. Invest indifferently? (e.g., DeMiguel et al., 2009)

- We derive a set of rules to answer the above
- Our research provides a number of rational justification for common ad-hoc practices
    - Risk-Parity
    - Naive allocation
    - Hierarchical allocation (decentralized portfolio choice)

The Framework - Full Information

- Under full information, the MV portfolio is given by

$$\xi = f(\mu, \Sigma) = \alpha_0 + \frac{1}{A}\alpha_1, \quad (1)$$

where

  - μ and Σ are the mean vector and covariance matrix of asset returns, respectively
  - α_0 is the global minimum variance portfolio (GMV)
  - α_1 is an arbitrage portfolio (weights sum to 0) that depends on μ and Σ
  - A is the investor's risk aversion

The Framework - Full Information

- Under full information, the MV portfolio is given by

$$\xi = f(\mu, \Sigma) = \alpha_0 + \frac{1}{A}\alpha_1, \tag{1}$$

where

  - μ and Σ are the mean vector and covariance matrix of asset returns, respectively
  - α_0 is the global minimum variance portfolio (GMV)
  - α_1 is an arbitrage portfolio (weights sum to 0) that depends on μ and Σ
  - A is the investor's risk aversion

Estimation Error

- In practice, μ and Σ are unknown and are evaluated ex-ante
- The result of which induces estimation error into the paradigm

The Framework - Estimation Error

- Let m and S denote the sample estimates of μ and Σ, respectively
  - using a sample of the recent n periods

The Framework - Estimation Error

- Let m and S denote the sample estimates of μ and Σ, respectively
  - using a sample of the recent n periods
- The estimated portfolio, thus, is given by

$$X = f(m, S) = X_0 + \frac{1}{A} X_1 \tag{2}$$

with $X_0 = \hat{\alpha}_0$ and $X_1 = \hat{\alpha}_1$

The Framework - Estimation Error

- Let m and S denote the sample estimates of μ and Σ, respectively
  - using a sample of the recent n periods
- The estimated portfolio, thus, is given by

$$X = f(m, S) = X_0 + \frac{1}{A} X_1 \tag{2}$$

with $X_0 = \hat{\alpha}_0$ and $X_1 = \hat{\alpha}_1$

- If R is the vector of asset returns for the $n + 1$ period, then

$$r_p = X' R = r_0 + \frac{1}{A} r_1 \tag{3}$$

denotes the ex-post portfolio return

The Framework - Estimation Error

- Let m and S denote the sample estimates of μ and Σ, respectively
  - using a sample of the recent n periods
- The estimated portfolio, thus, is given by

$$X = f(m, S) = X_0 + \frac{1}{A} X_1 \tag{2}$$

with $X_0 = \hat{\alpha}_0$ and $X_1 = \hat{\alpha}_1$

- If R is the vector of asset returns for the $n+1$ period, then

$$r_p = X'R = r_0 + \frac{1}{A} r_1 \tag{3}$$

denotes the ex-post portfolio return

To draw economic conclusions about the impact of estimation error on the MVEF, we need to analyze the distribution of r_p

Constructing MVEF under Estimation Error

- Demonstration of the MVEF full information versus estimation risk
- MVEF was derived using the FF-48 industry data between Jan 1970 and Dec 2015

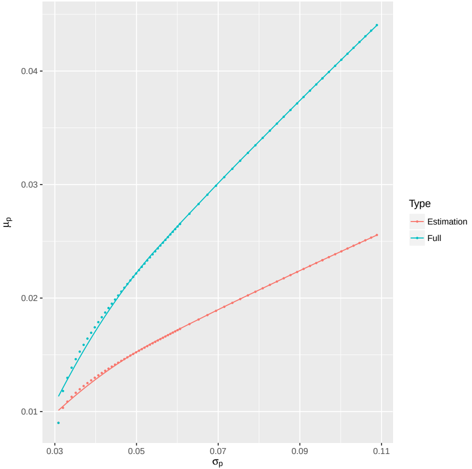


Figure: 10 years of data

Constructing MVEF under Estimation Error

- Demonstration of the MVEF full information versus estimation risk
- MVEF was derived using the FF-48 industry data between Jan 1970 and Dec 2015

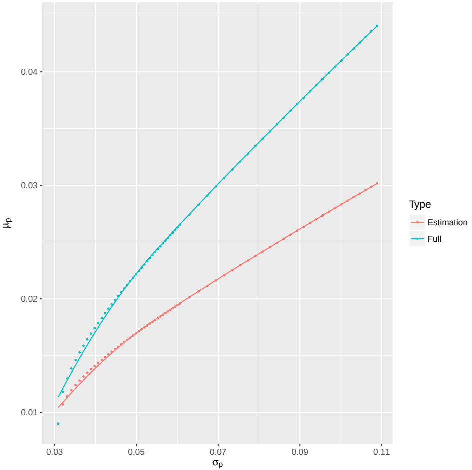


Figure: 20 years of data

Constructing MVEF under Estimation Error

- Demonstration of the MVEF full information versus estimation risk
- MVEF was derived using the FF-48 industry data between Jan 1970 and Dec 2015

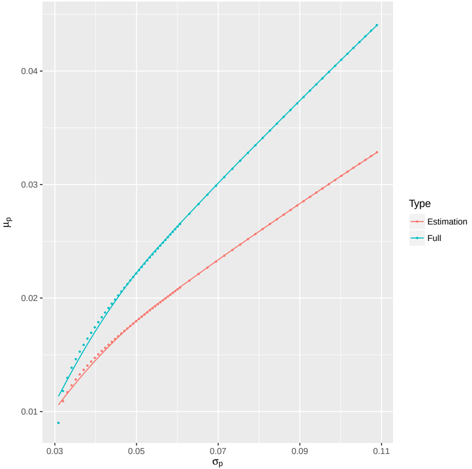


Figure: 30 years of data

- Demonstration of the MVEF full information versus estimation risk
- MVEF was derived using the FF-48 industry data between Jan 1970 and Dec 2015

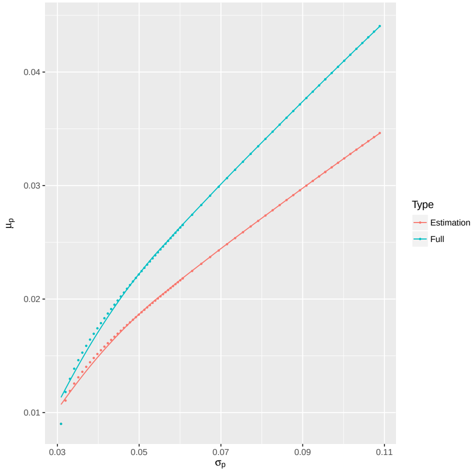


Figure: 40 years of data

Decision Rules under Estimation Risk

Step 1: MV versus GMV

- The GMV portfolio is preferable to any portfolio on the MVE frontier, if[a]

$$\frac{\sigma_\mu^2}{\sigma^2} < \frac{(1-\rho)}{n-1} \tag{4}$$

with σ_μ^2 denoting the cross-sectional variation among the mean returns

[a] Condition is simplified for the case when correlation and volatilities are uniform

Decision Rules under Estimation Risk

Step 1: MV versus GMV

- The GMV portfolio is preferable to any portfolio on the MVE frontier, if[a]

$$\frac{\sigma_\mu^2}{\sigma^2} < \frac{(1-\rho)}{n-1} \tag{4}$$

with σ_μ^2 denoting the cross-sectional variation among the mean returns

[a] Condition is simplified for the case when correlation and volatilities are uniform

Step 2: GMV versus Naive

- The naive portfolio is preferable to GMV if[a]

$$\frac{\sigma_N^2}{\sigma_0^2} < \frac{n}{n-d+1} \tag{5}$$

where σ_N^2 (σ_0^2) is the naive (GMV) volatility

[a] Condition is simplified for the case when the mean returns are uniform

Implementation

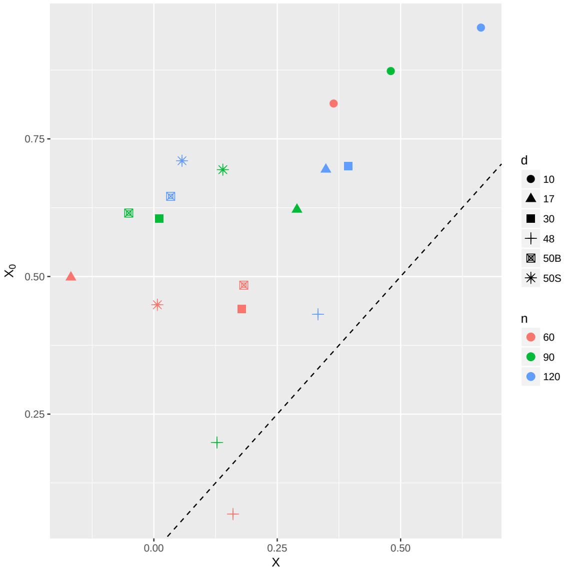
For a given dataset (d assets), do the following:

1. Starting at t (Aug 1986), use the recent $n \in \{60, 90, 120\}$ months to estimate (m, S)
2. Compute the MV X, GMV X_0, and naive X_N portfolios[a]
   - X chosen as the maximum SR portfolio on the MVEF
3. Realize the next period return of each portfolio
4. Repeat the above steps until the end (Dec 2015) on a rolling basis
5. Finally, summarize performance using out-of-sample SR

[a] Constraints on exposure to maximum/minimum individual asset allocation were deployed in the mixed strategy analysis only with respect to Jagannathan & Ma, 2003; Levy & Levy, 2014.

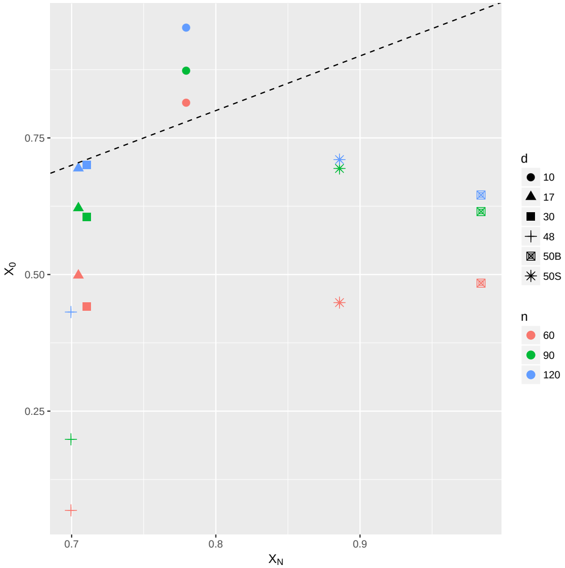
Out-of-Sample SR: GMV versus MV

- y and x axis denote the SR of GMV and MV, respectively
- Dashed line is a 45-degrees line
- Data is distinguished using shapes
- Colors highlight sample size



Out-of-Sample SR: GMV versus Naive

- y and x axis denote the SR of GMV and Naive, respectively
- Dashed line is a 45-degrees line
- Data is distinguished using shapes
- Colors highlight sample size



Implications from Condition 1

- y-axis is the difference in SR between MV and GMV
- x-axis is the LHS from Condition (4), i.e. σ_μ / σ
- Data is distinguished using colors
- Solid line is a fitted linear regression

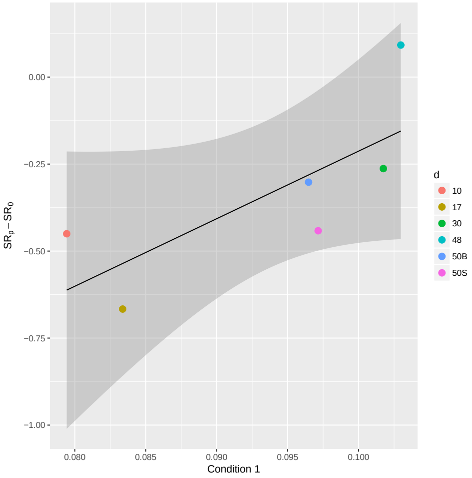


Figure: $n = 60$ months

Implications from Condition 1

- y-axis is the difference in SR between MV and GMV
- x-axis is the LHS from Condition (4), i.e. σ_μ/σ
- Data is distinguished using colors
- Solid line is a fitted linear regression

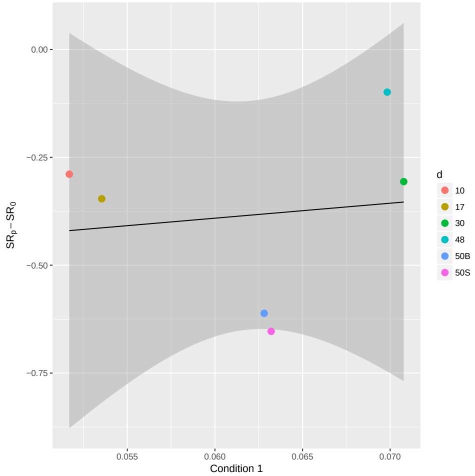


Figure: $n = 120$ months

Implications from Condition 2

- y-axis is the difference in SR between GMV and Naive
- x-axis is the LHS from Condition (5), i.e. σ_N / σ_0
- Datasets are distinguished using colors
- Solid line is a fitted linear regression

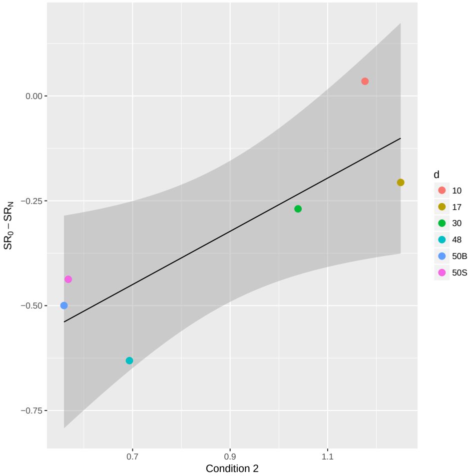


Figure: $n = 60$ months

Implications from Condition 2

- y-axis is the difference in SR between GMV and Naive
- x-axis is the LHS from Condition (5), i.e. σ_N/σ_0
- Datasets are distinguished using colors
- Solid line is a fitted linear regression

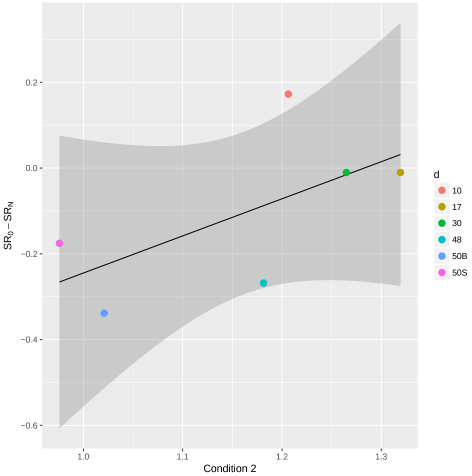
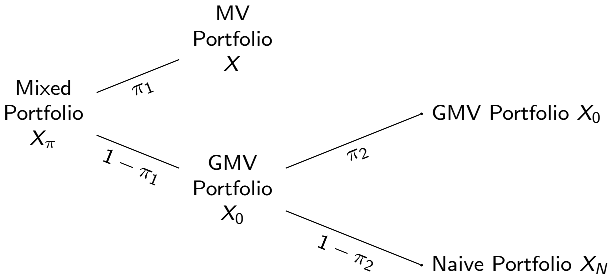
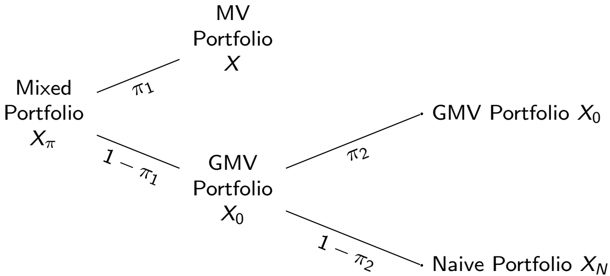


Figure: $n = 120$ months

Mixed Strategy



Mixed Strategy



Calibration

```
> f.star <- function(PI) {
+       Pi <- PI[1]
+       Pi2 <- PI[2]
+       X_pi <- Pi*X + (1-Pi)*( X0*Pi2   + (1-Pi2)*X_N  )
+       ret.pi <- as.matrix(RET12) %*% X_pi # RET12 - last 12 monthly returns
+       return(-mean(ret.pi)/sd(ret.pi)) # negative SR
+       }
>
> Pi.star <- nlminb(runif(2), f.star,lower = c(0,0),upper = c(1,1))[[1]]
```

Mixed Strategy - Performance Comparison

- y-axis is the difference in SR between the mixed strategy and each of the three portfolios
- From left to right, x-axis corresponds to the MV, GMV, and Naive portfolios
- Datasets are distinguished using colors
- Graph is created using `geom_violin`

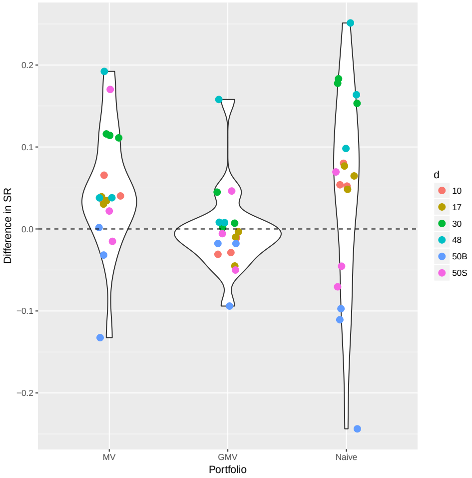


Figure: All datasets and sample sizes

- *y*-axis is the difference in SR between the mixed strategy and each of the three portfolios
- From left to right, *x*-axis corresponds to the MV, GMV, and Naive portfolios
- Datasets are distinguished using colors
- Graph is created using `geom_violin`

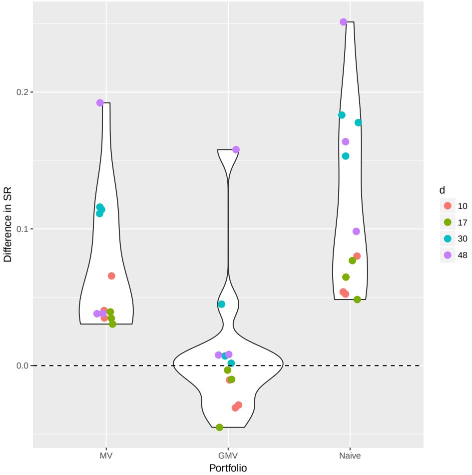


Figure: All sample sizes excluding stocks

Mixed Strategy - Performance Comparison

- y-axis is the difference in SR between the mixed strategy and each of the three portfolios
- From left to right, x-axis corresponds to the MV, GMV, and Naive portfolios
- Datasets are distinguished using colors
- Graph is created using `geom_violin`

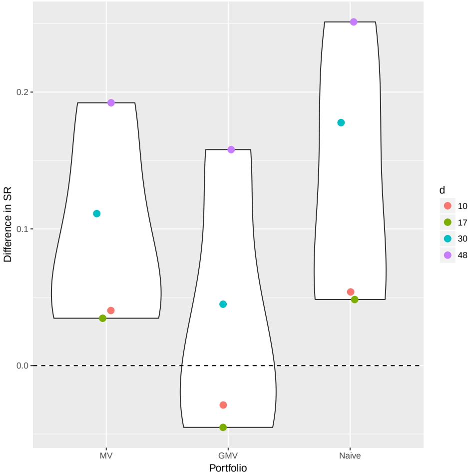


Figure: $n = 60$ excluding stocks

Mixed Strategy - Performance Comparison

- y-axis is the difference in SR between the mixed strategy and each of the three portfolios
- From left to right, x-axis corresponds to the MV, GMV, and Naive portfolios
- Datasets are distinguished using colors
- Graph is created using `geom_violin`

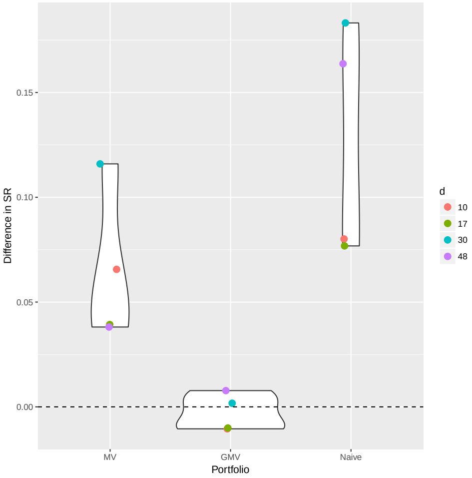


Figure: $n = 120$ excluding stocks

Concluding Remarks

"*Diversification is protection against ignorance. It makes little sense if you know what you are doing.*"

-Warren Buffett

Concluding Remarks

> "*Diversification is protection against ignorance. It makes little sense if you know what you are doing.*"
>
> -Warren Buffett

- **Optimization** (**naive allocation**) makes sense if you **know** (**don't know**) what you are doing
  - Industries are less prone to estimation error than individual stocks
  - Potential optimization among industries is more evident
  - Naive strategy dominates across stocks
  - Evidence supports hierarchical asset allocation

Concluding Remarks

> "*Diversification is protection against ignorance. It makes little sense if you know what you are doing.*"
>
> -Warren Buffett

- **Optimization** (**naive allocation**) makes sense if you **know** (**don't know**) what you are doing
  - Industries are less prone to estimation error than individual stocks
  - Potential optimization among industries is more evident
  - Naive strategy dominates across stocks
  - Evidence supports hierarchical asset allocation

- *Further results are in progress...*

Thank You!

Stay in touch...

Email:	`simaan.finance@gmail.com`
Linkedin:	`https://www.linkedin.com/in/majeed-simaan-85383045`
GitHub:	`https://github.com/simaan84`

Appendix - Empirical Results **Without** Position Constraints

n	d	X	X_0	X_N	X_π	π_1	π_2	σ_μ/σ	σ_N/σ_0
60	10	0.36	0.81	0.78	0.47	0.62	0.48	0.08	1.18
90	10	0.48	0.87	0.78	0.59	0.58	0.48	0.06	1.21
120	10	0.66	0.95	0.78	0.75	0.56	0.46	0.05	1.21
60	17	-0.17	0.50	0.70	-0.16	0.65	0.51	0.08	1.25
90	17	0.29	0.62	0.70	0.37	0.65	0.53	0.07	1.30
120	17	0.35	0.69	0.70	0.36	0.65	0.47	0.05	1.32
60	30	0.18	0.44	0.71	0.18	0.72	0.54	0.10	1.04
90	30	0.01	0.61	0.71	0.09	0.68	0.52	0.08	1.21
120	30	0.39	0.70	0.71	0.54	0.65	0.52	0.07	1.26
60	48	0.16	0.07	0.70	0.13	0.70	0.61	0.10	0.69
90	48	0.13	0.20	0.70	0.09	0.68	0.57	0.08	1.04
120	48	0.33	0.43	0.70	0.46	0.65	0.54	0.07	1.18
60	50B	0.18	0.48	0.98	0.18	0.73	0.59	0.10	0.56
90	50B	-0.05	0.62	0.98	0.01	0.66	0.50	0.08	0.90
120	50B	0.03	0.65	0.98	0.20	0.64	0.48	0.06	1.02
60	50S	0.01	0.45	0.89	-0.05	0.72	0.71	0.10	0.57
90	50S	0.14	0.69	0.89	0.17	0.71	0.50	0.07	0.85
120	50S	0.06	0.71	0.89	0.15	0.67	0.53	0.06	0.98

- Table reports the out-of-sample SR for each portfolio.

Appendix - Empirical Results **With** Position Constraints

n	d	X	X_0	X_N	X_π	π_1	π_2	σ_μ/σ	σ_N/σ_0
60	10	0.79	0.86	0.78	0.83	0.37	0.70	0.08	1.09
90	10	0.80	0.86	0.78	0.83	0.32	0.70	0.06	1.10
120	10	0.79	0.87	0.78	0.86	0.27	0.72	0.05	1.09
60	17	0.72	0.80	0.70	0.75	0.47	0.73	0.08	1.15
90	17	0.74	0.77	0.70	0.77	0.41	0.70	0.07	1.14
120	17	0.74	0.79	0.70	0.78	0.40	0.70	0.05	1.14
60	30	0.78	0.84	0.71	0.89	0.48	0.73	0.10	1.25
90	30	0.75	0.86	0.71	0.86	0.45	0.74	0.08	1.23
120	30	0.78	0.89	0.71	0.89	0.43	0.70	0.07	1.24
60	48	0.76	0.79	0.70	0.95	0.51	0.74	0.10	1.36
90	48	0.76	0.79	0.70	0.80	0.50	0.73	0.08	1.34
120	48	0.83	0.86	0.70	0.86	0.46	0.72	0.07	1.32
60	50B	0.87	0.83	0.98	0.74	0.47	0.71	0.10	1.18
90	50B	0.87	0.89	0.98	0.87	0.59	0.64	0.08	1.19
120	50B	0.92	0.90	0.98	0.89	0.50	0.60	0.06	1.18
60	50S	0.79	0.91	0.89	0.96	0.34	0.83	0.10	1.23
90	50S	0.82	0.85	0.89	0.84	0.48	0.70	0.07	1.21
120	50S	0.83	0.87	0.89	0.82	0.46	0.67	0.06	1.23

- Table reports the out-of-sample SR for each portfolio
- π_1 (π_2) denote the proportion of time that $X \succ X_0$ ($X_0 \succ X_N$)
- $X_\pi = \pi_1 X + (1 - \pi_1)(\pi_2 X_0 + (1 - \pi_2) X_N)$

References I

[]Bernstein, P. L. 2011. *Capital ideas evolving*. John Wiley & Sons.

[]Cochrane, J. H. 2014. A mean-variance benchmark for intertemporal portfolio theory. *The Journal of Finance*, *69*(1), 1–49.

[]DeMiguel, V., Garlappi, L., & Uppal, R. 2009. Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *Review of Financial Studies*, *22*(5), 1915–1953.

[]DeMiguel, V., Nogales, F. J., & Uppal, R. 2014. Stock return serial dependence and out-of-sample portfolio performance. *Review of Financial Studies*, *27*(4), 1031–1073.

[]Grolemund, G., & Wickham, H. 2011. Dates and times made easy with lubridate. *Journal of Statistical Software*, *40*(3), 1–25. Retrieved from http://www.jstatsoft.org/v40/i03/

[]Hlavac, M. 2015. stargazer: Well-formatted regression and summary statistics tables [Computer software manual]. Cambridge, USA. Retrieved from http://CRAN.R-project.org/package=stargazer (R package version 5.2)

[]Jagannathan, R., & Ma, T. 2003. Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance*, *58*(4), 1651–1684.

[]Kan, R., & Zhou, G. 2007. Optimal portfolio choice with parameter uncertainty. *Journal of Financial and Quantitative Analysis*, *42*(03), 621–656.

[]Ledoit, O., & Wolf, M. 2003. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of empirical finance*, *10*(5), 603–621.

[]Ledoit, O., & Wolf, M. 2017. Nonlinear shrinkage of the covariance matrix for portfolio selection: Markowitz meets goldilocks. *The Review of Financial Studies*, *30*(12), 4349–4388.

References III

[]Levy, H., & Levy, M. 2014. The benefits of differential variance-based constraints in portfolio optimization. *European Journal of Operational Research*, *234*(2), 372–381.

[]Michaud, R. O. 1989. The markowitz optimization enigma: is' optimized'optimal? *Financial Analysts Journal*, *45*(1), 31–42.

[]Simaan, M., Simaan, Y., & Tang, Y. 2017. Estimation error in mean returns and the mean-variance efficient frontier. *International Review of Economics and Finance*.

[]Tu, J., & Zhou, G. 2011. Markowitz meets talmud: A combination of sophisticated and naive diversification strategies. *Journal of Financial Economics*, *99*(1), 204–215.

[]Wickham, H. 2011. The split-apply-combine strategy for data analysis. *Journal of Statistical Software*, *40*(1), 1–29. Retrieved from `http://www.jstatsoft.org/v40/i01/`

[]Wickham, H. 2016. *ggplot2: Elegant graphics for data analysis*. Springer-Verlag New York. Retrieved from `http://ggplot2.org`