Rational Explanation for Rule-of-Thumb Practices in Asset Allocation

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“Markowitz came along, and there was light”

-William F. Sharpe in Bernstein, 2011
Figure: Fama-French 17 Industry Portfolios
In/Out Sample MV Efficient Frontier (MVEF)

Figure: Fama-French 30 Industry Portfolios
In/Out Sample MV Efficient Frontier (MVEF)

Figure: Fama-French 48 Industry Portfolios
Figure: Largest 50 Market Cap S&P 500 Stocks
In/Out Sample MV Efficient Frontier (MVEF)

Figure: Second Largest 50 Market Cap S&P 500 Stocks
“Though Markowitz derived the [MVEF] more than 60 years ago, we still have no settled way to compute that frontier in real-world situations.” - Cochrane, 2014
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Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead.

**Figure:** The sampling distribution of the MVEF using 10 years of data
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**Figure:** The sampling distribution of the MVEF using 20 years of data
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Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead.

**Figure:** The sampling distribution of the MVEF using 30 years of data
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Simaan et al., 2017, for instance, show that for each point on the MVEF, there is a joint sampling distribution instead.

**Figure:** The sampling distribution of the MVEF using 40 years of data
Estimation Error

- Estimation error $\rightarrow$ poor out-of-sample performance, (see e.g. Michaud, 1989)
- Out-of-Sample Expected Utility (see e.g., Kan & Zhou, 2007)
- Shrinkage approaches?
  - Short-sale constraints (Jagannathan & Ma, 2003)
  - “Markowitz Meets Goldilocks” (Ledoit & Wolf, 2017)
- Should investors optimize?
  - $1/N$ naive portfolio by DeMiguel, Garlappi, & Uppal, 2009
  - “Markowitz meets Talmud” (Tu & Zhou, 2011)
This Research...

How to choose a portfolio under estimation error?

1. Bother estimating mean returns? (e.g., DeMiguel, Nogales, & Uppal, 2014)
2. Focus on variance alone? (e.g., Ledoit & Wolf, 2003)
3. Invest indifferently? (e.g., DeMiguel et al., 2009)

We derive a set of rules to answer the above

Our research provides a number of rational justification for common ad-hoc practices

- Risk-Parity
- Naive allocation
- Hierarchical allocation (decentralized portfolio choice)
The Framework - Full Information

- Under full information, the MV portfolio is given by

\[ \xi = f(\mu, \Sigma) = \alpha_0 + \frac{1}{A} \alpha_1, \]  

(1)

where

- \( \mu \) and \( \Sigma \) are the mean vector and covariance matrix of asset returns, respectively
- \( \alpha_0 \) is the global minimum variance portfolio (GMV)
- \( \alpha_1 \) is an arbitrage portfolio (weights sum to 0) that depends on \( \mu \) and \( \Sigma \)
- \( A \) is the investor’s risk aversion
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Estimation Error

- In practice, \( \mu \) and \( \Sigma \) are unknown and are evaluated ex-ante
- The result of which induces estimation error into the paradigm
The Framework - Estimation Error

- Let $m$ and $S$ denote the sample estimates of $\mu$ and $\Sigma$, respectively
  - using a sample of the recent $n$ periods

For a sample period $n+1$, the estimated portfolio is given by

$$X = X_0 + AX_1$$

with $X_0 = \hat{\alpha}_0$ and $X_1 = \hat{\alpha}_1$.

The vector of asset returns for the $n+1$ period is $R$, and the ex-post portfolio return is

$$r_p = X' R = r_0 + Ar_1$$

To draw economic conclusions about the impact of estimation error on the MVEF, we need to analyze the distribution of $r_p$.
Let \( m \) and \( S \) denote the sample estimates of \( \mu \) and \( \Sigma \), respectively using a sample of the recent \( n \) periods.

The estimated portfolio, thus, is given by

\[
X = f(m, S) = X_0 + \frac{1}{A} X_1
\] (2)

with \( X_0 = \hat{\alpha}_0 \) and \( X_1 = \hat{\alpha}_1 \).
The Framework - Estimation Error

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- If $R$ is the vector of asset returns for the $n + 1$ period, then
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To draw economic conclusions about the impact of estimation error on the MVEF, we need to analyze the distribution of \( r_p \).
Constructing MVEF under Estimation Error

- Demonstration of the MVEF full information versus estimation risk
- MVEF was derived using the FF-48 industry data between Jan 1970 and Dec 2015

Figure: 10 years of data
Constructing MVEF under Estimation Error

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Figure: 20 years of data
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Figure: 30 years of data
Constructing MVEF under Estimation Error

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Figure: 40 years of data
Step 1: MV versus GMV

The GMV portfolio is preferable to any portfolio on the MVE frontier, if

$$\frac{\sigma^2_{\mu}}{\sigma^2} < \frac{(1 - \rho)}{n - 1}$$

(4)

with $$\sigma^2_{\mu}$$ denoting the cross-sectional variation among the mean returns.

\(^a\) Condition is simplified for the case when correlation and volatilities are uniform.
Decision Rules under Estimation Risk

Step 1: MV versus GMV

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with \(\sigma^2_\mu\) denoting the cross-sectional variation among the mean returns.

\(^a\) Condition is simplified for the case when correlation and volatilities are uniform.

Step 2: GMV versus Naive

- The naive portfolio is preferable to GMV if

\[
\frac{\sigma^2_N}{\sigma^2_0} < \frac{n}{n - d + 1}
\]

where \(\sigma^2_N\) (\(\sigma^2_0\)) is the naive (GMV) volatility.

\(^a\) Condition is simplified for the case when the mean returns are uniform.
Implementation

For a given dataset (d assets), do the following:

1. Starting at $t$ (Aug 1986), use the recent $n \in \{60, 90, 120\}$ months to estimate $(m, S)$

2. Compute the MV $X$, GMV $X_0$, and naive $X_N$ portfolios\(^a\)
   - $X$ chosen as the maximum SR portfolio on the MVEF

3. Realize the next period return of each portfolio

4. Repeat the above steps until the end (Dec 2015) on a rolling basis

5. Finally, summarize performance using out-of-sample SR

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\(^a\)Constraints on exposure to maximum/minimum individual asset allocation were deployed in the mixed strategy analysis only with respect to Jagannathan & Ma, 2003; Levy & Levy, 2014.
Out-of-Sample SR: GMV versus MV

- $y$ and $x$ axis denote the SR of GMV and MV, respectively.
- Dashed line is a 45-degrees line.
- Data is distinguished using shapes.
- Colors highlight sample size.
Out-of-Sample SR: GMV versus Naive

- $y$ and $x$ axis denote the SR of GMV and Naive, respectively.
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Implications from Condition 1

- $y$-axis is the difference in SR between MV and GMV
- $x$-axis is the LHS from Condition (4), i.e. $\sigma_\mu/\sigma$
- Data is distinguished using colors
- Solid line is a fitted linear regression

Figure: $n = 60$ months
Implications from Condition 1

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Figure: $n = 120$ months
Implications from Condition 2

- $y$-axis is the difference in SR between GMV and Naive
- $x$-axis is the LHS from Condition (5), i.e. $\sigma_N/\sigma_0$
- Datasets are distinguished using colors
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**Figure:** $n = 60$ months
Implications from Condition 2

- $y$-axis is the difference in SR between GMV and Naive
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Figure: $n = 120$ months
Mixed Strategy

Mixed Portfolio $X_{\pi}$

$\pi_1$

$1 - \pi_1$

MV Portfolio $X$

GMV Portfolio $X_0$

$\pi_2$

$1 - \pi_2$

GMV Portfolio $X_0$

Naive Portfolio $X_N$

Calibration

\[
\text{f.star <- function(PI) } \text{\{ }
+ \text{ Pi <- PI[1]}
+ \text{ Pi2 <- PI[2]}
+ \text{ X_pi <- Pi*X + (1-Pi)*( X0*Pi2 + (1-Pi2)*X_N )}
+ \text{ ret.pi <- as.matrix(RET12) %*% X_pi # RET12 - last 12 monthly returns}
+ \text{ return(-mean(ret.pi)/sd(ret.pi)) # negative SR}
\text{\}}
\]

\[
> \text{ Pi.star <- nlminb(runif(2), f.star,lower = c(0,0),upper = c(1,1))[[1]]}
\]
Mixed Strategy

Calibration

```r
> f.star <- function(PI) {
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```
Mixed Strategy - Performance Comparison

- y-axis is the difference in SR between the mixed strategy and each of the three portfolios
- From left to right, x-axis corresponds to the MV, GMV, and Naive portfolios
- Datasets are distinguished using colors
- Graph is created using geom_violin

Figure: All datasets and sample sizes
Mixed Strategy - Performance Comparison

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Figure: All sample sizes excluding stocks
Mixed Strategy - Performance Comparison

- **y-axis** is the difference in SR between the mixed strategy and each of the three portfolios.
- From left to right, **x-axis** corresponds to the MV, GMV, and Naive portfolios.
- Datasets are distinguished using colors.
- Graph is created using `geom_violin`.

Figure: \( n = 60 \) excluding stocks.
Mixed Strategy - Performance Comparison

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Figure: $n = 120$ excluding stocks
“Diversification is protection against ignorance. It makes little sense if you know what you are doing.”

-Warren Buffett
Concluding Remarks

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- Optimization (naive allocation) makes sense if you know (don’t know) what you are doing
  - Industries are less prone to estimation error than individual stocks
  - Potential optimization among industries is more evident
  - Naive strategy dominates across stocks
  - Evidence supports hierarchical asset allocation
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- *Further results are in progress...*
Thank You!

Stay in touch...

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GitHub: https://github.com/simaan84
### Appendix - Empirical Results Without Position Constraints

The table reports the out-of-sample SR for each portfolio. The symbols $X > X_0$ (or $X > X_N$) denote the proportion of time that $X_\pi > X_0$ (or $X_\pi > X_N$).

**Table:**

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<th>$X$</th>
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<td>0.70</td>
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</table>

- Table reports the out-of-sample SR for each portfolio
- $\pi_1$ ($\pi_2$) denote the proportion of time that $X \succ X_0$ ($X_0 \succ X_N$)
- $X_\pi = \pi_1X + (1 - \pi_1)(\pi_2X_0 + (1 - \pi_2)X_N)$


