Conditional Autoregressive Value-at-Risk: all flavors of CAViaR.

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June 1st, 2018

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Introduction.

This papers aimed to:

1. Investigate the statistical behavior of **13 parametric CAViaR models for 27 stock's indices** with respect to the Bias-Variance dilemma.
2. Our findings pointed out the **Adaptive model** should be chosen when no prior information is available, since it presented the **smallest MSE in 23 of 27 assets**.
3. We provided an empirical golden rule for choosing the CAViaR structure:

$$CAViaR_t(\beta) = CAViaR_{t-1}(\beta_1) + \beta_1\{[1 + \exp(G[Y_{t-1} - CAViaR_{t-1}(\beta)]$$

where $G \in \mathbb{R}$ is an arbitrary constant to be chosen. We used $\tau = 0.05, G = 1$ in our analysis.

Method and Analysis

1. Split the data into *training* (70% first observations) and *validation* (30% last observations).
2. Using the follow objective function:

$$\mathcal{M} = \min_{\boldsymbol{\theta}} \frac{1}{T} \sum_{t=1}^{T} \left[\tau - \mathbb{1}\left(Y_t - CAViaR_t(\boldsymbol{\theta})\right)\right]\left[CAViaR_t(\boldsymbol{\theta})\right] \quad (1)$$

   estimate the vector of parameters $\boldsymbol{\theta}$ that minimize Equation 1 using only the *training* dataset.
3. Create a resample dataset using the (Politis and Romano, 1994)'s Bootstrap of size $B = 5000$ for both *training* and *validation*.

1. Using (Politis and Romano, 1994)'s Bootstrap estimate the *hits* measure defined by

$$hits^{(b)} = \sum_{t=1}^{T} \frac{\mathbb{1}\left[Y_t^{(b)} < CAViaR_t(\boldsymbol{\theta})\right]}{T}$$

for both *training* and *validation* where b is the b-th bootstrap sample, T is the sample size of the dataset and $\mathbb{1}(\cdot)$ is the indicator function.

2. The last step is to estimate the Mean Square Error (MSE) of the *training* and *validation* calculating the sample variance and bias of $hits$ using $b = 1, \ldots, B$ and then

$$\widehat{MSE} = \widehat{\mathbb{V}}(hits) + \widehat{Bias}(hits)^2$$

The MSE measure is then used to evaluate the model with respect to its consistency *in-sample* and *out-sample*.

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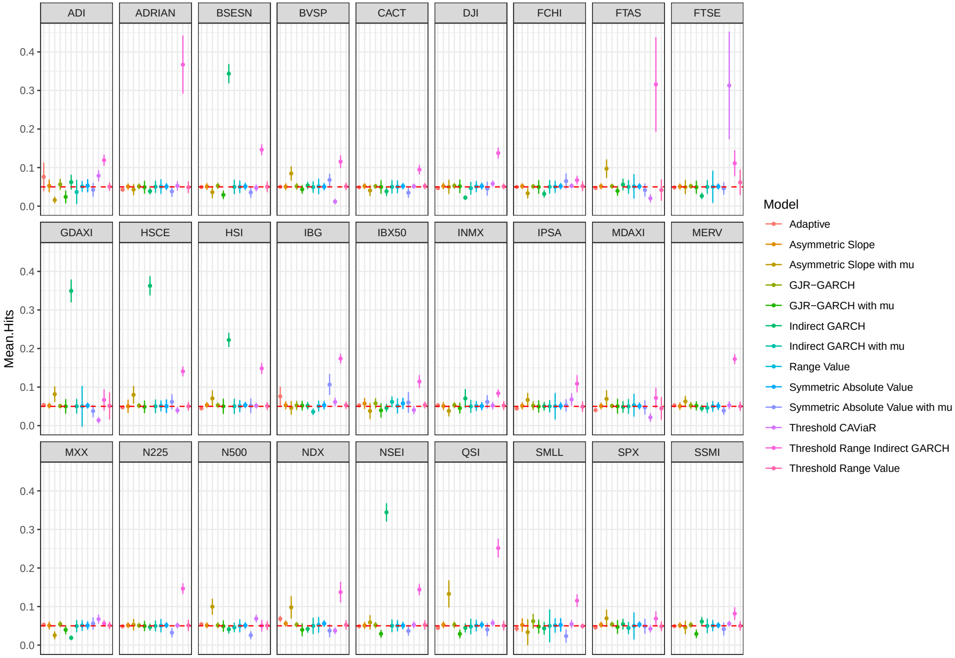
Results.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MERV
MXX
N225
N500
NDX
NSEI
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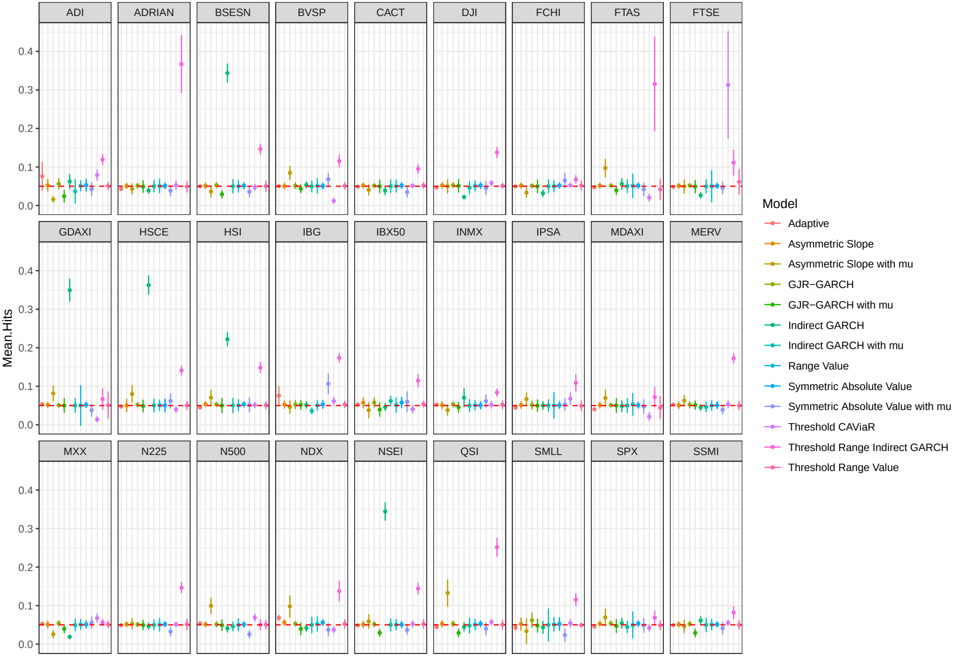
SPX
SSMI
Mean.Hits
0.0
0.1
0.2
0.3
0.4
Model
Adaptive
Asymmetric Slope
Asymmetric Slope with mu
GJR−GARCH
GJR−GARCH with mu
Indirect GARCH
Indirect GARCH with mu
Range Value
Symmetric Absolute Value
Symmetric Absolute Value with mu
Threshold CAViaR
Threshold Range Indirect GARCH
Threshold Range Value



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N500
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NSEI
QSI
SMLL
SPX
SSMI
Mean.Hits
0.0
0.1
0.2
0.3
0.4
Model
Adaptive
Asymmetric Slope
Asymmetric Slope with mu
GJR−GARCH
GJR−GARCH with mu
Indirect GARCH
Indirect GARCH with mu
Range Value
Symmetric Absolute Value
Symmetric Absolute Value with mu
Threshold CAViaR
Threshold Range Indirect GARCH
Threshold Range Value

Conclusion.

Based on our empirical evidence if the analysts do not have much information about the data, they can used the Adaptive model, since it presented the smallest MSE in 23 of 27 assets.

We also observed that the **quality of the model's fit strongly depends on the data**, and this can be caused due to the financial stylized facts of the returns (Cont, 2001) which can change a lot between financial data.

Since the presented models assume a steady parametric form, some models can be better than others depending on the data used. This suggest that the use of **nonparametric models can be better to estimate the CAViaR in most of the cases**.

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Figure: Machine Learning Lab in Finance and Organizations

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Politis, D. N. and Romano, J. P. (1994).
The stationary bootstrap.
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