Conditional Autoregressive Value-at-Risk: all flavors of CAViaR.

Pedro Albuquerque

University of Brasília

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Introduction.

Method and Analysis

Results.

Conclusion.

Pedro Albuquerque
pedra@unb.br
University of Brasília

Conditional Autoregressive Value-at-Risk: all flavors of CAViaR.
Introduction.
This paper aimed to:

1. Investigate the statistical behavior of 13 parametric CAViaR models for 27 stock’s indices with respect to the Bias-Variance dilemma.

2. Our findings pointed out the Adaptive model should be chosen when no prior information is available, since it presented the smallest MSE in 23 of 27 assets.

3. We provided an empirical golden rule for choosing the CAViaR structure:

\[
\text{CAViaR}_t(\beta) = \text{CAViaR}_{t-1}(\beta_1) + \beta_1 \left\{ 1 + \exp(G[Y_{t-1} - \text{CAViaR}_{t-1}(\beta)] \right\}
\]

where \(G \in \mathbb{R}\) is an arbitrary constant to be chosen. We used \(\tau = 0.05, G = 1\) in our analysis.
Method and Analysis
1 Split the data into *training* (70% first observations) and *validation* (30% last observations).

2 Using the following objective function:

\[ M = \min_{\theta} \frac{1}{T} \sum_{t=1}^{T} [\tau - 1(Y_t - CAViaR_t(\theta))] [CAViaR_t(\theta)] \]  

(1)

estimate the vector of parameters \( \theta \) that minimize Equation 1 using only the *training* dataset.

3 Create a resample dataset using the (Politis and Romano, 1994)'s Bootstrap of size \( B = 5000 \) for both *training* and *validation*. 
1. Using (Politis and Romano, 1994)'s Bootstrap estimate the hits measure defined by

\[
\text{hits}^{(b)} = \sum_{t=1}^{T} \frac{\mathbb{1} \left[ Y_t^{(b)} < \text{CAViaR}_t(\theta) \right]}{T}
\]

for both training and validation where \( b \) is the \( b \)-th bootstrap sample, \( T \) is the sample size of the dataset and \( \mathbb{1}(\cdot) \) is the indicator function.

2. The last step is to estimate the Mean Square Error (MSE) of the training and validation calculating the sample variance and bias of hits using \( b = 1, \ldots, B \) and then

\[
\hat{\text{MSE}} = \hat{\text{V}}(\text{hits}) + \hat{\text{Bias}}(\text{hits})^2
\]

The MSE measure is then used to evaluate the model with respect to its consistency in-sample and out-sample.
Results.
<table>
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<tr>
<th>Model</th>
<th>ADI</th>
<th>ADRIAN</th>
<th>BSESN</th>
<th>BVSP</th>
<th>CACT</th>
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- **Model**
  - Adaptive
  - Asymmetric Slope
  - Asymmetric Slope with mu
  - GJR−GARCH
  - GJR−GARCH with mu
  - Indirect GARCH
  - Indirect GARCH with mu
  - Range Value
  - Symmetric Absolute Value
  - Symmetric Absolute Value with mu
  - Threshold CAViaR
  - Threshold Range Indirect GARCH
  - Threshold Range Value

- **Variables**
  - GDXI
  - HSCE
  - HSI
  - IBG
  - IBX50
  - INMX
  - IPSA
  - MDAXI
  - MERV
  - MXX
  - N225
  - N500
  - NDX
  - NSEI
  - QSI
  - SMLL
  - SPX
  - SSMI
Conclusion.
Based on our empirical evidence if the analysts do not have much information about the data, they can use the Adaptive model, since it presented the smallest MSE in 23 of 27 assets.

We also observed that the quality of the model’s fit strongly depends on the data, and this can be caused due to the financial stylized facts of the returns (Cont, 2001) which can change a lot between financial data.

Since the presented models assume a steady parametric form, some models can be better than others depending on the data used. This suggests that the use of nonparametric models can be better to estimate the CAViaR in most of the cases.
Figure: Machine Learning Lab in Finance and Organizations

Empirical properties of asset returns: stylized facts and statistical issues.

*Quantitative Finance*, 1:223--236.

Politis, D. N. and Romano, J. P. (1994).

The stationary bootstrap.