Q-Gaussian Probability Default Model

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June 1st, 2018
Probability of default and Merton Model

➢ Probability of default
  ▪ An estimate of the likelihood that borrower will not be able to repay its debt

➢ Where the PD models are used?
  ▪ Assessment of the credit risk used by analysts and investors
  ▪ Valuation of bond prices
  ▪ Calculation of the economic capital to ensure firm’s solvency
  ▪ Calculations of the capital adequacy dictated by financial regulators

➢ Many PD models follow Merton’s intuition
  ▪ Distance between the expected value of the company’s assets $V$ and the default point $D$ measured in sigma (volatility)

  ▪ Industry standard Merton’s $DD(t)$:
    $$DD(t) = \frac{\log\left(\frac{V_A}{D}\right)}{\delta_A \sqrt{t}}$$

  ▪ Probability of default:
    $$PD(t) = P \left[ V_A \leq D \right] = \cdots = \Phi\left(-DD\right)$$
    $\Phi$ – cumulative normal distribution below the default line

  ▪ Deficiencies of Merton model: yields negligible PD values for investment grade companies above BBB –
Merton model of default - visualization

\[ \log(V/D) \]  

**Inversed log-leverage ratio** = \( \log(V/D) \);  

\[ V = D + E \]

\[ \text{Gaussian distribution of } \log(V/D) \text{ at the horizon } T \]

**Constant Volatility of V**

\[ \text{Distance-to-Default (DD) = 3 Standard deviations} \]

\[ \text{Constant Default Point} = \log(D) \]

\[ PD \text{ depends only on DD} \]

**PD** = likelihood of hitting the default barrier at \( x = \log(V/D) = 0 \) at the time horizon \( t \), conditional on the initial position \( x_0 \) at \( t = 0 \).
Q-Gaussian

- Probability distribution arising from the maximization of the Tsallis entropy

- Q-Gaussian distribution is applied in finance and economy due to its heavy tails (for 1 < q < 3)
Empirical study based on 650 North American entities from industrial sector

- Example: time series of financial assets UPS & UTX (‘06–’12)
  - Daily time series of the issuer’s market capitalization $E$, the accounting book value of the total debt $D$ and the market value of asset $V$, estimated by the direct proxy method: $V = E + D$.

![Graphs showing time series of total assets, equity, and debt for UPS and UTX.](image)
Empirical Study: Time series of financial assets UPS & UTX (‘06-’12)

Time series of a daily value of log-asset-returns estimated by the direct proxy method:

\[ V = E + D. \]

\[ \nu_{i,j} = \ln\left(\frac{V_{i,j}}{V_{i-1,j}}\right) \]

UPS

UTX

Abs [\( \nu_{i,j} \)]
Empirical Study: Q–Q Plots for q-Gaussian and Gaussian distributions

q-Gaussian with $1 < q < 3$

Distributions of daily log-asset-returns estimated by the direct proxy method from July 11, 2006 to June 21, 2012.
Q-Gaussian model of Default. Model Validation 1-Year PD

ROC curve:
- 361 non-defaulted firms and 29 defaulters
- between 2007 and 2012
- AUC = 0.97

1-year PD forecast for investment grade issuers (2008-2009)

UTX (A)

Q-Gaussian

Merton

UPS (AAA, AA-)

S&P Global
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Conclusions and future work - R package

Conclusion

- Generally, q-Gaussian model of default can provide a much stronger prediction signal for the corporate default
  - the q-PD trend arise sharply before the downgrade happen, and is much higher than m-PD.

- Merton Probability Default model is based on assumption of the Gaussian distribution of log leverage returns, while empirical analysis point to fat tail distributions like q-Gaussian

- q-Gaussian PD may be used as a supplemental model to classical Merton model and help companies with averting credit risk measure degradation and protect investors

Work underway

- Creating an R package
- Adding set of financial data from S&P Global products to test and demonstrate functionality of the q-Gaussian PD model
Project Overview

- S&P Global Market Intelligence combines broad data, powerful analytics, and deep sector intelligence to give our clients unrivaled insight into the companies and markets they follow.

- Working with Educational Institutions
  - Capstone Project with Columbia Business School: Q-Gaussian Probability Default Model
    - Market Intelligence - Yuri Katz, Mohammed Hadi, and Thomas Zakrzewski
    - Columbia Business School - Students: Sheng Zhang, Yiqing Su, Khyati Jain, Zhirui Zhang, Yash Rane, Dylan Cohen, Zian Cheng, and Hugo Ducruc, under consultation of Professor Souleymane Kachani

- References
  - http://www.acrn-journals.eu/resources/jofrp201404g.pdf
Thank you

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