Q-Gaussian Probability Default Model

Thomas Zakrzewski (Tom Z.,)
Distinguished Engineer
Head of Architecture and Digital Design
S&P Global Market Intelligence Risk Services

Yuri Katz, PhD
Director Quantitative Analysis
S&P Global Market Intelligence

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Probability of default and Merton Model

➢ Probability of default

- An estimate of the likelihood that borrower will not be able to repay its debt

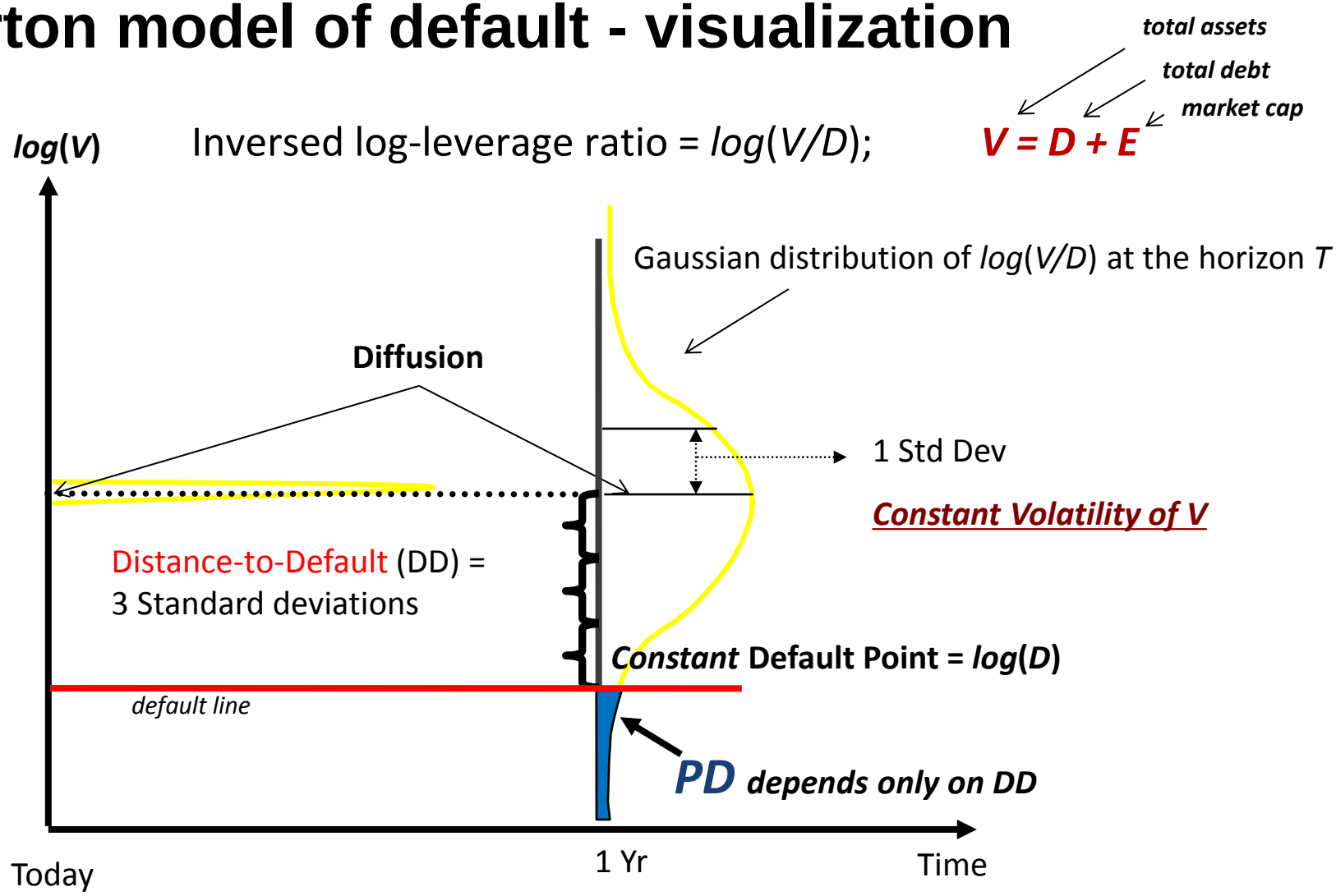
➢ Where the PD models are used?

- Assessment of the credit risk used by analysts and investors
- Valuation of bond prices
- Calculation of the economic capital to ensure firm's solvency
- Calculations of the capital adequacy dictated by financial regulators

➢ Many PD models follow Merton's intuition

- Distance between the expected value of the company's assets *V* and the default point *D measured in sigma (volatility)*
- Industry standard Merton's DD(t): $$\mathrm{DD(t)} = \frac{\log(\frac{V_A}{D})}{\delta_A \sqrt{t}}$$
- Probability of default: $$PD(t) = P\,[V_A \leq D] = \cdots = \Phi(-DD)$$ Φ – *cumulative normal distribution below the default line*
- Deficiencies of Merton model: yields negligible PD values for investment grade companies above BBB –

Merton model of default - visualization



PD = likelihood of hitting the default barrier at $x = Log(V/D) = 0$ at the time horizon t, conditional on the initial position x_0 at $t = 0$.

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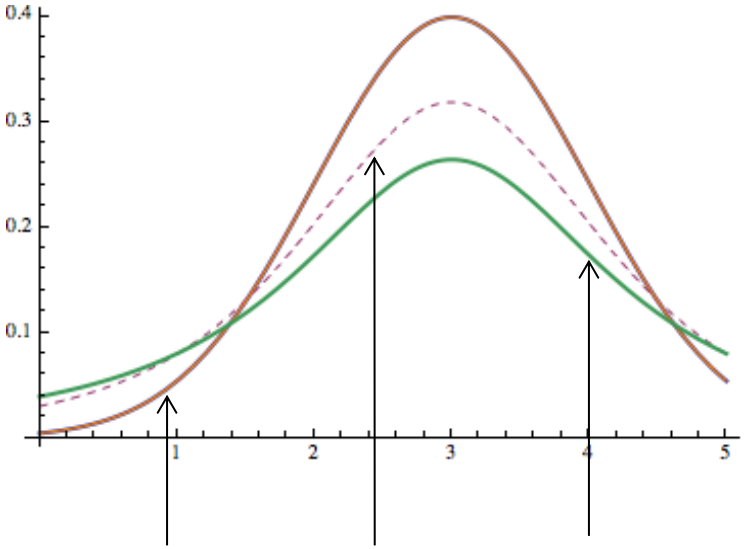
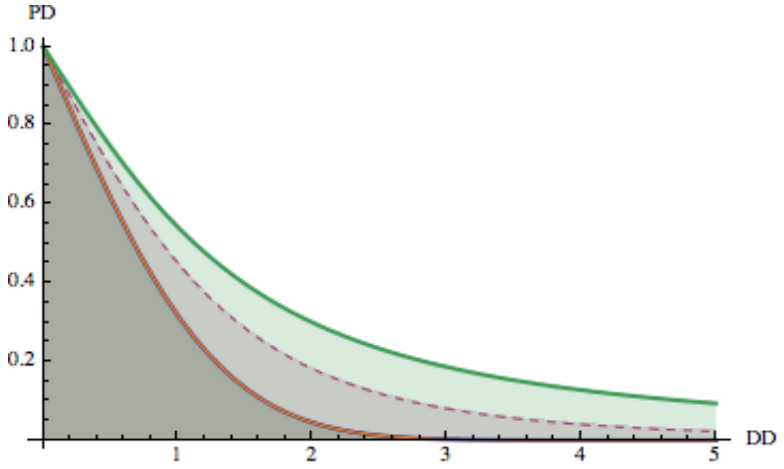
Q-Gaussian

- **Probability distribution arising from the maximization of the Tsallis entropy**

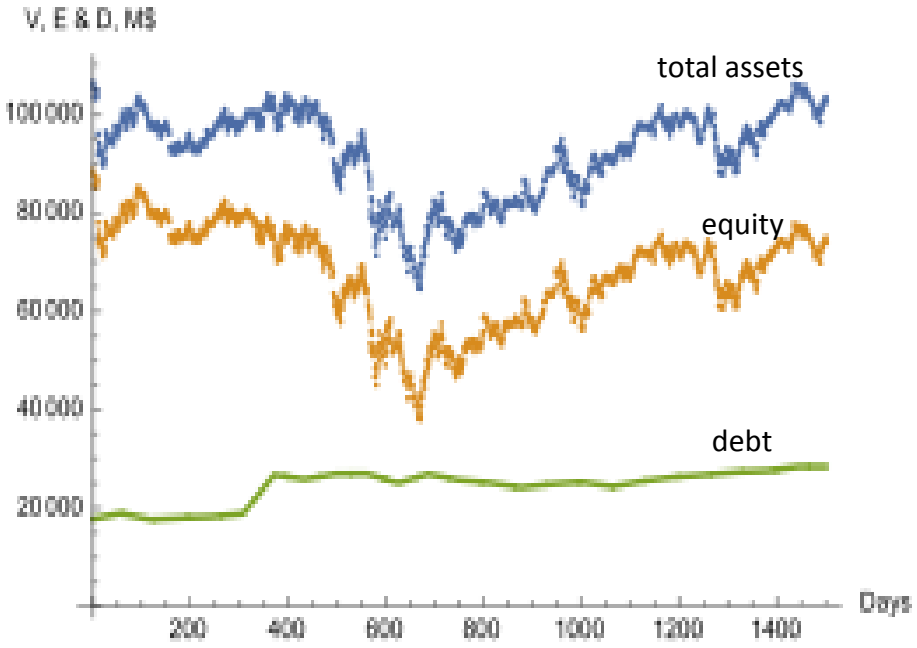
- **Q-Gaussian distribution is applied in finance and economy due to its heavy tails (for 1 < q < 3)**



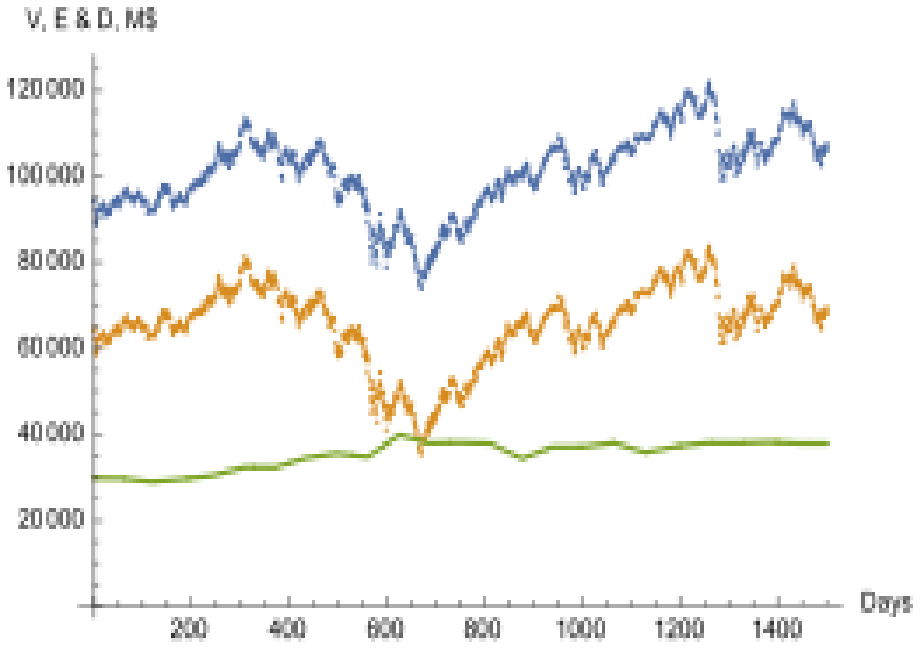


Empirical study based on 650 North American entities from industrial sector

- **Example: time series of financial assets UPS & UTX (‘06-’12)**
  Daily time series of the issuer’s market capitalization *E*, the accounting book value of the total debt *D* and the market value of asset *V*, estimated by the direct proxy method: $V = E + D$.



UPS

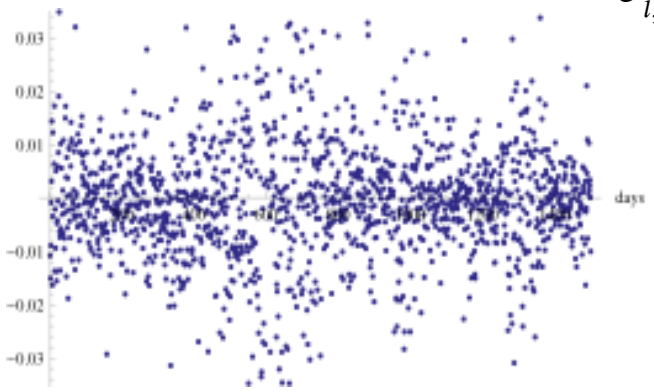


UTX

Empirical Study: Time series of financial assets UPS & UTX ('06-'12)

Time series of a daily value of log-asset-returns estimated by the direct proxy method:
$V = E + D$.

$$\upsilon_{i,j} = \ln(V_{i,j} / V_{i-1,j})$$



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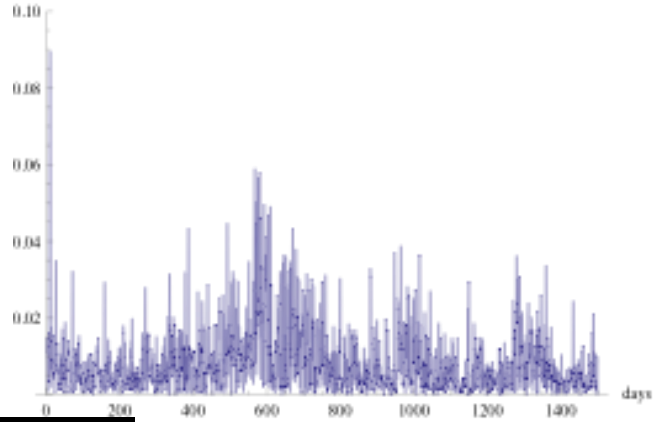
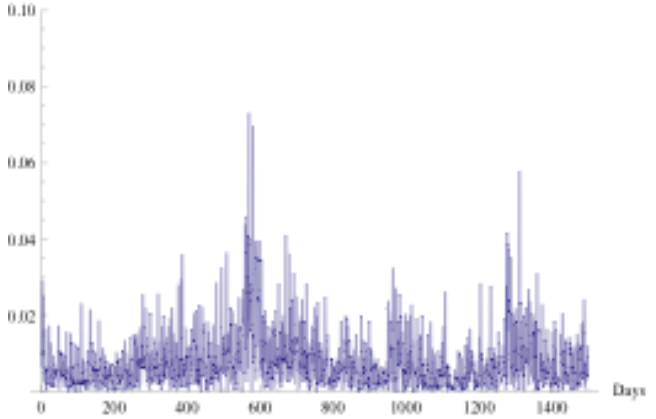
UTX



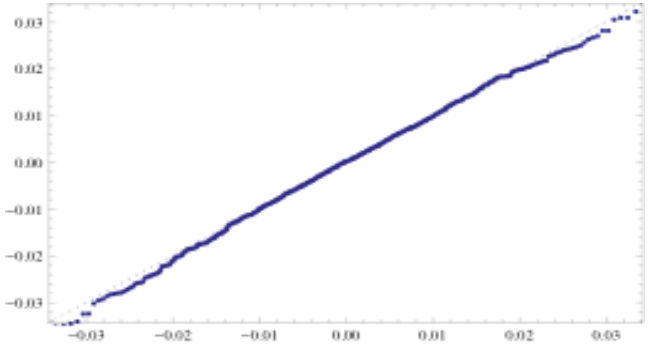
$$Abs\,[\upsilon_{i,j}]$$



Empirical Study: Q–Q Plots for q-Gaussian and Gaussian distributions

q-Gaussian with $1 < q < 3$

UTX



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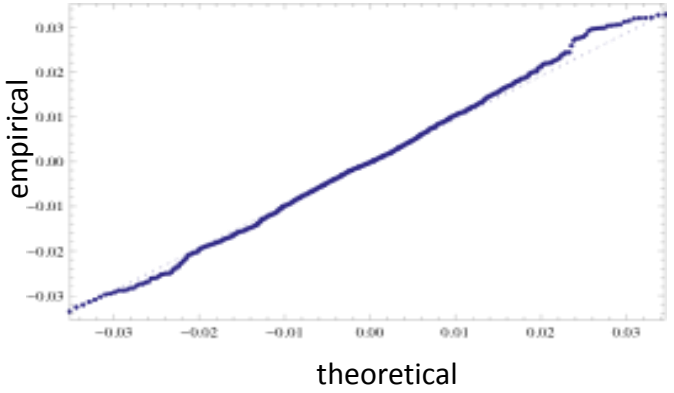
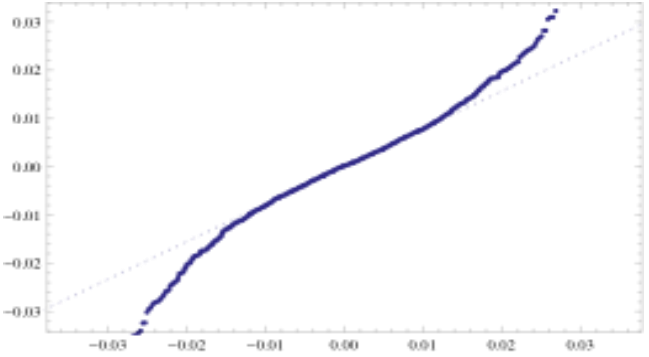
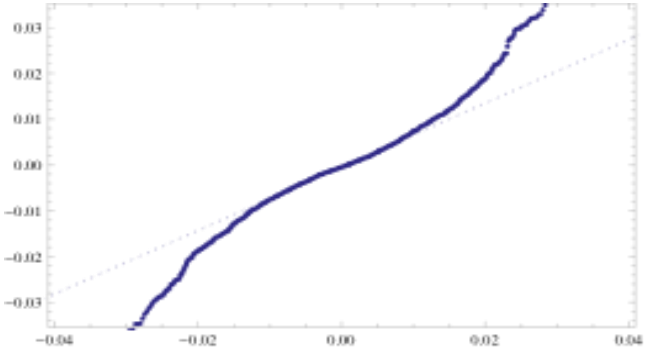


Distributions of daily log-asset-returns estimated by the direct proxy method from July 11, 2006 to June 21, 2012.



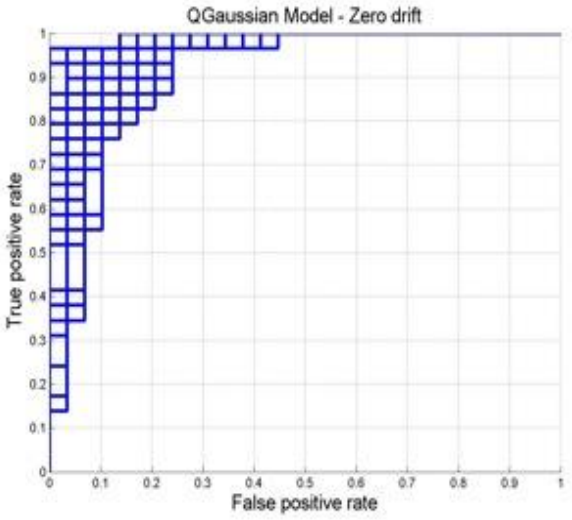
Gaussian



Q-Gaussian model of Default. Model Validation 1-Year PD

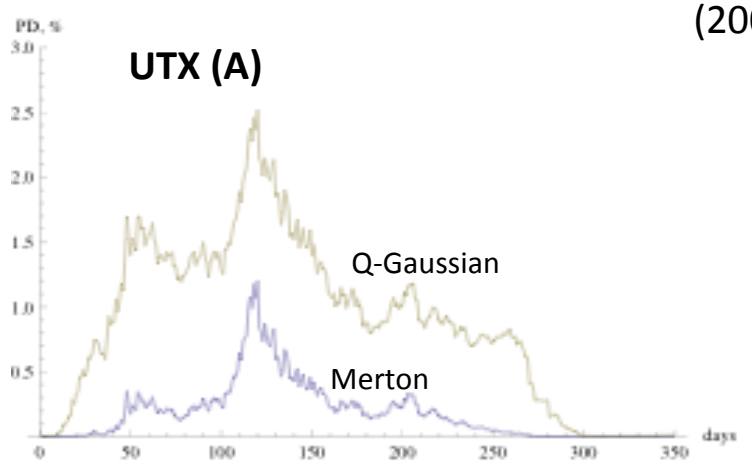
ROC curve:

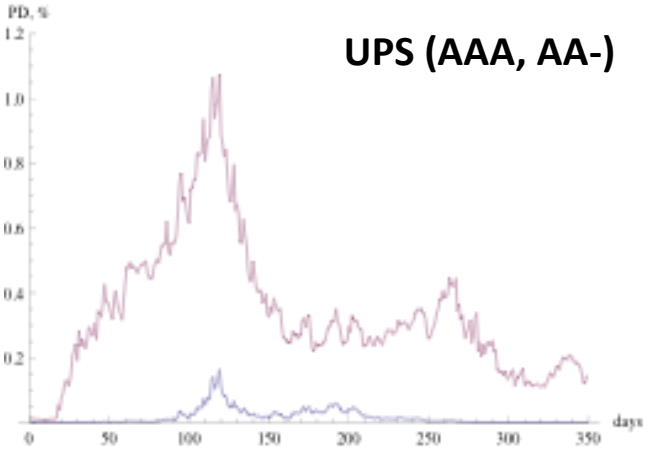
- 361 non-defaulted firms and 29 defaulters
- between 2007 and 2012
- AUC = 0.97



1-year PD forecast for investment grade issuers

(2008 -2009)





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Conclusions and future work - R package

➢ Conclusion

- Generally, q-Gaussian model of default can provide a much stronger prediction signal for the corporate default
  - the q-PD trend arise sharply before the downgrade happen, and is much higher than m-PD.

- Merton Probability Default model is based on assumption of the Gaussian distribution of log leverage returns, while empirical analysis point to fat tail distributions like q-Gaussian

- q-Gaussian PD may be used as a supplemental model to classical Merton model and help companies with averting credit risk measure degradation and protect investors

➢ Work underway

- Creating an R package
- Adding set of financial data from S&P Global products to test and demonstrate functionality of the q-Gaussian PD model

Project Overview

- **S&P Global Market Intelligence combines broad data, powerful analytics, and deep sector intelligence to give our clients unrivaled insight into the companies and markets they follow.**

- **Working with Educational Institutions**
  - Capstone Project with Columbia Business School: Q-Gaussian Probability Default Model
    - Market Intelligence - Yuri Katz, Mohammed Hadi, and Thomas Zakrzewski
    - Columbia Business School - Students: Sheng Zhang, Yiqing Su, Khyati Jain, Zhirui Zhang, Yash Rane, Dylan Cohen, Zian Cheng, and Hugo Ducruc, under consultation of Professor Souleymane Kachani

- **References**

  - http://www.acrn-journals.eu/resources/jofrp201404g.pdf
  - https://www.sciencedirect.com/science/article/pii/S0378437114002313
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Thank you

Thomas Zakrzewski (Tom Z.,)
Distinguished Engineer
Head of Architecture and Digital Design
S&P Global Market Intelligence
Risk Services
T: +1 212.438.8458
Thomas.Zakrzewski@spglobal.com

Yuri Katz, PhD
Director of Quantitative Analysis
S&P Global Market Intelligence

T: +1 212-438-2325
Yuri.Katz@spglobal.com