Correlated Idiosyncratic Volatility Shocks

Xiao Qiao
SummerHaven Investment Management, LLC [1]

Alex Wang
University of Chicago Booth School of Business

[1] The views expressed are those of the authors and do not necessarily reflect official positions of SummerHaven Investment Management, LLC.

Motivation

Volatility research is important for both academics and practitioners

- Academics: Implications for volatility modeling and asset pricing
- Practitioners: Risk management and pricing derivatives

Recent focus on the behavior of idiosyncratic volatility

- Ang et al. (2006) show stocks with high idiosyncratic vol earns lower average returns
- Herskovic et al. (2014) demonstrate idiosyncratic volatilities contain a common component, priced in the cross section of stock returns

Research Question

What accounts for the common factor in idiosyncratic volatility?

- Comovement in time-varying idiosyncratic vol (TVV)
- Correlated volatility innovations (VIN)

Are both components important in describing the data?

Which component is priced in the cross-section of equity returns?

This Paper

1. Document correlated volatility shocks as a source of comovement
2. A statistical model for commonality in idiosyncratic volatility
   - Dynamic Factor Correlation (DFC)
   - Fits characteristic-sorted portfolios better than the DCC model (Engle 2002)
   - DFC nests the models of Bollerslev (1990) and Engle and Kelly (2012)
3. Asset pricing implications of components of idiosyncratic volatility
   - TVV and VIN are both priced in the cross section
   - Univariate sorts for loadings on VIN and TVV show average return spreads between 3.31% and 1.62% per year
   - TVV and VIN carry distinct premiums from each other, not subsumed by market equity (ME)

Data

- Monthly data from Ken French's website
  - Fama and French (1992, 2015) factors
  - Portfolios formed on market equity (ME), book-to-market (BE/ME), long-term reversal (LT Rev), operating profitability (OP), investment (Inv), momentum (Mom), and short-term reversal (ST Rev)
  - Bivariate portfolios formed on the above characteristics
- Monthly stock returns, prices, and shares outstanding are from CRSP

Idiosyncratic Volatility Shocks are Correlated

- Factor models for mean returns, univariate GARCH models to idiosyncratic returns
- H_0: Cross-sectionally uncorrelated GARCH innovations
- GARCH residuals are not autocorrelated, but cross-sectionally correlated

Correlations of GARCH residuals:

	Univariate Portfolios						
	ME	BE/ME	LT Rev	OP	Inv	Mom	ST Rev
Max	0.52	0.22	0.54	0.28	0.26	0.57	0.53
Min	0.05	0.02	0.01	-0.02	-0.01	0.08	0.03
Avg	0.19	0.12	0.24	0.09	0.09	0.28	0.18
	Bivariate Portfolios						
	ME, BE/ME	ME, OP	ME, Inv	ME, Mom	ME, ST Rev	ME, LT Rev	
Max	0.75	0.49	0.39	0.71	0.63	0.50	
Min	-0.04	-0.03	-0.05	0.02	0.01	0.01	
Avg	0.12	0.13	0.10	0.22	0.19	0.14	

Dynamic Factor Correlation (DFC)

Start with standard factor model with GARCH volatility

$$r_{i,t} = \mathbf{f}_t'\beta_i + a_{i,t}, \quad h_{i,t} = \mathbb{E}[a_{i,t}^2|\mathcal{F}_{t-1}]$$
$$e_{i,t} = a_{i,t}/\sqrt{h_{i,t}}$$

Impose factor structure on standardized residuals

$$e_{i,t} = \frac{q_{i,t}}{s_{i,t}}$$
$$\text{Where } \; q_{i,t} = v_t\xi_i + \sigma_i\epsilon_{i,t}, \quad s_{i,t}^2 = \mathbb{E}_{t-1}[q_{i,t}^2]$$
$$v_{t|t-1} \sim \mathcal{N}(0, h_{v,t})$$

Empirically, use $h_{v,t} = \frac{1}{N}\sum_{i=1}^{N} e_{i,t-1}^2 = \overline{\mathbf{e}_{t-1}^2}$

The Need for a New Model

Monte Carlo simulations of the DFC model

N = 3	T = 1000		T = 5000		N = 10	T = 1000		T = 5000	
	DFC	DCC	DFC	DCC		DFC	DCC	DFC	DCC
	Panel A: $\xi = (0.1, 0.1, 0.1)'$					Panel D: $\xi = 0.1 \cdot \mathbf{1}_{10}$			
RMSE	0.030	0.032	0.011	0.014	RMSE	0.019	0.029	0.009	0.016
MAE	0.023	0.026	0.009	0.010	MAE	0.014	0.023	0.007	0.013
	Panel B: $\xi = (0.2, 0.3, 0.5)'$					Panel E: $\xi = (0.2, 0.3, 0.4, 0.5, 0.6)' \otimes \mathbf{1}_2'$			
RMSE	0.029	0.035	0.017	0.022	RMSE	0.022	0.028	0.016	0.020
MAE	0.023	0.027	0.013	0.017	MAE	0.017	0.022	0.012	0.015
	Panel C: $\xi = (0.5, 0.5, 0.5)'$					Panel F: $\xi = 0.5 \cdot \mathbf{1}_{10}$			
RMSE	0.039	0.053	0.029	0.045	RMSE	0.037	0.042	0.032	0.037
MAE	0.032	0.042	0.025	0.037	MAE	0.031	0.037	0.028	0.031

- Engle's (2002) DCC cannot capture the common component in idiosyncratic volatility shocks

Empirical Performance: DFC vs. DCC

	ME	BE/ME	LTRev	OP	Inv	Mom	STRev
$\mathcal{L}$(DFC)	31813.6	27972.5	26164.0	17596.8	17535.7	26146.3	26032.7
$\mathcal{L}$(DCC)	31535.3	27602.1	25918.7	17430.4	17474.6	25746.7	25771.5

DFC model is related to other multivariate GARCH models:

- $\forall t, h_{v,t} = 1$: Special case of Bollerslev's (1990) CCC
- $\xi_i = \xi_j \equiv \bar{\xi}$: DECO of Engle and Kelly (2012)

Decomposing the Common Factor in Idio Vol

$$\underbrace{\sigma^2_{i,t} - \mathbb{E}_{t-1}[\sigma^2_{i,t}]}_{\text{Unexpected Idiosyncratic Volatility}} = \frac{\sigma^2_{i,t} - \mathbb{E}_{t-1}[\sigma^2_{i,t}]}{\mathbb{E}_{t-1}[\sigma^2_{i,t}]} \cdot \mathbb{E}_{t-1}[\sigma^2_{i,t}]$$

$$= \underbrace{\tilde{\nu}_{i,t}}_{\text{Volatility Innovation}} \cdot \underbrace{\mathbb{E}_{t-1}[\sigma^2_{i,t}]}_{\text{Time-Varying Volatility}}$$

Herskovic et al. (2014): CIV_t is priced in the cross section

$$CIV_t = \frac{1}{N}\sum_{i=1}^{N}\left(\sigma^2_{i,t} - \mathbb{E}_{t-1}[\sigma^2_{i,t}]\right) \approx -\frac{\bar{\tilde{\nu}} \cdot \bar{\sigma}^2}{N} + \bar{\sigma}^2 \underbrace{\frac{1}{N}\sum_{i=1}^{N} w_{i,t}\tilde{\nu}_{i,t}}_{VIN_t} + \bar{\tilde{\nu}} \underbrace{\frac{1}{N}\sum_{i=1}^{N} w_{i,t}\mathbb{E}_{t-1}[\sigma^2_{i,t}]}_{TVV_t}$$

Univariate Quintiles on VIN and TVV Betas

VIN-beta	1 (Low)	2	3	4	5 (High)	5-1	t(5-1)
			Panel A: One-way sorts on VIN-beta				
$\mathbb{E}[R] - r_f$	9.72%	8.70%	8.75%	7.63%	6.41%	-3.31%	-1.72
α_{CAPM}	2.64%	1.74%	1.42%	-0.34%	-3.12%	-5.75%	-3.11
α_{FF}	1.63%	0.51%	0.47%	-0.45%	-2.35%	-3.98%	-2.22
TVV-beta	1 (Low)	2	3	4	5 (High)	5-1	t(5-1)
			Panel B: One-way sorts on TVV-beta				
$\mathbb{E}[R] - r_f$	9.36%	8.79%	8.11%	8.72%	7.74%	-1.62%	-0.91
α_{CAPM}	1.14%	1.97%	1.29%	1.14%	-1.30%	-2.44%	-1.36
α_{FF}	1.14%	1.33%	0.34%	0.41%	-1.52%	-2.67%	-1.49

5×5 Portfolios on VIN and TVV Betas

- Long position in portfolio with largest VIN and TVV betas
- Short position in portfolio with smallest betas

Model	Intercept	RMRF	SMB	HML	WML	R^2
Panel A: Raw Return	-7.99%					
	(-2.72)					
Panel B: CAPM	-10.13%	0.29				3.11%
	(-3.47)	(5.15)				
Panel C: FF 3-factor	-8.51%	0.14	0.57	-0.34		10.40%
	(-2.98)	(2.42)	(6.62)	(-3.83)		
Panel D: Carhart 4-factor	-13.18%	0.19	0.59	-0.23	0.43	15.66%
	(-4.63)	(3.37)	(7.08)	(-2.58)	7.16	

- Similar effect as the CIV factor of Herskovic et al. (2014)

5 × 5 Portfolios on VIN Beta and ME

VIN-beta	1 (Low)	2	3	4	5 (High)	5-1	t(5-1)
Average excess returns of bivariate sorts on ME and VIN-beta							
1 (Small)	15.95%	13.60%	13.29%	11.65%	10.62%	-5.33%	-2.66
2	16.45%	12.71%	11.91%	11.32%	9.31%	-3.10%	-1.71
3	11.57%	10.97%	11.05%	10.62%	8.34%	-3.23%	-1.69
4	10.51%	10.52%	9.67%	10.33%	8.95%	-1.56%	-0.88
5 (Big)	8.52%	8.54%	8.54%	6.70%	7.26%	-1.26%	-0.71
5-1	-7.43%	-5.06%	-4.75%	-4.95%	-3.36%	-	-
t(5-1)	-3.54	-2.71	-2.35	-2.28	-1.35	-	-

5 × 5 Portfolios on TVV Beta and ME

TVV-beta	1 (Low)	2	3	4	5 (High)	5-1	t(5-1)
Average excess returns of bivariate sorts on ME and TVV-beta							
1 (Small)	15.37%	13.61%	12.07%	13.65%	10.15%	-5.22%	-2.82
2	12.58%	11.38%	9.83%	11.79%	9.11%	-3.47%	-2.21
3	10.54%	10.53%	10.98%	10.36%	10.31%	-0.23%	-0.14
4	10.00%	10.81%	10.35%	9.58%	9.18%	-0.81%	-0.58
5 (Big)	8.48%	7.75%	8.21%	7.83%	7.94%	-0.54%	-0.35
5-1	-6.88%	-5.86%	-3.85%	-5.81%	-2.21%	-	-
t(5-1)	-2.87	-3.03	-1.96	-2.93	-0.94	-	-

Conclusion

- Correlated volatility shocks contribute to comovement in idiosyncratic volatility
- Dynamic Factor Correlation (DFC) directly models correlated vol shocks and fits the data better than DCC
- DFC reduces to well-known multivariate volatility models under certain restrictions
- Common factor in idio vol can be decomposed into VIN and TVV
- VIN and TVV both capture some expected return variation in the cross section